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The AEF is the Cradle of The Modern "Right-Rate School" of Option Pricing Theory

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Abstract

Responding to the need for a more fruitful orientation for option pricing research, this paper outlines developments in the modern "right-rate school" of option pricing which embraces a critical role for risk preferences as an essential part of its theoretical foundation. It is controversial since it is counter to the theoretically orthodox risk-neutral valuation framework. The contentious theory was nurtured by the supportive environment that prevails in the Academy of Economic and Finance (AEF) meetings. This paper outlines the history of the development of the school through a review of the papers presented at the AEF meetings.

JEL Codes: G10, G13
Keywords: Options, Right-Rate School

Introduction

A new guiding premise is needed in option pricing research. This paper will review the development of a prospective source of guidance, the modern "right-rate school" of option pricing theory. The theory embraces a critical role for risk preferences (reflected in the return to the stock and the option) as its theoretical basis. Early option pricing researchers such as Sprenkle (1961) and Samuelson (1965), also incorporated a role for both the return on the stock and the return on the market in option pricing theory. However, with the advent of the Black Scholes model (1973), those roles were obliterated. Risk-neutral valuation was heralded as "the single most important tool" for the analysis of derivatives (Hull (1989). Risk-neutral valuation is a direct consequence of the fact that no variables such as the stock return that are affected by risk preferences appear in the Black Scholes Model. It is considered to be a corollary of the validity of the Black Scholes model. Few challenges to the new Black Scholes model were raised initially because in most cases the heralded model worked well (Rubinstein, 1985). Two professors associated with its origin (Scholes and Merton) were awarded the Nobel prize in 1997. The profession largely turned its back on many of the theoretical developments of the 1950s and 1960s in favor of a foundation based upon the Black Scholes model and its analog, risk neutral valuation. Extensive early empirical tests (Rubinstein, 1985) supported this orientation.

By the 1980s there was increased evidence that the model was no longer working satisfactorily. A distinct pattern of volatility estimates implied by the model which were basic to the validity of the Black Scholes model ceased to appear in empirical tests (Bates, 2003). The model required constant volatility across an option chain (options on the same stock on the same day) with different exercise prices. The patterns, particularly in the equities markets, were systematic and unexplained by the theory (Rubinstein, 1994; Bates, 2003). The equity pattern has been described as a smirk (Hull, 2009). In the light of this, option models which embraced multiple alternative stock distributions such as the Cox (1976) and Cox and Ross (1976) constant elasticity of variance model or the Heston (1993) stochastic volatility model were increasingly embraced and tested. But none of these appeared to solve this new riddle (Bates, 2003).

Moore (1997; 1999; 2001; 2003; 2005) suggested that the problem might be related to the absence of the risk preference variables in the option modeling process. This reintroduction of risk preference into option pricing theory is called the modern "right-rate school" of option pricing. If this orientation is true, it undermines a significant part of the status quo option theory which embraced risk neutral valuation. It also undermines the dominant theoretical paradigm in option pricing theory, the Black Scholes option pricing model and its accompanying logic as well as any spinoff models. A contrary view can be seen in modern textbooks (Hull, 2009; 2016) which suggest explanations for the volatility smirk like asymmetric stock distributions, leverage and crashaphobia which may be considered consistent with the risk neutral framework. Thus, the theories will need to battle for theoretical supremacy.

Thus, the modern "right-rate school" of option pricing theory is one of the most significant, but disruptive developments in option pricing theory in the last few decades. It threatens to disrupt the traditional focus of options research which operates on a foundation of risk neutral pricing. It proposes a completely new explanation for the patterns of the implied volatility smile (Hull, 2009; Bates, 2003) or smirk (Hull, 2009) that were documented in the Black Scholes model (Macbeth and Merville, 1979; 1980) by suggesting that differences in rates are the driving force (Moore, 2003). The validity of risk neutral valuation or pricing (Hull, 1989) is challenged. The general failure of almost all proposed alternative option pricing models is explained because they are all derived from the risk neutral assumptions which are the foundation of the Black Scholes model. Finally, it significantly diminishes the role of the 1973 Black Scholes option pricing model in financial economics and option pricing theory.

That such a disruptive force could be nurtured and have nearly all its origins in the strongly supportive and friendly climate of the Academy of Economics and Finance (AEF) is probably not a surprise. After risk neutral pricing rose to theoretical prominence, theories like the modern "right-rate school" were not well received. Consequently, serious dissent from the status quo risk neutrality framework needed a less judgmental outlets like the Academy of Economics and Finance. Merton (2006) says that Samuelson had lamented his regret that Samuelson (1965) had been "a near miss" in the face of the Black Scholes triumph. Sprenkle's (1961) paper was in effect dumped into the ashbins of history by the rise of risk neutral pricing. Moore's papers presented at the AEF represented the reemergence for the role of the stock return and the option return in option pricing theory. Earlier models like Samuelson (1965) were consistent with discounted cashflow pricing being applied to options. The papers show that Moore had an unabating respect for Samuelson. Samuelson died not realizing that his work like the Sprenkle (1961) work was much less of a "near miss" than he thought. The assumptions of the "right-rate school" are supported by the work reported at the AEF. Economic history should elevate the value of Samuelson's contributions from the early 1950s and 1960s option research which included his students as well as his own work. As the chief protagonist of the "right-rate school" for over a twenty-year period, Moore's works presented at the AEF suggests papers like Samuelson's and Sprinkle's work deserves more assiduous study. An idea grows, like money grows--where it is well treated. In this article, an outline of how interactions with the organization led to the continued development of the "right-rate" theory is provided.

The Modern "Right-Rate School" of Option Pricing Theory

The "right-rate school" of option pricing theory has as its foundation the generic Feynman-Kac formulation, (Alghassi et al. 2022; Cairoli and Baule, 2017; Janson and Tysk, 2006) which in the context of options is

$$\frac{\partial O_F}{\partial t} + a\,(s,t)\frac{\partial^2 O_F}{\partial S^2} + b(s.t)\frac{\partial O_F}{\partial S} + f(t) = c\,(s,t)O_F \; for \; t < T, \tag{1}$$

$$O_F(s,T) = \; O_F(s) \tag{2}$$

where a is the Brownian motion term, b is the drift rate expression and c is the discount rate term. O_F is the option value while s is the stock price. The second expression states that the option is a function of the value at expiration and the stock price. The f(t) is a functional which is often set to zero depending upon how a functional is thought to be related to the particular situation. The nomenclature for time is t while the expression for the maturity date is T. The terminal boundary condition in call options is probably better described

$$O_F(s,T) = \; O_F(s-x,0) \tag{3}$$

where s-x is the stock price minus the exercise price x.

The implied ansatz derived from the Black Scholes hedging argument suggests that given a lognormal distribution and stock return and option return equal to the risk-free rate that the coefficients for the generic Feynman Kac formulation (Alghassi et al. 2022) with f(t) = 0 should be

$$a = \frac{\sigma^2}{2}S^2 \qquad b = rS \qquad \text{and } c = r \tag{4}$$

which leads to the expression

$$\frac{\partial O_F}{\partial t} + \frac{\sigma^2}{2}S^2\,\frac{\partial^2 O_F}{\partial S^2} + rS\,\frac{\partial O_F}{\partial S} = \; rO_F \tag{5}$$

When rearranged this expression is the Black Sholes partial differential equation (PDE) that is often seen in textbooks as the Black Scholes partial differential equation (Hull, 2009).

The "right-rate school" uses a different ansatz which is inspired by the expression following from the application of Ito's Lemma (1951) to the underlying stock return process. Assume like many others (Sprinkle, 1961; Samuelson, 1965; Black and Scholes, 1973) that there is a single risky asset which follows a geometric Brownian motion:

$$\frac{dS}{S} = u\,dt + \sigma\,\mathrm{dz} \quad \text{or} \quad dS = u\,S\,dt + \sigma\,S\,\mathrm{dz} \tag{6}$$

with constant drift terms arithmetic mean return u. It is useful to be aware that dz, the Weiner process term, has the following properties: 1) The mean of dz = 0, 2) The standard deviation of dz = $\sqrt{dt}$, and 3) The variance of dz = dt. Applying Ito's lemma (Ito, 1951; Hull, 1989, p. 82) leads to the description of the option price process as:

$$dO = \left(\frac{\partial O}{\partial S}uS + \frac{\partial O}{\partial t} + \frac{1}{2}\frac{\partial^2 O}{\partial S^2}\ \sigma^2 S^2\right)dt + \frac{\partial O}{\partial S}\ \ \sigma S dz \tag{7}$$

The first part of the expression is the first moment. Therefore, it can be seen as the expectation term while the second term is the variance. The mathematics for Ito's lemma (1951), although an approximation of a Taylor series is unchallenged. The "right-rate school" argues there is no need to proceed to a different partial differential equation as the basis for option pricing theory. In a greatly expanded paper on the issue (Moore, 2023) explains in greater detail why the Black Scholes arguments fails to hold up. The "right-rate school" thus focuses upon the logical ansatz which flows from the application of Ito's lemma to a lognormal stock price. It rejects the proposition that the effects of the stock return and the effects of the option effectively cancel each other out as proposed by the current dominant status quo research paradigm.

Since the mean of dz = 0, it would be expected that it would have no effect on the expectation solution that would follow in the application of the Feynman-Kac theorem. Focusing in on the first moment term and dropping the variance term gives us:

$$dO = \left(\frac{\partial O}{\partial S}uS + \frac{\partial O}{\partial t} + \frac{1}{2}\frac{\partial^2 O}{\partial S^2}\ \sigma^2 S^2\right)dt \tag{8}$$

The drift term can be expressed as either the arithmetic mean u or the geometric mean α. The difference is the geometric mean α requires the convexity correction. This involves the subtractions of $\frac{1}{2}\ \sigma^2$ from the arithmetic mean u. This logic suggests an ansatz for coefficients in the generic Feynman Kac formulation with f(t) = 0 and with

$$\text{a} = \frac{\sigma^2}{2}S^2 \qquad \text{b} = \alpha S \qquad \text{and c} = \beta\ so\ \ \text{dO} = \beta\ O \tag{9}$$

Substituting these values in the generic Feynman Kac formulation with the drift rate, a, set at αS (the return on the stock times the stock price S and the discount rate, c, set at β the option price return and setting the f(t) value equal to 0 results in a partial differential equation to model the process as:

$$\frac{\partial O_F}{\partial t} \ + \ \frac{\sigma^2}{2}S^2\ \ \frac{\partial^2 O_F}{\partial S^2} + \alpha S\frac{\partial O_F}{\partial S} = \beta\ O_F\ for\ t < T \tag{10}$$

The solution domain should have the same boundary conditions for the solution domain as Black Scholes (1973):

$$O_F\ (S_T\ , t)\ \approx O(S_t - e^{rt}X) \tag{11}$$

$$O_F\ (0\ , t) = 0 \tag{12}$$

$$O_F\ (S_T\ , T) = \max\ (S - X, 0) \tag{13}$$

The Feynman Kac theorem provides the linkage from this partial differential equation to an expectation solution. Equation (10) is thus the fundamental partial differential equation of the "right-rate school." It is interesting to note that equation 10 follows from equation 4 in the Black Scholes (1973) paper. It is also very similar to equation 27 in Samuelson (1965) except that the coefficients in Samuelson's equation seem to reflect a normal rather than lognormal process. The solution is not analytical since the analytical solvable "heat equation" used by Black and Scholes (1973) only allows a single rate.

The use of the general path integral algorithm (Alghassi et al., 2022; Cairoli and Baule, 2017; Janson and Tysk, 2006) is probably the best approach. Richard Feynman discovered that certain partial differential equations like those in the general Feynman Kac formulation above could best be solved by "averaging" over paths. His discovery led to the reformulation of the quantum theory in terms of his "path integrals". The Alghassi et al. (2022) technique offers one method of solving these certain partial differential equations by simulating random paths of the stochastic process with greater speed, although it is not easy to program. The technique represents a faster solution than the traditional Monte Carlo methods. The Moore's R algorithm (Moore, 2003) discussed later is very easy to program even with a simple Excel spreadsheet but it approximates a solution. It also requires an appropriate value for the "combined rate". As the Moore (2003) algorithm uses the same thermodynamic mathematics as Black Scholes, it has its own limitations. The use of thermodynamics mathematics means that only a single rate can be used in the formulation of the solution, but Moore solves this by using a single varying "combined" stock return/option rate.

Of course, the "right-rate school "of options pricing would include spinoff models derived from this base case including such augmentation as mean reversion, jumps, stochastic elements etc. More realistic models must include mean reversion factors and distributional abnormalities since these characteristics have been well established in the literature (Heston, 1993). However, it would be good to establish the difficulties with the base case model before more complicated models are considered.

History of the "Right-Rate School" at the Academy of Economics and Finance

Moore (1997)

The empirical work which was the beginning of the modern "right-rate school" can be traced to a paper, "Option Pricing: An Expectations Model" presented by Moore (1997) at the twenty-fourth annual meeting of the Academy of Economics and Finance at Layfette, Louisiana in 1997. The paper is discussed in Moore (1999). In it, a model was proposed where the terms for a and c in the generic Feynman-Kac formulation are set to α , the return on the stock. The generic Feynman-Kac formulation model is

$$\frac{\partial O_F}{\partial t} + a\,(s,t)\frac{\partial^2 O_F}{\partial S^2} + b(s.t)\frac{\partial O_F}{\partial S} + f(t) = c\,(s,t)O_F \; for \; t < T \tag{14}$$

$$O_F(s,T) = \; O_F(s) \tag{15}$$

Assume a lognormal process, setting the stock return and option return equal to α , substitute, and dropping f(t) yields

$$\frac{\partial O_F}{\partial t} + \; \frac{\sigma^2}{2}S^2 \; \frac{\partial^2 O_F}{\partial S^2} + \alpha S \; \frac{\partial O_F}{\partial S} = \; \alpha O_F \tag{16}$$

Think of this equation as the Samuelson differential equation because Samuelson (1965) argued that α and β cannot be that far apart and suggested strong reasons why they should not diverge. But this exact equation cannot be found in a perusal of the Samuelson (1965) paper, so any inference that Samuelson thought it was a valid empirical model is not strictly correct. This responsibility lies with Moore. Samuelson did perform extensive analytical but not empirical work on the rational price of warrants if $\alpha = \beta$. In any event, in this version of the Feynman Kac equation, Moore proposed using α as a factor in the drift and discount rate in the context of a lognormal process. Using this assumption, one can use the Black Scholes solution because the required solution to this equation is mathematically equivalent to the solution to the Black Sholes equation. Their solution flows as a matter of mathematics since the solution to all such differential equations where the discount and drift rates are the same can be derived from the same thermodynamics' origin (Black and Scholes, 1973). This result is a natural outcome of the Feynman Kac theorem. The clever thermodynamic solution used in the Black Scholes is a specific application of the general Feynman Kac mathematics (Janson and Tysk, 2006).

One can use that the Feynman Kac theorem general solution separately, through path integration methods, or find the same expectation solution via the mathematics of the thermodynamics based Black Scholes solution. The Black Scholes formula (1973) as a subset of the more general Feynman Kac formulation should produce the same Feynman Kac path expectation. The problem is represented as being associated with the generic differential equation where r can be any value as:

$$\frac{\partial O_F}{\partial t} + \; \frac{\sigma^2}{2}S^2 \; \frac{\partial^2 O_F}{\partial S^2} + rS \; \frac{\partial O_F}{\partial S} = \; rO_F \tag{17}$$

where r is a generic rate, and the Black and Scholes solution now contains the generic r. The famous Black Scholes formula used one unique r, the risk-free rate. In this 1997 paper, Moore chose to use α for r. Moore stated that this model is founded on the assumption of a present value model which used a single rate for both the growth rate and the discount factor. In mathematical terms, this means the solution can be found via a generic Black Scholes solution. All specific applications with equal discount and growth rates can be solved with generic model solution which was originally solved in work associated with thermodynamics. By setting r = α in this paper, this solution is evident. The computations are easily done on a spreadsheet programmed with the traditional Black Scholes model but with an ex-post estimated return on the stock market as the rate input.

Using a sample of 300 S and P 500 Index options from the Wall Street Journal, Moore (1997) estimated the BS model prices using estimates of the market return prevalent during the period. The sample was a bit problematic because he used the last price listed by the Wall Street Journal on that particular day. This implied that the data had a "synchronicity" problem. Expecting that the bias would be reduced by substituting alpha for the risk-free rate, he was surprised to find the overall performance of this expectations model was about the same as the Black Scholes model. In testing the two models, he found that he could not statistically differentiate the two models. What was intriguing was that the bias pattern was flipped so that the implied volatility was much like a mirror image of the Black Scholes model and had an increasing rather than the traditional decreasing "implied volatility" slope. To date, there has been no other direct comparison of the Samuelson/Moore equation-based model compared to using the Black Scholes model

The results of the model bias from the expectation model in this paper led to the conclusion that there would never be a constant single rate that would produce a flat volatility curve across an option chain. Rather, the implied volatility patterns suggested that the effects of α and β would need to be combined. In order to use the thermodynamic mathematics, a single rate combining the two rates into a variable rate would be necessary.

Moore (1999)

Moore (1999) presented a paper, "Option Pricing: A Single Factor Expectations Model", at the twenty-sixth annual meeting of the Academy of Economics and Finance in Little Rock Arkansas in February 1999. A major difficulty of the Moore (1997) model was the assumption of a constant stock return rate, α , should be used in the generic solution to the "thermodynamic generic model" (Black Scholes generic model) regardless of strike price. Such an assumption denied that the option return shifts dramatically across the option chain while the return on the stock is constant as the exercise price changes. Such an assumption fails to properly treat the varying discount factor, the return on the option which varying incrementally across the domain. Moore suggests using a varying factor which "combines" the effects of the drift rate and the discount rate to obtain a "combined" rate, the Moore's R. The next three papers, (Moore, 1999; 2001; 2003) are attempts to define the combined rate. Suggesting that the generic thermodynamics solution (Black Scholes model solution) is a general solution for any generic r chosen was not new, but the novel concept in the paper was to apply the model with a different combined rate to each point in the domain. The procedure is validated by the logic dictating that any rate logically associated with a domain point is mathematically valid since the general thermodynamic solution is valid at that point. This implies that a different "combined rate" across the option chain with differing r is valid continuously at each and every point on the domain.

In the paper, Moore notes, "Since the Moore (1997) model and the Black Scholes model are essentially the same except for the assumption of what the proper discount rate for the model, this implies that the Black Scholes model is a present value model where the cash growth factor and the discount factor are the same. In the Black Scholes model, the rate is the risk-free discount rate."

In this paper, Moore suggested a leveraging measure in the spirit of Modigliani and Miller (1958). He proposes an operationally defined "option return" expression representing the leverage of the "option return" using the partial derivative of the stock relative to the option in a CAPM like expression. The factor for the leverage of the "option return" defined as:

$$Ro = (1/ N(d1))(Rm-Rf) + Rf \tag{18}$$

Moore's expression is not valid as an option return, but only as a leveraging factor. Deep in the money options where N(d1) = one makes the expression, Ro, equal to Rs for S and P 500 options where the beta is one. The beta of an option is used in a CAPM model to calculate expected return. The beta of an option is also the elasticity of the option relative to the stock times the stock's beta. The elasticity of an option is stock price divided by the call price times N(d1), the partial derivative of the stock relative to the option.

The expression, Ro, is a reflection of the leverage of the option on the stock return which increases as the exercise price increases (as one examines options in the direction of being out of the money). Moore uses the leverage of Ro to further define a hybrid rate combining the return on the stock, Rs and the leverage from his operationally defined, Ro. With Rs being the stock return, he suggests the combined rate, Moore's R, can be found by plugging his estimates into an expression:

$$R(\text{Moore's R}) = Rs/Ro *Rs \tag{19}$$

This combined rate, R (Moore's R) is plugged into the Black Scholes equation to obtain the Single Factor Model estimate. This is reasonable since the generic r-based solution used by Black and Scholes is valid mathematically for any generic r.

The value of the model was illustrated by using the last price settlement data on S and P 500 from the Wall Street Journal on March 20, 1997. Some of the option settlement dates were obviously stale. The t bill rate was .051 while the market return over the period was estimated to be .07. The combined rate was calculated as described above. The N(d1) factor was from Black Scholes (1973) model using the risk-free interest rate. Then the combined rate was plugged into the general Black Scholes formula with the combined rate as the interest rate. A regression was run of the form,

$$Pi = a + b1\ Ai + e(t) \tag{20}$$

where Pi was the model price and Ai was the actual price. The two models were tested to see if the model price and the actual price were the same. That is the hypothesis that b1 = 1 was tested. The Black Scholes model was rejected with a beta coefficient of 1.05 while the variable Moore model was not rejected with a coefficient of 1.009. The intercept, a, coefficient was found not to be statistically different than 0 in the Moore model but was statistically different than zero for the Black Scholes model. The conclusion is that the hypothesis that the model price and the actual price was the same for the Single Factor model could not be rejected. However, this hypothesis is rejected for the Black Scholes model. The implied volatility of the Moore's R variable model was flat while the Black Scholes model was downward sloping. Moore concludes proclaiming, "However, the theoretical purity implied by our illustration, one common variance across all strike prices with an implied variance closer to the actual variance means these models deserves closer scrutiny from a theoretical perspective."

In general, the paper can be seen as one which showed the value of using a leveraged factor for the option return. The search for a better approach continued in the two following papers.

Moore (2001)

At the 2001 meeting of the Academy of Economics and Finance, Moore (2001) presented the paper, "An Option Model Using the Solution Provided by the Bachelier-Einstein-Dynkin Derivation of the Fokker-Planck Equation with a Varying R". The paper proposed the use of the marginal implied rate of return on a portfolio of the assets held in a Bachelier-Einstein-Dynkin derivation of the Fokker-Planck equation equilibrium as the "combined rate" input interest rate to be used in the generic Black Scholes formula. The rate is derived from a long position of ϕ(d1) stocks and a short position of ϕ(d2) bonds. Each position is multiplied by the return on that asset. This implies a portfolio return of

$$R(p) = \phi\,(d_1) \text{ x } \alpha - \phi\,(d_2) \text{ x } Rf \qquad (21)$$

This portfolio return is the "Moore's R". The procedure is reasonable because the thermodynamics solution that Black and Scholes use is not unique to the choice of the rate, r. In theory the $\phi\,(d_1)$ term is the partial derivative defined by a Moore's R type model as opposed to a Black Scholes model, but there is not a large difference between the two.

It is well known that the Black-Scholes option pricing model is more accurate when its prices at the money options (McBeth and Merville, 1979; 1980) is used. Assuming the Black-Scholes option pricing model correctly prices options at the money, the following procedure was used:

1. Find the at-the-money option.
2. Extract the implied volatility from the at-the-money option.
3. Set up the *Moore's R* equation equal to the risk-free rate.
4. Solve for the expected return on the stock and plug that expected stock return into a spreadsheet to calculate the *Moore's R* at each strike price; and
5. Compare the actual and model price fit via regression.

The improvement in the fit over the Black Scholes model was examined using a simple OLS procedure. A regression of actual and model option prices using data for the options on Microsoft on November 13, 1996 was run. The regression was:

$$\text{Pi} = \text{a} + \text{b1 Ai} + \text{e(t)} \qquad (22)$$

where Pi represents the model price, a is the intercept, b1 is the regression coefficient, Ai is the actual option price and e(t) corresponds to the error term. The models were examined as to the hypothesis that b1 = 1. The results showed that the Black Scholes model was rejected as a model with a b1 coefficient of 1.04 while the Moore model was almost a perfect fit with a b1 coefficient of 1.008. The a term in the Moore model was also not statistically different than 0. This was a nearly perfect fit for the proposed Moore model.

Another set of options, the S and P 500 index option on January 2, 1990, was tested using the same procedure and regression. These results did not have the perfect fit of the Microsoft data. Analysis of individual model bias across the chain showed that the very deepest in the money options were causing the bias problem. When the deepest in the money options were removed the proposed model did have a remarkable fit. These are the very options that are often deleted from many option studies. But this was still troublesome! The conclusion must be that the simple Moore's R algorithm has trouble with the very deep in the money options.

Moore (2003)

Moore presented a paper, "Extracting Overall Cap rates from Options Using the Bachelier Einstein - Dykin solution of the Fokker-Plank Equation" at the 2003 AEF meeting. The paper was included in the Proceedings of the Thirtieth Annual Meeting of the Academy of Economic and Finance at Savanna Georgia in February 2003. The paper relies on the result that the general solution used by Black and Scholes (1973) which is an application of the Bachelier-Einstein Dykin solution of the Fokker-Plank equation to partial differential equations of the form:

$$\frac{\partial O_F}{\partial t} + \frac{\sigma^2}{2} S^2 \frac{\partial^2 O_F}{\partial S^2} + rS \frac{\partial O_F}{\partial S} = rO_F \tag{23}$$

can be used for any r proposed. In short, the Black Scholes formula can be used for any proposed r. But here Moore wanted to use it to numerically find the "combined rate." The empirically derived combined rate r is still called Moore's R (Moore, 2001; 2005). Conceptually, it is a merged rate growing the cashflows at the stock return and discounting them at the option return. This paper solves the combined rate numerically by setting the actual option price equal to the Black Scholes formula with a given appropriate volatility. In discussing his expectations for the "solved rate", Moore (2003) declares, "Overall, our analysis suggests that if a single rate is to model the combined effects of cashflow growth and discount at a leveraged rate, then the rate must vary in a systematic way. The rate of return on the stock is a constant regardless of the stock price. The discount rate on the related to the option increases as the strike price places the option more out of the money. Consequently, a combined rate must decrease as the strike price increases".

Using this method, he extracted the pattern of "implied" combined rates which he analyzed as to functional form. Discussing the observed pattern, he articulates, "In general, our prior hypothesis of the pattern of the overall rate is supported. The deep in the money options have higher overall cap rates than out of the money options. Most interesting is that the pattern has a smooth functional form."

Moore noted that the pattern across the option chain was that the overall rate fell as one moved toward the out of the money direction. It fell functionally, systematically, smoothly and consistently from the point where the partial derivative of the stock price to the call price called N(d1) in the Black Scholes model was about one. Consequently, the out of the money options had very small "combined rates" consistent with the option discount factors from the option rate being very large. However, at the point where N(d1) was about one, the expected pattern in terms of the values of the stock return and option return did not seem to predict the slope of the "combined rate" curve. The cap rate straightened out and then started declining. Like the Moore (2001) paper, the pattern of the combined rate also known as Moore's R was explained by examining the predicted effect of the return on the option which rises as one moves in the out of the money direction versus the return on the stock. It explains the general slope of the Moore's R curve up to the point where the option is extremely likely to be exercised. Although the slope was as expected up until this point, the pattern disappeared and reversed itself at the very deepest in the money options.

This implies there may be more to the story than the simple interaction between the stock return and the option return variables. In Table 1 and Figure 1 in the Moore (2003) paper, the combined stock return/option return rate falls as one moves toward the out of the money position as one moves away from the critical point. Moore simply says, "…As the probability of the option being exercised approaches 1, the curve flattens out."

This minor exception to the expected pattern implies the need for future research. But overall, the general pattern was consistent with the return on the option being an increasing function of the degree of leverage which increases as the option return increases as one moves further and further out of the money. As the option discount rate is large, the overall Moore's R "combined rate" is very low for the deep out of the money options.

The paper showed that empirically applying the Bachelier Einstein Dykin solution of the Fokker-Plank Equation differential equation continuously provides a reasonable approximation of the combined effects of growing cashflows at the return on the stock and discounting them at the return on the option. However, the pattern where there is a very high certainty of option exercise shows that the simple stock return/option return relationship still needs additional analysis. Analysis of individual observations of model bias across the chain showed the same deviate pattern as the Moore (2001) paper, the very deepest in the money options results are not explained by a simple combined rate analysis. Although there is the temptation to delete these observations as has been done in many option studies, deleting data is always troublesome. The correct conclusion was likely that the simple Moore's R algorithms have trouble with the very deep in the money options. A possible explanation is that with certainty of exercise, the value of an option , $O_f\ (S_t\ , t)\ is\ (S_t - e^{-rt}X)$.

Moore (2005)

"Extracting Expected Stock Return Correlates from Option Prices" was presented at the 2005 Meeting of the Academy of Economics and Finance. In it, Moore (2005) combined Moore's R concepts his 2001 and 2003 papers. By combining the two he was able to obtain estimates of the underlying stock returns associated with the option chain. Merton (1990, p.282) claims that such attempts are doomed to failure. Moore did not think so but he was cautious as evidenced by the title of the paper. In the paper, he extracts Moore's R via the 2003 paper and then sets it equal to the Moore's R expression in the 2001 paper. The expression in his 2001 paper for Moore's R was defined as:

$$R(p) = \phi(d_1) \times \alpha - \phi(d_2) \times Rf \tag{24}$$

where the hedge ratio term , $\phi(d_1)$, is derived from a Moore's R type model. Setting the expression in equation 24 equal to Moore's R solved numerically using the 2003 paper technique allows an estimate of the underlying stock return. Surprisingly, the technique produced shockingly consistent results across the option chain. One must be cautious that any extracted stock return estimate from options are "true stock return estimators", as there is reason to be leery of the thermodynamic mathematics of Black Scholes. Thus, these model estimates are called stock return correlates. Future research will be needed to see how close these correlates are to actual stock returns. These expected stock returns are to a large degree much like the model constructed "implied volatility" which have no real counterpart in the real world. However, they were consistent across the option chain suggesting they have real economic meaning. The method of inquiry was:

1. Find the at-the-money option.
2. Extract the implied volatility from the at-the-money option.
3. *The Moore's R* estimate at the money is the risk-free rate using the Moore (2001) paper Moore's R estimator substitute the known risk-free rate into it and solve for the expected return on the stock.
4. Repeat for all other options but used the extracted overall cap rate (Moore, 2003) as the Moore's R and set it equal to the expression in the 2001 paper.
5. Solve for the Stock return at each price.
6. Analyze the consistency of the stock return estimates across the option chain.

Using the outlined procedure, the results following the method were examined using data from Januay 2, 1990 on S and P 500 index options. The results showed remarkable consistency in the estimates of stock returns. Because of concerns about the stock volatility, sensitivity analysis was done to test the effect of various proposed standard deviations. It was likely that the actual volatility might be slightly different than the at the money implied volatility. The sensitivity analysis shows that slight deviation in the standard deviation produced substantially improved consistency of the results.

Moore and Simpson (2023)

Marc Simpson presented a paper "Why the Black Sholes Model Worked – and Then It Didn't" at the 2023 Meeting of the Academy of Economics and Finance. The paper was a remarkable supporting piece of evidence for the "Right-Rate School". In the paper the authors show that the differential equation for the Black Scholes model is

$$\frac{\partial O_F}{\partial t} + \frac{\sigma^2}{2} S^2 \frac{\partial^2 O_F}{\partial S^2} + rS \frac{\partial O_F}{\partial S} = rO_F \tag{25}$$

Contrary to the current orthodox view, the "right-rate school" of option pricing takes the position that this equation implies that the risk-free rate, the return on the stock and the return on the option are all equal to the risk-free rate. Moore and Simpson (2023) examine the economic history of the return on stock and the risk-free rate. They find that in the initial testing period the short-term interest rate expectations and the expectations of stock returns were not much different. This in fact implies that there was no expected risk premium prevailing in the market. Another way of saying this is that the return on the stock, the return on the option and the risk-free rate are economically the same in terms of the expectations relevant to the traded option. They claim this may be why the Black Scholes model seemed to work during the tests of the 1970s data (Rubinstein, 1985) when it was initially tested. It also explains why in the 1980s when the two rates diverged more significantly the Black Scholes model no longer worked in the equities market (Rubinstein, 1994). His is an ironic quirk in economic history and possibly one of the most interesting cases of serendipity in scientific history!

The paper is a critical piece of evidence in favor of the "right-rate school" of option pricing. The implication of the paper is that the "right-rate school" is the more precise way of modeling option pricing since it compresses to the Black Scholes model when the risk-free rate and the return on the market are equal. Because a rate explanation can explain the shift in the volatility smile in the early 1980s, it must lead to a serious reexamination of the Black Scholes model as the focal paradigm of option pricing. Given the evidence in papers such as this one, it appears that the "right-rate school" is the more correct orientation.

Discussion and Conclusion

The "right-rate school" of option pricing is today one of the most disruptive theories in all of financial economics. This paper outlines the major developments in the theory that occurred at the Academy of Economics and Finance meetings. The theory challenges the validity of the Black Scholes model, the associated hedging arguments associated with it, and the general risk neutral pricing framework. It suggests that a risk-free hedge is not possible since a combination of a call option hedge with a fractional stock cannot guarantee a constant rate. Although it denies the absolute validity of the Cox and Ross (1976) risk neutral pricing arguments, it does not deny the usefulness of the risk neutral argument as an approximation. It explains the volatility smirk as a consequence of rational differences in rates (higher option returns as one moves to in the direction of out the money options) rather than some mysterious and unknown volatility processes (Moore, 2003). It focuses option research away from the predominate emphasis on the stock distribution to a large extent by focusing on the fact that the implied volatility smile is due to differing option returns across the option chain (Moore, 2003). In short, it requires rewriting much of the knowledge previously characterizing option pricing.

It is notable that studies using limited resources can challenge with statistical validity the predominant view of much of the status quo option's research. But the status quo has suggested a rocky foundation for decades. What is more remarkable is that an organization which is open and non-critical in its approach to scholarly research played such an important role in its development. These papers presented at the AEF are certainly interesting and should spur additional research as well as further inquiry into the topics. Some of them may lead to significant advances in option pricing theory. Without such an atmosphere of collegiality and open inquiry as exist in the Academy of Economics and Finance, the "right-rate school" of option pricing and similar theoretical developments would not have a chance to advance.

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Pricing Mortgage-Backed Securities with a Skewness-Adjusted Binomial Interest Rate Model

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Abstract

In a 2006 statistical study, Johnson, Zuber, and Gandar found a large number of periods of increasing and decreasing interest rate cases in which skewness was significant. Their findings, in turn, point to the importance of using a skewness-adjusted binomial interest rate model when pricing bond and bond derivatives when the underlying interest rate is expected to increase or decrease. This paper explains how a skewness-adjusted binomial model can be applied to the pricing of mortgage backed securities whose discount rate, as well as cash flows, are sensitive to interest-rate risk.

JEL: G12, G13
Keywords: Binomial; Skewness; MBS

Introduction

Johnson, Zuber, and Gandar (2006) tested for the significance of skewness in the logarithmic returns for U.S. Treasury yields using the D'Agostino, Belanger, and D'Agostino_(1999) statistical tests of normality. They found a large number of periods of increasing and decreasing interest rate cases in which skewness was significant. In an update of that study, Johnson and Sen (JS) (2018) applied the D'Agostino, Belanger, and D'Agostino tests of normality for the logarithmic returns for 1-year, 2-year, 5-year and 10-year U.S. Treasury yields for the period from February 15, 1977 to December 19, 2017, using rolling 250-day periods. They similarly found a significant number of periods of increasing interest rates characterized by a positive mean and negative skewness and a significant number of decreasing interest rate periods characterized by a negative mean and positive skewness. Their findings, as well as other earlier studies (Koedijk, Stork, and Casper, 1992; Kon, 1990; Aggarwal and Rao, 1990; Turner and Weigel, 1992), point to the importance of using a skewness-adjusted binomial interest rate model when pricing bond and bond derivatives when the underlying interest rate is expected to increase or decrease.

There are two general approaches to modeling stochastic interest rate movements using a binomial model – the equilibrium model (Rendelman and Bartter, 1980; Cox, Ingersoll, and Ross, 1981) and the calibration model (Black, Derman, and Toy, 1990; Ho and Lee, 1986; Heath, Jarrow, and Morton, 2008). Both models assume that the interest rate's logarithmic return is normally distributed. Câmara and Chung (2006) and Johnson, Pawlukiewicz, and Mehta (JPM) (1997) have extended the Cox, Ross, and Rubinstein (CRR) (1979) binomial option pricing model to include skewness. In the JPM skewness-adjustment model, the upward (u) and downward parameters (d) and the probability of an increase in one period (q) values defining a binomial process are found by setting the equations for the binomial distribution's expected value, variance, and skewness equal to their respective empirical values, and then solving the resulting equation system simultaneously for u, d, and q. Johnson and Sen (2018) also adjusted the Black-Derman-Toy (BDT) binomial-interest rate model to account for skewness and showed the BDT model loses its arbitrage-free feature when the variability conditions are not adjusted to account for skewness.

The valuation of mortgage-backed securities (MBS) is one of the more complex fixed-income securities because of the difficulty in estimating cash flows due to the prepayment options of the mortgage borrowers. One common approach used to determine the possible values of an MBS is vector analysis. Vector analysis involves generating a matrix of MBS values based on different discount rates and prepayment speeds. One way to estimate different vectors is to use a binomial interest rate tree. The purpose of this paper is to apply JPM's skewness-adjusted binomial model to the pricing of mortgage backed securities—securities whose discount rate, cash flows, and expected interest rate paths are sensitive to interest-rate risk.

Skewed Binomial Distributions

Binomial distributions of spot rates, S, and their corresponding logarithmic returns, $g_n = \ln(S_n/S_0)$, are shown in Exhibit 1. The exhibit shows three end-of-the-period distributions resulting from a binomial process in which the number of periods to expiration is n = 30 and the initial spot interest rate is $S_0 = 0.10$. The probability distribution in the top exhibit is generated from a binomial process in which the upward parameter (u) is equal to 1.02, the downward parameter (d) is equal to 1/1.02 ($|\ln u| = |\ln d|$) and the probability of an increase in one period (q) is equal to 0.5. In this case, the distribution approaches a normal distribution with $E(g_n) = 0$, $V(g_n) = 0.01176821$, and skewness, $Sk(g_n)$, equal to zero:

$$E(g_n) = \sum_{j=0}^{n} p_{nj} g_{nj}$$

$$E(g_n) = n[q \ln u + (1-q)\ln d] = n\,E(g_1) \tag{1}$$

$$E(g_{30}) = 30[0.5 \ln(1.02) + (1-0.5)\ln(1/1.02) = 0$$

$$V(g_n) = E[g_n - E(g_n)]^2 = \sum_{j=0}^{n} p_{nj}[g_n - E(g_n)]^2$$

$$V(g_n) = nq(1-q)[\ln u/d)]^2 = n\,V(g_1) \tag{2}$$

$$V(g_{30}) = 30\,(0.5)(1-0.5)\,[\ln(1.02/1/1.02)]^2 = 0.01176821$$

$$Sk(g_n) = E[g_n - E(g_n)]^3 = \sum_{j=0}^{n} p_{nj}[g_n - E(g_n)]^3$$

$$Sk(g_n) = n[q(1-q)^3 - q^3(1-q)]\,[\ln u/d)]^3 = n\,Sk(g_1) \tag{3}$$

$$Sk(g_{30}) = 30[(0.5)(1-0.5)^3 - (0.5)^3(1-0.5)]\,[\ln(1.02/(1/1.02))]^3 = 0$$

where: g_1 = the logarithmic return for one period; j = the number of increases in n periods; p_{nj} = the probability of j increases in n periods that is defined as:

$$p_{nj} = \frac{n!}{(n-j)!\,j!}\;q^j\,(1-q)^{n-j}$$

The middle distribution reflects an increasing interest rate case in which u = 1.02, d = 0.99 ($|\ln u| > |\ln d|$), and q = 0.75. Here the resulting distribution is negatively skewed with a positive mean: $E(g_{30}) = 0.37018$, $V(g_{30}) = 0.005013$, and $Sk(g_{30}) = -0.00007483$. The bottom distribution in Exhibit 1 reflects a decreasing interest rate case in which u = 1.01, d = 0.980392 ($|\ln u| < |\ln d|$), and q = 0.25; the distribution is positively skewed with a negative mean: $E(g_{30}) = -0.37018$, $V(g_{30}) = 0.004979$, and $Sk(g_{30}) = 0.000074$.

The first distribution shown in Exhibit 1 has an expected value equal to zero for the case in which q = 0.5. This property is the result of assuming not only that there is an equal probability of an increase or decrease in each period, but also that u and d parameters are inversely proportional, or equivalently that the proportional increase in each period (lnu) is equal in absolute value to the proportional decrease (lnd). However, if the distribution of the logarithmic return at the end of n periods had, for example, a positive expected value and zero skewness, then the underlying binomial process would have been characterized by the proportional increases in each period exceeding in absolute value the proportional decreases, with the probability of the increase in each period being 0.5; if the distribution also had negative skewness, then the probability of the increase in one period would have exceeded 0.5. On the other hand, if the distribution of the logarithmic return had a negative expected value and zero skewness, then the underlying binomial process would have been characterized by the proportional decreases in each period exceeding in absolute value the proportional increases and with q = 0.5; if the distribution also had a positive skewness, then q would have been less than 0.5.

Skewness-adjusted Binomial Interest Rate Tree

A binomial process that converges to an end-of-the-period distribution of logarithmic returns that is normal will have equal probabilities of the underlying security increasing or decreasing each period, whereas one that converges to a distribution that is skewed will not. As noted, in the JPM option pricing model, the u and d parameters and q values defining a binomial process are found by setting the equations for the binomial distribution's expected value, variance, and skewness equal to their respective empirical values, and then solving the resulting equation system simultaneously for u, d, and q:

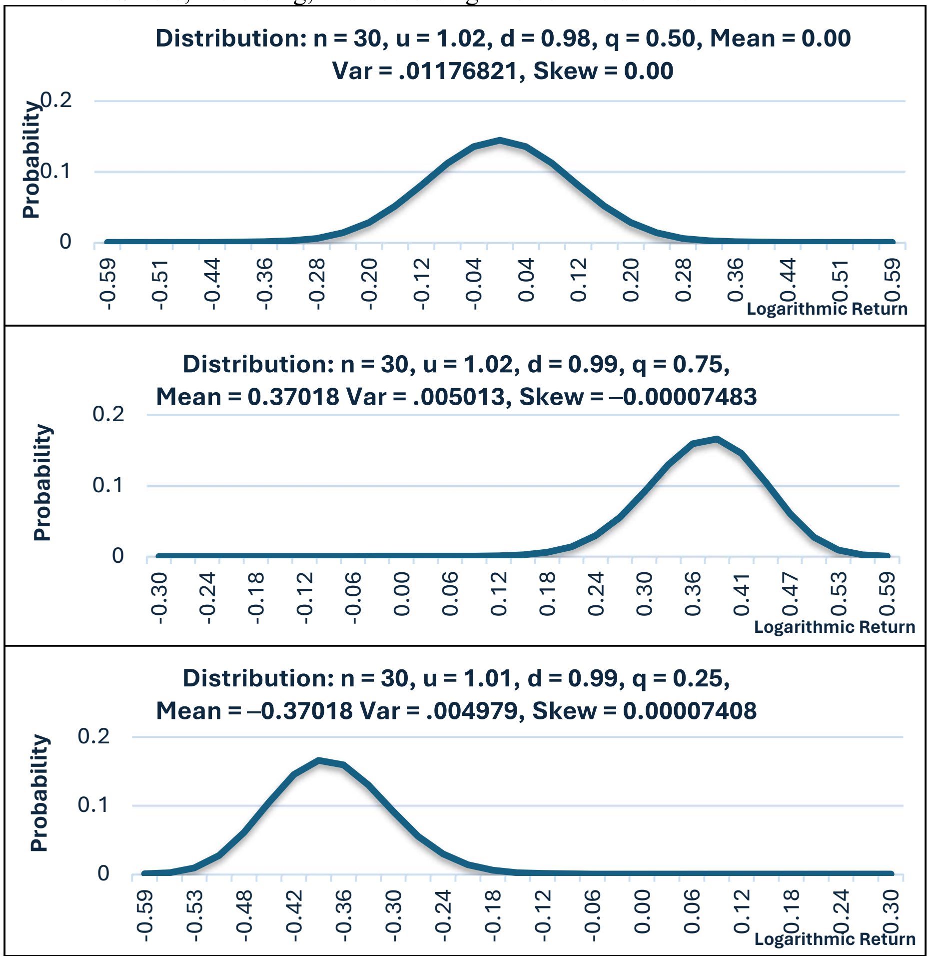
$$E(g_n) = n[q \ln u + (1-q)\ln d] = \mu_e \tag{4}$$

$$V(g_n) = nq(1-q)[\ln u/d)]^2 = V_e \tag{5}$$

$$Sk(g_n) = n[q(1-q)^3 - q^3(1-q)]\,[\ln u/d)]^3 = \delta_e \tag{6}$$

where: μ_e, V_e, δ_e = the empirical values of the mean, variance, and skewness for a period equal in length to n periods.

Exhibit 1: Stable, increasing, and decreasing interest rate distributions



The values of u, d, and q that satisfy this system of equations are:

$$u = e^{\frac{\mu_e}{n} + \left[\frac{(1-q)V_e}{n\,q}\right]^{1/2}} \tag{7}$$

$$d = e^{\frac{\mu_e}{n} - \left[\frac{qV_e}{(1-q)\ n}\right]^{1/2}} \tag{8}$$

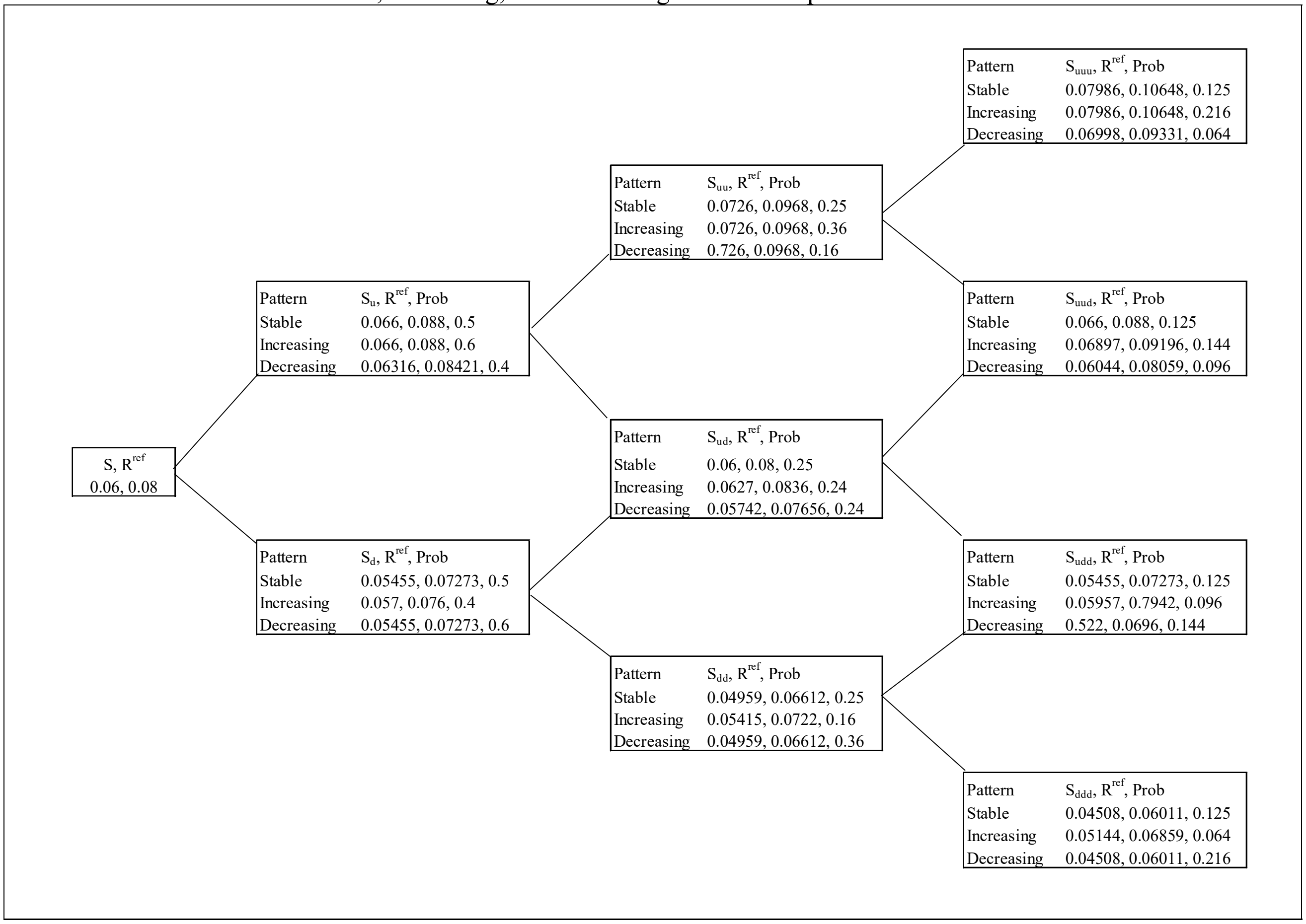
$$q = \frac{1}{2} \mp \frac{1}{2}\left[\frac{4V_e^3}{n\delta_e^2} + 1\right]^{-1/2} \tag{9}$$
$$-\text{ if } \delta_e > 0;\ +\text{ if } \delta_e < 0$$

If δ_e is positive (negative), then q is less (greater) than 0.5; if skewness is zero, then q = 0.5 and Equations (7) and (8) simplify to the Cox, Ross, and Rubinstein (CRR) binomial option pricing model formulas for u and d:

$$u = e^{\frac{\mu_e}{n} + \left[\frac{V_e}{n}\right]^{1/2}} \tag{10}$$

$$d = e^{\frac{\mu_e}{n} - \left[\frac{V_e}{n}\right]^{1/2}} \tag{11}$$

Exhibit 2: Binomial trees for stable, increasing, and decreasing interest rate patterns



Stable: $\mu_e = 0.00$, $V_e = 0.036336$, $\delta_e = 0.00$; u = 1.1, d =.09091, and q =0.5
Increasing: $\mu_e = 0.14667516$, $V_e = 0.020633$, $\delta_e = -0.00060497$; u = 1.1, d = 0.95, and q = 0.6
Decreasing: $\mu_e = -0.14667513$, $V_e = 0.020633$, $\delta_e = 0.00060497$; u = 1.0526316, d = 0.9091, and q = 0.4

As an example, suppose the spot interest rate is currently at 6% and there is a market expectation of increasing rates over the next four years with the expected distribution of logarithmic returns of spot rates having the following estimated parameters of $\mu_e = 0.14667516$, $V_e = 0.020633$, and $\delta_e = -0.00060497$. The u, d, and q values for a binomial interest rate tree that would calibrate a binomial distribution to this distribution would be u = 1.1, d = 0.95, and q = 0.6.

$$u = e^{\frac{\mu_e}{n} + \left[\frac{(1-q)V_e}{n\,q}\right]^{1/2}} = e^{\frac{0.14667516}{4} + \left[\frac{0.020633\,(1-0.6)}{(4)(0.6)}\right]^{1/2}} = 1.10$$

$$d = e^{\frac{\mu_e}{n} - \left[\frac{qV_e}{n\,(1-q)}\right]^{1/2}} = e^{\frac{0.14667516}{4} - \left[\frac{0.020633\,(0.6)}{(4)(1-0.6)}\right]^{1/2}} = 0.95$$

$$q = \frac{1}{2} \mp \frac{1}{2}\left[\frac{4V_e^3}{n\delta_e^2} + 1\right]^{-\frac{1}{2}} = \frac{1}{2} + \frac{1}{2}\left[\frac{4(0.020633)^3}{(3)\,(-0.00060497)^2} + 1\right]^{-1/2} = 0.6$$

In contrast to an increasing rate case, suppose the market expects declining rates over the next four years, with the expected distribution of logarithmic returns of spot rates having the following estimated parameters of $\mu_e = -0.14667513$, $V_e = 0.020633$, and $\delta_e = 0.00060497$. The u, d, and q values for a three-period binomial interest rate tree that calibrate a binomial distribution to this distribution are u = 1.0526316, d = 0.9091, and q = 0.4. Finally, under a stable case where $\mu_e = 0.00$, $V_e = 0.036336$, and $\delta_e = 0.00$, u = 1.1, d =.09091, and q =0.5.

Exhibit 2 shows three binomial interest rate trees defined for a one-year spot rate and a mortgage refinancing rate, R_t^{Ref}, with a maturity between 7 and 10 years and with the length of each period for the tree being one year. In the top row of each box in the exhibit, the one-year spot rates and the refinancing rates reflect a binomial process for the stable interest rate pattern: $\mu_e = 0.00$, $V_e = 0.036336$, and $\delta_e = 0.00$; u = 1.1, d =.09091, and q =0.5). The middle row in each box shows the one-year spot and refinancing rates for an increasing interest rates scenario: $\mu_e = 0.14667516$, $V_e = 0.020633$, and $\delta_e = -0.00060497$; u = 1.1, d = 0.95, and q = 0.6. Finally, the last row in each box shows the decreasing rate case: $\mu_e = -0.14667513$, $V_e = 0.020633$, and $\delta_e = 0.00060497$; u = 1.0526316, d = 0.9091, and q = 0.4.

MBS Valuation

In using the binomial interest rate tree approach to price an MBS or the collateral of mortgages underlying an MBS issue, the assumption about the underlying stochastic process is important in determining the value. If rates are expected to decrease in the future, then the value of an MBS will be greater given expected lower discount rates and greater earlier cash flows resulting from the expected increase in the prepayment of principal. Under this interest rate scenario, an MBS should be valued by a binomial model that is characterized by a positive expected logarithmic return and possibly negative skewness. In contrast, if rates are expected to increase in the future, then the value of an MBS will be lower given higher expected discount rates and lower earlier cash flows due to expected slower prepayment. In this case, the appropriate binomial model would be one that is characterized by a negative expected return and possibly positive skewness. The impact of these different interest rate scenarios on the value of an MBS can be examined by valuing the security using a Monte Carlo simulation approach that uses a binomial model that captures these different cases.

With each of the binomial processes shown in Exhibit 2, there are four possible rates at the end of the third period for the spot and refinancing rate and eight possible interest rate paths. The cash flows from an MBS or a pool of mortgages depend on prepayment. Most analysts use a prepayment model in which the conditional prepayment rate (CPR) is determined by the seasonality of the mortgages and a refinancing incentive that ties the interest rate paths to the proportion of the mortgage collateral prepaid. To illustrate, consider an MBS formed from a mortgage pool with a par value of $1 million, weighted average coupon rate (WAC) = 8%, and weighted average maturity (WAM) = 10 years. To fit this example to the three-period binomial trees, assume that the mortgages in the pool all make annual cash flows, that all have a balloon payment at the end of year 4, and that the pass-through rate on the MBS formed from the mortgage portfolio is equal to the WAC of 8%. This mortgage pool can be viewed as a four-year asset with a principal payment made at the end of year four that is equal to the original principal less the amount paid down. A simple prepayment model to apply to this mortgage pool is shown in Table 1. The prepayment model assumes the annual CPR is equal to 5% if the current refinancing rate is equal to the WAC of 8% or greater ($X = WAC_t - R_t^{Ref} \leq 0$). If the refinancing rate is less than the WAC of 8% ($X = WAC_t - R_t^{Ref} > 0$), though, the model assumes that the CPR will exceed 5% and that it will increase within certain ranges as X (= $WAC_t - R_t^{Ref}$) increases.

Table 1: Conditional prepayment model

X = WAC − R^{Ref} Range	**WAC = 8% CPR**
X < 5%	5%
0.0% < X < 0.5%	10%
0.5% < X < 1.0%	20%
1.0% < X < 1.5%	30%
1.5% < X < 2.0%	40%
2.0% < X < 2.5%	50%
2.5% < X < 3.0%	60%
X > 3.0%	70%

The cash flows (CF) for each of the eight interest rate paths for the stable interest rate case are presented in Exhibit 3. As shown, the cash flows for path 1 (the path with three consecutive decreases in rates) consist of $335,224 in year 1 (interest = $80,000, scheduled principal = $69,029, and prepaid principal = $186,194, reflecting a CPR of 0.20), $324,764 in year 2, with $205,540 being prepaid principal (CPR = 0.30), $257,259 in year 3, with $173,802 being prepaid principal (CPR = 0.40), and $281,560 in year 4. The year 4 cash flow with the balloon payment is equal to the principal balance at the beginning of the year and the 8% interest on that balance. In contrast, the cash flows for path 8 (the path with three consecutive interest rate increases) are smaller in the first three years and larger in year 4, reflecting the low CPR of 5% in each period.

Given the cash flows of each path, the value of a path is:

$$V_0^{\text{Path i}} = \sum_{t=1}^{nM} \frac{CF_t}{(1+Z)^t}$$

where: Z_t = Discount rate = S_t + k; k = Risk premium = Option-adjusted spread

The discount rate, Z, is the risk-adjusted spot rate. This rate is equal to the riskless spot rate, S_t, plus a risk premium, k. Assuming the underlying mortgages are insured against default, the risk premium would only reflect the additional return needed to compensate MBS investors for the prepayment risk they are assuming. This premium is referred to as the option-adjusted spread (OAS). For this example, the OAS is assumed to be 2% greater than the one-year, riskless spot rates. From these current and future one-year spot rates, the current 1-year, 2-year, 3-year, and 4-year equilibrium spot rates are obtained for each path by using the geometric mean (Exhibit 3, Column 10). Thus, the set of spot rates used to discount the cash flows for path 1 are:

$$Z_1 = 0.08$$

$$Z_2 = [(1.08)(1.074546)]^{1/2} - 1 = 0.077269$$

$$Z_3 = [(1.08)(1.074546)(1.069588)]^{1/3} - 1 = 0.0747029$$

$$Z_4 = [(1.08)(1.074546)(1.069588)(1.06508)]^{1/4} - 1 = 0.72289$$

Using these rates, the value of path 1 is $1,010,465:

$$V_0^{\text{Path 1}} = \frac{\$335{,}224}{(1.08)^1} + \frac{\$324{,}764}{(1.077269)^2} + \frac{\$257{,}259}{(1.0747029)^3} + \frac{\$281{,}560}{(1.072289)^4} = \$1{,}010{,}465$$

The risk-adjusted spot rates and values for the eight paths are shown in Columns 10 and 11 in Exhibit 3. The theoretical value of the MBS is the weighted average of the values of all of the interest rate paths, with the weights being the probabilities of attaining each path. With q = 0.5, the probability of attaining any path in this three-period example is 0.125, yielding a theoretical value of $997,457.

The MBS value of $997,457 is obtained from a binomial tree that reflects a stable interest rate pattern with an expected logarithmic return and skewness of zero. Suppose, however, the decreasing interest rates scenario were expected (μ_e = −0.14667513, V_e = 0.020633, and δ_e = 0.00060497; u = 1.0526316, d = 0.9091, and q = 0.4). Exhibit 4 shows the cash flows from the mortgage pool, discount rates, values, and probabilities for each path. The theoretical value of the MBS pool under this scenario is $1,003,871, exceeding the value obtained in the previous case. This larger value reflects the decreasing interest rate case in which there are a greater number of paths with larger CPRs and with greater probabilities associated with those paths. Note, if the probability of the decrease in each period were 0.8 instead of 0.6, then the theoretical value would be even higher ($1,006,880). This is a case of greater positive skewness (0.0003025), reflecting greater decreasing rates. In this case, the size of the CPRs is greater in more paths and the probabilities for decreasing interest rates paths are greater.

Instead of an expected decreasing or stable interest rates patterns, suppose the increasing interest rates scenario were expected (μ_e = 0.14667516, V_e = 0.020633, δ_e = −0.00060497; u = 1.1, d = 0.95, and q = 0.6). Exhibit 5 shows the cash flows of the mortgage pool, discount rates, values, and probabilities for the eight paths for this scenario. In this increasing binomial interest rate case, the theoretical value of the MBS pool is only $991,462. This lower value reflects a greater number of paths with lower CPRs and greater probabilities associated with those paths. Note that if the probability of the increase in each period were 0.8 instead of 0.6, then the theoretical value would be even lower ($985,161). This is a case of a greater negative skewness (-0.0003025), reflecting greater increasing rates. In this case, the size of the CPRs is smaller in more paths and the probabilities for increasing interest rates paths are greater.

Exhibit 3: Valuation of MBS for stable interest rate pattern

1 **Year**	2 **R^{ref}**	3 **Balance**	4 **Interest**	5 **Sch Pr**	6 **CPR**	7 **Pre Pr**	8 **CF**	9 **$Z_{1,t-1}$**	10 **Z_{t0}**	11 **Value**	12 **Prob.**
1	0.073	$1,000,000	$80,000	$69,029	0.20	$186,194	$335,224	0.080	0.0800	$310,392	0.5
2	0.066	$744,776	$59,582	$59,641	0.30	$205,540	$324,764	0.075	0.0773	$279,846	0.5
3	0.060	$479,594	$38,368	$45,089	0.40	$173,802	$257,259	0.070	0.0747	$207,255	0.5
4		$260,703	$20,856				$281,560	0.065	0.0723	$212,972	
						Path 1			**Value**	**$1,010,465**	**0.125**
1	0.073	$1,000,000	$80,000	$69,029	0.20	$186,194	$335,224	0.080	0.0800	$310,392	0.5
2	0.066	$744,776	$59,582	$59,641	0.30	$205,540	$324,764	0.075	0.0773	$279,846	0.5
3	0.073	$479,594	$38,368	$45,089	0.20	$86,901	$170,358	0.070	0.0747	$137,245	0.5
4		$347,604	$27,808				$375,413	0.075	0.0747	$281,461	
						Path 2			**Value**	**$1,008,945**	**0.125**
1	0.073	$1,000,000	$80,000	$69,029	0.20	$186,194	$335,224	0.080	0.0800	$310,392	0.5
2	0.080	$744,776	$59,582	$59,641	0.05	$34,257	$153,480	0.075	0.0773	$132,253	0.5
3	0.073	$650,878	$52,070	$61,192	0.20	$117,937	$231,200	0.080	0.0782	$184,465	0.5
4		$471,749	$37,740				$509,489	0.075	0.0773	$378,301	
						Path 3			**Value**	**$1,005,411**	**0.125**
1	0.088	$1,000,000	$80,000	$69,029	0.05	$46,549	$195,578	0.080	0.0800	$181,091	0.5
2	0.080	$884,422	$70,754	$70,824	0.05	$40,680	$182,258	0.086	0.0830	$155,393	0.5
3	0.073	$772,918	$61,833	$72,666	0.20	$140,050	$274,550	0.080	0.0820	$216,742	0.5
4		$560,202	$44,816				$605,018	0.075	0.080	$444,494	
						Path 4			**Value**	**$997,720**	**0.125**
1	0.073	$1,000,000	$80,000	$69,029	0.20	$186,194	$335,224	0.0800	0.0800	$310,392	0.5
2	0.080	$744,776	$59,582	$59,641	0.05	$34,257	$153,480	0.0745	0.0773	$132,253	0.5
3	0.088	$650,878	$52,070	$61,192	0.05	$29,484	$142,747	0.0800	0.0782	$113,892	0.5
4		$560,202	$44,816				$605,018	0.0860	0.0801	$444,494	
						Path 5			**Value**	**$1,001,031**	**0.125**
1	0.088	$1,000,000	$80,000	$69,029	0.05	$46,549	$195,578	0.080	0.080	$181,091	0.5
2	0.080	$884,422	$70,754	$70,824	0.05	$40,680	$182,258	0.086	0.083	$155,393	0.5
3	0.088	$772,918	$61,833	$72,666	0.05	$35,013	$169,512	0.080	0.082	$133,820	0.5
4		$665,240	$53,219				$718,459	0.086	0.083	$522,269	
						Path 6			**Value**	**$992,574**	**0.125**
1	0.088	$1,000,000	$80,000	$69,029	0.05	$46,549	$195,578	0.080	0.080	$181,091	0.5
2	0.096	$884,422	$70,754	$70,824	0.05	$40,680	$182,258	0.086	0.083	$155,393	0.5
3	0.088	$772,918	$61,833	$72,666	0.05	$35,013	$169,512	0.093	0.086	$132,277	0.5
4		$665,240	$53,219				$718,459	0.086	0.086	$516,247	
						Path 7			**Value**	**$985,008**	**0.125**
1	0.088	$1,000,000	$80,000	$69,029	0.05	$46,549	$195,578	0.080	0.080	$181,091	0.5
2	0.096	$884,422	$70,754	$70,824	0.05	$40,680	$182,258	0.086	0.083	$155,393	0.5
3	0.106	$772,918	$61,833	$72,666	0.05	$35,013	$169,512	0.093	0.086	$132,277	0.5
4		$665,240	$53,219				$718,459	0.100	0.090	$509,741	
						Path 8			**Value**	**$978,502**	**0.125**
										Theoretical Value	**$997,457**

Exhibit 4: Valuation of MBS for decreasing interest rate pattern

1 Year	2 R^{ref}	3 Balance	4 Interest	5 Sch Pr	6 CPR	7 Pre Pr	8 CF	9 $Z_{1,t-1}$	10 Z_{t0}	11 Value	12 Prob.
1	0.073	$1,000,000	$80,000	$69,029	0.2	$186,194	$335,224	0.080	0.080	$310,392	0.6
2	0.066	$744,776	$59,582	$59,641	0.3	$205,540	$324,764	0.075	0.077	$279,846	0.6
3	0.060	$479,594	$38,368	$45,089	0.4	$173,802	$257,259	0.070	0.075	$207,255	0.6
4		$260,703	$20,856				$281,560	0.065	0.072	$212,972	
						Path 1			**Value**	**$1,010,465**	**0.216**
1	0.073	$1,000,000	$80,000	$69,029	0.2	$186,194	$335,224	0.080	0.080	$310,392	0.6
2	0.066	$744,776	$59,582	$59,641	0.3	$205,540	$324,764	0.075	0.077	$279,846	0.6
3	0.070	$479,594	$38,368	$45,089	0.3	$130,352	$213,808	0.070	0.075	$172,250	0.4
4		$304,154	$24,332				$328,486	0.072	0.074	$246,818	
						Path 2			**Value**	**$1,009,306**	**0.144**
1	0.0727	$1,000,000	$80,000	$69,029	0.2	$186,194	$335,224	0.080	0.080	$310,392	0.6
2	0.0766	$744,776	$59,582	$59,641	0.1	$68,513	$187,737	0.075	0.077	$161,771	0.4
3	0.0696	$616,621	$49,330	$57,972	0.3	$167,595	$274,896	0.077	0.077	$219,855	0.6
4		$391,055	$31,284				$422,339	0.072	0.076	$315,032	
						Path 3			**Value**	**$1,007,050**	**0.144**
1	0.0842	$1,000,000	$80,000	$69,029	0.05	$46,549	$195,578	0.080	0.080	$181,091	0.4
2	0.0766	$884,422	$70,754	$70,824	0.10	$81,360	$222,938	0.083	0.082	$190,576	0.6
3	0.0696	$732,238	$58,579	$68,841	0.30	$199,019	$326,439	0.077	0.080	$259,002	0.6
4		$464,378	$37,150				$501,528	0.072	0.078	$371,126	
						Path 4			**Value**	**$1,001,794**	**0.144**
1	0.0727	$1,000,000	$80,000	$69,029	0.20	$186,194	$335,224	0.080	0.080	$310,392	0.6
2	0.0766	$744,776	$59,582	$59,641	0.10	$68,513	$187,737	0.075	0.077	$161,771	0.4
3	0.0806	$616,621	$49,330	$57,972	0.05	$27,932	$135,234	0.077	0.077	$108,156	0.4
4		$530,717	$42,457				$573,175	0.080	0.078	$424,282	
						Path 5			**Value**	**$1,004,602**	**0.096**
1	0.0842	$1,000,000	$80,000	$69,029	0.05	$46,549	$195,578	0.080	0.080	$181,091	0.4
2	0.0766	$884,422	$70,754	$70,824	0.10	$81,360	$222,938	0.083	0.082	$190,576	0.6
3	0.0806	$732,238	$58,579	$68,841	0.05	$33,170	$160,590	0.077	0.080	$127,415	0.4
4		$630,227	$50,418				$680,645	0.080	0.080	$499,829	
						Path 6			**Value**	**$998,910**	**0.096**
1	0.0842	$1,000,000	$80,000	$69,029	0.05	$46,549	$195,578	0.080	0.080	$181,091	0.4
2	0.0886	$884,422	$70,754	$70,824	0.05	$40,680	$182,258	0.083	0.082	$155,801	0.4
3	0.0806	$772,918	$61,833	$72,666	0.05	$35,013	$169,512	0.086	0.083	$133,371	0.6
4		$665,240	$53,219				$718,459	0.080	0.083	$523,195	
						Path 7			**Value**	**$993,458**	**0.096**
1	0.0842	$1,000,000	$80,000	$69,029	0.05	$46,549	$195,578	0.080	0.080	$181,091	0.4
2	0.0886	$884,422	$70,754	$70,824	0.05	$40,680	$182,258	0.083	0.082	$155,801	0.4
3	0.0933	$772,918	$61,833	$72,666	0.05	$35,013	$169,512	0.086	0.083	$133,371	0.4
4		$665,240	$53,219				$718,459	0.090	0.085	$518,615	
						Path 8			**Value**	**$988,878**	**0.064**
										Theoretical Value	**1,003,871**

Exhibit 5: Valuation of MBS for increasing interest rate pattern

1 Year	2 R^{ref}	3 Balance	4 Interest	5 Sch Pr	6 CPR	7 Pre Pr	8 CF	9 $Z_{1,t-1}$	10 Z_{t0}	11 Value	12 Prob.
1	0.076	$1,000,000	$80,000	$69,029	0.1	$93,097	$242,127	0.080	0.080	$224,191	0.4
2	0.072	$837,873	$67,030	$67,097	0.2	$154,155	$288,282	0.077	0.078	$247,844	0.4
3	0.069	$616,621	$49,330	$57,972	0.3	$167,595	$274,896	0.074	0.077	$220,021	0.4
4		$391,055	$31,284				$422,339	0.071	0.076	$315,492	
						Path 1			**Value**	**$1,007,548**	**0.064**
1	0.076	$1,000,000	$80,000	$69,029	0.1	$93,097	$242,127	0.080	0.080	$224,191	0.4
2	0.072	$837,873	$67,030	$67,097	0.2	$154,155	$288,282	0.077	0.078	$247,844	0.4
3	0.079	$616,621	$49,330	$57,972	0.1	$55,865	$163,166	0.074	0.077	$130,595	0.6
4		$502,785	$40,223				$543,008	0.080	0.078	$402,581	
						Path 2			**Value**	**$1,005,210**	**0.096**
1	0.076	$1,000,000	$80,000	$69,029	0.1	$93,097	$242,127	0.080	0.080	$224,191	0.4
2	0.084	$837,873	$67,030	$67,097	0.05	$38,539	$172,665	0.077	0.078	$148,445	0.6
3	0.079	$732,238	$58,579	$68,841	0.1	$66,340	$193,760	0.083	0.080	$153,857	0.4
4		$597,057	$47,765				$644,822	0.080	0.080	$474,289	
						Path 3			**Value**	**$1,000,782**	**0.096**
1	0.088	$1,000,000	$80,000	$69,029	0.05	$46,549	$195,578	0.080	0.080	$181,091	0.6
2	0.084	$884,422	$70,754	$70,824	0.05	$40,680	$182,258	0.086	0.083	$155,393	0.4
3	0.079	$772,918	$61,833	$72,666	0.1	$70,025	$204,524	0.083	0.083	$161,058	0.4
4		$630,227	$50,418				$680,645	0.080	0.082	$496,490	
						Path 4			**Value**	**$994,032**	**0.096**
1	0.076	$1,000,000	$80,000	$69,029	0.1	$93,097	$242,127	0.080	0.080	$224,191	0.4
2	0.084	$837,873	$67,030	$67,097	0.05	$38,539	$172,665	0.077	0.078	$148,445	0.6
3	0.092	$732,238	$58,579	$68,841	0.05	$33,170	$160,590	0.083	0.080	$127,518	0.6
4		$630,227	$50,418				$680,645	0.089	0.082	$496,315	
						Path 5			**Value**	**$996,469**	**0.144**
1	0.088	$1,000,000	$80,000	$69,029	0.05	$46,549	$195,578	0.080	0.080	$181,091	0.6
2	0.084	$884,422	$70,754	$70,824	0.05	$40,680	$182,258	0.086	0.083	$155,393	0.4
3	0.092	$772,918	$61,833	$72,666	0.05	$35,013	$169,512	0.083	0.083	$133,487	0.6
4		$665,240	$53,219				$718,459	0.089	0.084	$519,546	
						Path 6			**Value**	**$989,517**	**0.144**
1	0.088	$1,000,000	$80,000	$69,029	0.05	$46,549	$195,578	0.080	0.080	$181,091	0.6
2	0.097	$884,422	$70,754	$70,824	0.05	$40,680	$182,258	0.086	0.083	$155,393	0.6
3	0.092	$772,918	$61,833	$72,666	0.05	$35,013	$169,512	0.093	0.086	$132,277	0.4
4		$665,240	$53,219				$718,459	0.089	0.087	$514,839	
						Path 7			**Value**	**$983,600**	**0.144**
1	0.088	$1,000,000	$80,000	$69,029	0.05	$46,549	$195,578	0.080	0.080	$181,091	0.6
2	0.097	$884,422	$70,754	$70,824	0.05	$40,680	$182,258	0.086	0.083	$155,393	0.6
3	0.106	$772,918	$61,833	$72,666	0.05	$35,013	$169,512	0.093	0.086	$132,277	0.6
4		$665,240	$53,219				$718,459	0.100	0.090	$509,741	
						Path 8			**Value**	**$978,502**	**0.216**
										Theoretical Value	**$991,462**

Conclusion

The binomial interest rate model has become a useful practitioners' model for determining the values of bonds and bond derivatives, as well as the duration, convexity, and the option adjusted spreads of fixed-income instruments with embedded option features. The JPM skewness-adjusted binomial model is particularly applicable for cases in which interest rates are expected to be increasing (decreasing) in the future such that the end-of-the-period expected logarithmic return is positive (negative) and its skewness negative (positive). In such cases, the model estimated u and d parameters that reflect proportional increases (decreases) in each period that exceed in absolute value the proportional decreases (increases) and also where the probability of the increase (decrease) in one period is greater than 0.5. As shown in this paper, this skewness model is particularly applicable to the pricing of mortgage backed securities—securities whose discount rate, pcash flows, and expected interest rate paths are sensitive to interest-rate risk.

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The Impact of Student Background on Performance in Introductory Macroeconomics

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Abstract

This paper examines how various individual and academic characteristics such as hours worked, commute time, high school background in math and economics, high school GPA, SAT/ACT scores, race, gender, age, and course load impact student performance in introductory macroeconomics. The data was obtained from three sources: a survey of students, course records, and university student records. Significant independent variables in both the ordered logit model and the OLS model were the student's age, high school GPA, and hours worked. High school background in economics and math did not impact performance in introductory macroeconomics.

JEL Classification: A20, A22
Key Words: economic education, academic success, student background, introductory macroeconomics

Introduction

The introductory macroeconomics course at most universities is considered to be a "historically difficult course" due to the relatively high rates of students who earn grades of D or F or who withdraw from the class. This study seeks to determine the impact of both personal attributes and academic characteristics on grades earned in the introductory macroeconomics course, in order to obtain a better understanding of the characteristics of students who succeed in the course. Factors considered include hours worked, commute time, high school background in math and economics, high school GPA, SAT/ACT scores, race, gender, and course load. The data was obtained from three sources: a survey of students, course records, and university student records. Results are presented for two regression models: ordered logit and ordinary least squares (OLS).

Review of the Literature

The existing research literature suggests that many factors beyond the instructor's control do impact academic success in courses at the university level. Some of those external factors include the intelligence level of a student, how many hours a student works, the student's living situation, whether or not a student has taken an economics course in high school, and the student's math background. Previous research has investigated these and other factors. However, some of those studies yield conflicting results.

Gershenfeld et al. (2016) focused their study on underrepresented students and determined that first-semester GPA was a better predictor of college success, as defined by the six-year graduation rate, than other composite measures, such as an ACT score. Kassis and Boldt (2020) later concluded that both high school (or college) GPA and standardized test scores, as well as participation in an honors program, amount of earned credit hours, and access to financial support from their family were all positively associated with academic performance in introductory college economics courses. Anderson et al. (1994) again found grades to be an important determinant for success in college introductory economics courses, but their research indicated it was specifically those grades earned in the final year of high school.

Interestingly, Anderson et al. (1994) also concluded that completing a high school calculus class supports later success in economics courses. Hoag and Benedict (2010) similarly concluded that the range of math skills a student brings in has a statistically significant impact on their performance in economics courses. However, work done by Lopus and Maxwell (2001) and then Andrews et al. (2014) found that completion of high school economics courses did not impact performance in college economics courses.

Another external factor that's been studied is student residence in college. Turley and Wodtke (2010) determined that for most students, the type of residence in which they reside does not significantly impact first-year college performance. However, their research did suggest that black students living on campus have a significantly higher GPA.

Other studies have investigated the relationship between the number of hours a student works and the potential impact on academic achievement. Again, there are conflicting results. The work of Kalenkoski and Pabilonia (2010) found that the number of hours worked negatively impacted the GPA of first-semester students. Additionally, their research showed that this adverse effect was greater at two-year than four-year colleges. Body et al. (2014) also found a negative impact on student success, specifically for students who work more than 8 hours each week. On the other hand, this group also found that flexibility regarding employment could somewhat offset the adverse effect of hours worked. Based on this, they suggest that factors such as additional access to financial aid and improved flexibility in class offerings might help students perform at higher levels in college. However, Dundes and Marx (2006) indicated that minimizing work hours may not be optimal for students. These researchers found that undergraduates working 10-19 hours per week outperformed all others.

And lastly, some studies examined the impact of scheduling differences. For example, Dills and Hernandez-Julian (2008) found two scheduling scenarios to be superior for student performance. The first was classes held later in the afternoon and the second was for those classes that meet more often, such as three days a week for shorter time slots versus two days a week with longer sessions. However, Carrington (2010) yielded different results. Here, the focus was on intermediate accounting courses, and student success was not statistically different for students enrolled in compressed scenarios (such as shorter summer or one-day-per-week courses) compared to typical semester two-days-per-week courses. However, this study concluded that schedules of more frequent class sessions, such as three days per week, were significantly less successful in intermediate accounting compared to the more compressed offerings.

Data

Table 1 lists, describes, and provides summary statistics for each variable used in the regression analysis. The data was derived from three sources: a survey of students, course records, and university student records. Survey data was initially obtained from a total of 158 students in face-to-face and online sections of Principles of Macroeconomics courses taught by three instructors in Fall 2018. Questions on topics such as high school attended, living situation, work hours, high school math background, and previous economics courses were asked in the survey. A copy of the survey is included in the appendix. This survey data was supplemented with student data available from the university as well as course data. Information such as high school GPA, SAT/ACT scores, race, gender, age, and course load was obtained from university records, while the exam average was obtained from course records. Each student signed a waiver agreeing to participate in the study, and the research project received approval from the university's Institutional Review Board (IRB). Data on all the independent variables used in the analysis was available for 132 students. The summary statistics in Table 1 are for these 132 students.

Table 1: Data Description

Variable	Definition	Mean	Std. Dev.	Min	Max
GRADEWEIGHT	1=D, F or W; 2=C; 3=A or B	2.22	0.765	1	3
EXAMAVG (n=122)	Average grade on all exams	66.01	13.01	33.2	95.8
BUSMAJOR	1=Business Major	0.811	0.393	0	1
INS1	Dummy for Instructor1	0.045	0.209	0	1
INS2	Dummy for Instructor 2	0.727	0.447	0	1
HONORS	1=Honors section	0.053	0.225	0	1
ONLINE	1= Online section	0.098	0.299	0	1
11AMClASS	1=11am class	0.402	0.492	0	1
AGE	Age of student (in years)	20.60	2.222	18	35
MALE	1=Male	0.424	0.496	0	1
BLACK	1=Black	0.394	0.490	0	1
TOTALHRS	Total semester hours	13.92	1.921	9	18
HSGPA	High school GPA	3.246	0.456	2.24	4
SAT*	Student SAT score	959	95.13	790	1260
HOURSWORKED	Hours worked per week	14.27	13.85	0	45
COMMUTE	Commute time to campus (in minutes)	10.99	16.37	0	75
APECON	1=AP econ course	0.106	0.309	0	1
HSCALC	1=Highest high school math: calculus (AP or regular)	0.167	0.374	0	1
HSSTAT	1=High school stat course	0.265	0.443	0	1
COLMATH	1=College math course in high school	0.136	0.344	0	1
MICROFIRST	1=student completed Principles of Microeconomics	0.394	0.490	0	1
EARNEDHRS	Total institutional earned hours	33.79	25.58	0	124
RESREQ	1=Student's high school in zip code requiring on-campus residence during freshmen year	0.591	0.494	0	1
PUBLIC	1=Student attended a public high school	0.955	0.209	0	1

*ACT scores were adjusted to SAT equivalent. n=132

Regression Results

Tables 2 and 3 present the regression results on the impact of various student background and course variables on the probability of success in Principles of Macroeconomics. In Table 2, the measure of success is the student's final grade in the course. In the sample, 13.6% of students earned a final grade of A, 28.8% earned a grade of B, 37.1% earned a grade of C, 10.6% earned a D, 6.1% earned an F, and 3.8% withdrew from the class. Due to the small number of students earning an F or withdrawing from the course in this sample, grades of F and course withdrawals (W) are combined with grades of D into one category indicating a lack of success in the course. This is consistent with university policy which does not consider a letter grade of D in the course as a truly successful outcome as it may keep the student from being able to take upper-level courses and possibly prevent the student from achieving the necessary GPA to graduate. In addition, the DFW rate, which measures the percentage of students earning a grade of D or F or withdrawing from the course, is a metric that is often used when examining issues related to student progression. Due to the relatively low number of students earning an A, letter grades of A and B were combined into one grade category.

The dependent variable is GRADEWEIGHT, which is equal to 3 if the student earned a grade of A or a B, 2 if the student earned a C, and 1 if the student earned a D, F, or W. Given that the dependent variable is ordered, an ordered logit approach as described in Greene (1993) was used to estimate the model. The estimation equation is:

$$y^* = \beta' x + \varepsilon$$

where y^* is an unobserved measure of student success in Principles of Macroeconomics. The measure of success used is the student's letter grade:

$y = 1$ (grade D, F, or W) if $y^* \leq \mu_1$
$y = 2$ (grade C) if $\mu_1 < y^* \leq \mu_2$
$y = 3$ (grade B or A) if $y^* > \mu_2$

Table 2: Ordered Logit Regression

	Coef.	Std. Err.	z	P>z
BUSMAJOR	1.6944	0.6236	2.72	0.007***
INS1	-1.7383	1.5003	-1.16	0.247
INS2	0.4195	0.5816	0.72	0.471
HONORS	1.7579	1.2052	1.46	0.145
ONLINE	1.4739	1.1440	1.29	0.198
11AMCLASS	-0.3383	0.4665	-0.73	0.468
AGE	0.3884	0.1373	2.83	0.005***
MALE	0.3404	0.4106	0.83	0.407
BLACK	-0.2104	0.4835	-0.44	0.663
TOTALHRS	0.2995	0.1257	2.38	0.017**
HSGPA	2.5999	0.5491	4.73	0.000***
SAT	-0.0014	0.0023	-0.59	0.553
HOURSWORKED	-0.0329	0.0171	-1.93	0.054*
COMMUTE	0.0342	0.0165	2.08	0.038**
APECON	-0.0017	0.6973	0	0.998
HSCALC	-0.0557	0.5855	-0.1	0.924
HSSTAT	0.5695	0.5077	1.12	0.262
COLMATH	0.9489	0.7796	1.22	0.224
MICROFIRST	-0.2634	0.5319	-0.5	0.620
EARNEDHRS	0.0056	0.0103	0.55	0.586
RESREQ	0.7726	0.5271	1.47	0.143
PUBLIC	-0.9936	1.0898	-0.91	0.362
μ_1	18.7605	5.0087		
μ_2	21.3245	5.1114		
Pseudo R^2	0.2551			

Dependent Variable: GRADEWEIGHT. n=132
*Significance at the 0.1% level, ** Significance at the 0.05% level, *** Significance at the 0.01% level

The μ are unknown threshold values that will be estimated. In the ordered logit regression, a positive and significant β coefficient implies that an increase in the independent variable increases the probability of getting an A or a B in the Principles of Macroeconomics course and decreases the probability of getting a D, F, or W in the course. A negative and significant β

coefficient indicates that an increase in the independent variable would reduce the probability of getting an A or B in the course and increase the probability of getting a D, F, or W.

The regression results presented in Table 2 look at the sample of 132 students for which all independent variables were present in the data. These results indicate that a student's probability of earning an A or a B in Principles of Macroeconomics is positively and significantly related to their age, being a business major, the total number of hours taken during a semester, high school GPA, and commute time. On the other hand, working more hours lowers the probability of a student earning an A or a B (and increases the probability of earning a D, F, or W). None of the other variables in the regression had a significant impact on the student's probability of earning a high grade in the course. Of particular note is the fact that the only high school related variable that is significant is the high school GPA. The student's math and economic background and standardized test scores were not significant nor was whether they attended a public high school.

For the regression results in Table 3, the measure of success is the student's average proctored exam score (EXAMAVG). The number of exams differed in the various sections of the course as did the weight of exams in the calculation of the final grade[1]. This regression is estimated using a standard ordinary least squares (OLS) model. For this regression, students who did not take all the exams in the course were excluded, which resulted in a sample of 122 students. The results for this model suggest that the factors impacting success on proctored exams are somewhat different from the factors that impacted success in terms of the final letter grade in the course. Age and high school GPA were positive and significant in both regressions, and hours worked were negative and significant in both regressions. This suggests that older students and students with better high school grades were more likely to earn higher exam scores as well as better final grades while working more hours lowered both the probability of higher exam scores as well as getting better letter grades in the course. However, being a business major, taking more hours during a semester, and having a longer commute, which were positive and significant in the regression using final course grades as the measure of success, were not significant variables in the regression using proctored exam average as the measure of success. On the other hand, having higher standardized test scores, being enrolled in the honors section, and being a male had a positive and significant impact on the proctored exam average as did taking the class from Instructor 1. Taking the class in an online format had a negative and significant impact on the proctored exam average. None of these variables were significant in the regression that used final course grade as the measure of success.

Table 3: OLS Regression

	Coef.	Std. Err.	t	P>t
BUSMAJOR	4.0206	3.2509	1.24	0.219
INS1	19.7147	7.7605	2.54	0.013**
INS2	2.1031	2.9989	0.70	0.485
HONORS	9.6442	5.6664	1.70	0.092*
ONLINE	-19.8995	4.9426	-4.03	0.000***
11AMCLASS	-1.0103	2.4207	-0.42	0.677
AGE	1.4220	0.6611	2.15	0.034**
MALE	4.1109	2.1024	1.96	0.053*
BLACK	-1.4305	2.4976	-0.57	0.568
TOTALHRS	0.4182	0.6132	0.68	0.497
HSGPA	6.8679	2.4723	2.78	0.007***
SAT	0.0346	0.0124	2.79	0.006***
HOURSWORKED	-0.1537	0.0853	-1.80	0.075*
COMMUTE	0.0226	0.0711	0.32	0.751
APECON	1.1201	3.2095	0.35	0.728
HSCALC	0.3794	2.8041	0.14	0.893
HSSTAT	1.6339	2.5552	0.64	0.524
COLMATH	3.2914	3.4357	0.96	0.340
MICROFIRST	1.5613	2.5943	0.60	0.549
EARNEDHRS	-0.0530	0.0535	-0.99	0.324
RESREQ	-0.4610	2.5952	-0.18	0.859
PUBLIC	-1.2231	4.9396	-0.25	0.805
CONSTANT	-26.2174	25.2512	-1.04	0.302
R^2	0.4703			
Adjusted R^2	0.3526			

Dependent Variable: EXAMAVG. n=122

*Significance at the 0.1% level, **Significance at the 0.05% level, *** Significance at the 0.01% level

Conclusions

Regression results on the overall performance in the Principles of Macroeconomics courses examined in this research are summarized in Tables 2 and 3. In the ordered logit model (Table 2) with GRADEWEIGHT as the dependent variable, six independent variables were significant (BUSMAJOR, AGE, TOTALHRS, HOURSWORKED, COMMUTE, and HSGPA). In the OLS model (Table 3) with EXAMAVG as the dependent variable, eight independent variables were significant (INS1, HONORS, ONLINE, AGE, MALE, HSGPA, SAT, and HOURSWORKED). The results suggest that older students and students with a higher high school GPA were more likely both to get a higher letter grade and score better on the proctored exams, while students who worked more hours were less likely to get an A or a B in their Principles of Macroeconomics class and also scored lower on their proctored exams. A notable exception to anticipated expectations was the positive coefficient on TOTALHRS, which suggests that students taking more hours during the semester also performed better in their Principles of Macroeconomics course. The positive relationship between total course hours and overall performance in the course could reflect academically stronger students who are more likely to do well in the course choosing to take more hours per semester. However, taking more hours did not have a positive impact on proctored exam scores.

Other differences between the two regressions are also of interest. While business majors and students with longer commutes were more likely to get a higher letter grade, these characteristics were not associated with higher proctored exam grades. Since Principles of Macroeconomics is a required class for all business majors at this university and business majors must have a GPA of at least 2.0 after their first 45 hours in order to enroll in upper-level business classes, the difference in the impact of being a business major on final letter grade compared to exam average could reflect an incentive effect that motivates business students to focus more on earning a high letter grade rather than specifically on their exam average.

Although higher standardized test scores, being in an honors section, being male, and taking the class from instructor 1 were all associated with higher proctored exam scores, they had no impact on the probability of earning a higher letter grade in the course. The relationship between standardized test scores and proctored exam scores may reflect better test-taking skills or less test anxiety, which could impact performance on tests in general, but may not have as much of an impact on the letter grade. The positive impact of being male is consistent with the results in previous research by Anderson, Benjamin, and Fuss (1994).

While taking the course online was associated with lower exam scores, the online format had no impact on the probability of earning a higher final grade in the class. The instructor effect and the impact of taking the course online may be at least partially related to differences in the weighting of exams in the online sections. Instructor 1 only taught an online course, and the two proctored exams in this class accounted for 50% of the final grade. In the other online courses taught by Instructor 2, there was one proctored exam that was worth 20% of the final grade. The results indicate that online students overall had a lower proctored exams average than face-to-face students, but students in the section where proctored exams were a higher percentage of their final grades performed relatively better on their proctored exams. This result may reflect an incentive effect associated with the higher weighting of the exams in instructor 1's course motivating students to put more effort into scoring well on the high-stakes exams.

Some of the insignificant variables included in the regressions are also worth noting. Insignificant variables in the regressions summarized in Tables 2 and 3 included math-related variables (HSCALC, HSSTAT, and COLMATH) and previous economics course variables (MICROFIRST and APECON). Based on previous studies, there was an expectation that the math background of students and the completion of previous courses in economics such as AP high school economics or another economics course in college, such as Principles of Microeconomics, would positively impact performance in Principles of Macroeconomics. The results fail to show such a relationship for either measure.

The results presented in this paper suggest several avenues for future research. This study focuses on Principles of Macroeconomics at a single institution. It would be useful to see if these conclusions hold at other universities and in Principles of Microeconomics courses. The results suggest that having taken higher-level math in high school and previous economics courses both appear to have no impact on student performance in a college Principles of Macroeconomics course. This research could be expanded to include the grades in these courses to see if students who do better in their previous math and economics classes later perform better in Principles of Macroeconomics. It would also be useful to further explore the differences in the factors that impact the two measures of success – final course grades and proctored exam averages. The results suggest that course-related variables, such as instructor and course format (online and honors), seem to significantly impact student performance on exams, but not their performance in terms of final grades. Future research to examine how different grade weightings in a course may impact student incentives to succeed on exams relative to their overall course grade might provide additional insights into the results of this paper.

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Appendix
Student Survey

I. Background Questions

1. What is the name of your high school from which you graduated? ______________________________ .
2. Do you live on campus?
   _____ Yes
   _____ No (if you checked this box, indicate how long it takes you to drive to the UWG campus________________)
3. Do you work?
   _____ Yes (if you checked this box, indicate how many hours you work per week________________)
   _____ No

II. Questions on Economics and Mathematics Courses

4. Did you take an economics course as a high school student?
   _____ Yes (if yes, indicate what grade you were in when you took economics ________________)
   _____ No
5. If you took an economics course in high school, which of the following best describes that course?
   _____ A general economics course that covered topics such as macroeconomics and microeconomics
   _____ AP (Advanced Placement) Microeconomics
   _____ AP (Advanced Placement) Macroeconomics
   _____ College Principles of Macroeconomics (as a dual enrolled student)
   _____ College Principles of Microeconomics (as a dual enrolled student)
   _____ Other (please describe __________________________________)
6. What is the highest level of math you took in high school?

_____ Algebra 1	_____ Algebra 2	_____ Pre-Calculus	_____ AP Calculus
_____ Geometry	_____ Algebra /Trigonometry	_____ Calculus	

7. Did you take a high school statistics course? _____ Yes _____ No
8. While you were a high school student, did you take any math course(s) at the college level as a dual enrolled student?
   _____ Yes (if yes, which course(s) did you take?)____________________________________ . _____ No
9. Before taking this Principles of Macroeconomics course, did you take Principles of Microeconomics as a college student?
   _____ Yes _____ No
10. Have you previously taken a Principles of Macroeconomics course as a college student? _____ Yes _____ No

Post-financial Crisis Diversification Benefits of Real Estate

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Abstract

This study examines the empirical evidence on the diversification benefits of adding investable real estate exposure to an equity portfolio in the post-financial crisis period. The paper finds that one-half of real estate volatility is driven by idiosyncratic risk. This study finds that in the post-financial crisis period the correlation between real estate and stocks does not decline. This paper's findings conflict with the reported decline in correlation in the post-1987 stock market crash period. The paper concludes that real estate offers diversification benefits in the post-financial crisis period.

JEL Codes: G11, G15
Keywords: portfolio management, real estate exposure, equities, crisis

Introduction

This study examines recent empirical evidence to determine whether adding real estate exposure to a diversified equity portfolio can enhance portfolio performance in the post-financial crisis period. The extant literature reports several shifts in diversification benefits of real estate over the last 40 years, including immediately subsequent to the stock market crash of 1987. The correlation trend in the post-financial crisis period is compared to the post-1987 stock market crash trend.
Real estate returns have long been reported to be uncorrelated with equity returns, making real estate a potentially good diversification candidate. The empirical evidence in the extant literature supports the diversification benefits of real estate in the runup to the financial crisis. Yet there is still a dearth of evidence on the diversification benefits of real estate exposure in the post-financial crisis period for the investor who holds an internationally diversified developed markets portfolio.

The investability of real estate has been facilitated, since 2000, by real estate securitization in the form of real estate investment trusts (REITs). The two real estate vehicles examined herein are large, liquid, popular and investable alternatives by which investors can add real estate exposure via funds which invest in REITs. This paper presents direct evidence on the diversification benefits of real estate in the post-financial crisis period. This study also decomposes real estate volatility, and tests for a trend in correlation in the post-financial crisis period.

Background and Literature Review

Markowitz's (1952) mean-variance investor is in pursuit of a diversified portfolio which offers the best risk-return tradeoff—in this spirit this study examines the diversification benefit of augmenting an equity portfolio with real estate exposure. Goetzmann and Ibbotson (1990), and Goyenko and Keim (1992) report similar levels of return for stocks, and for REITs. In addition to these similar levels of return, both Ibbotson and Siegal (1984), and Brueggeman et al. (1984), report a low correlation between real estate and stock returns. The similar level of return and the low correlation suggest that the addition of real estate to an equity portfolio may offer diversification benefits. Furthermore, Chandrashekaran (1999) offers evidence that the addition of REITs does not diminish portfolio performance, while Bouri et al. (2022) report contagion across REITs and equities.

This analysis contributes to the literature's understanding of the diversification benefits of adding real estate exposure to a portfolio of developed market equities—as the standard portfolio today often includes both U.S. equities and other developed market equities. A developed market portfolio comprised of both U.S. equities as well as equities from the balance of the developed markets is used to construct a developed market "benchmark" portfolio—to be broadly consistent with the portfolio that a U.S. investor interested in exposure to developed market equities worldwide might hold. However, most of the extant literature examines the diversification benefits of adding real estate exposure to a portfolio which is restricted to U.S. stocks. Therefore, in addition to examining diversification benefits for a portfolio of developed market equities, this paper also reports on the diversification benefits for a portfolio comprised exclusively of U.S. stocks. Examining the empirical results for both a U.S. stock portfolio, and for an internationally diversified developed markets stock portfolio allows for the direct comparability of this paper's findings for a U.S. stock portfolio with the findings in the extant literature.

Understanding the diversification benefits of real estate is, of course, more complicated than simply observing similar levels of risk and return for equities and for real estate and concluding that real estate offers diversification benefits. Anderson et al. (2021) find structural changes in correlation across time while Goetzmann and Ibbotson (1990) and Huang and Zhong (2013) report that REIT returns are time-dependent. It follows that while REITs may earn similar average returns as stocks, experience similar risk levels, and exhibit a low correlation with equities yet the relationship between the returns of real estate and equities may not be stable. While Lee and Stevenson (2005) report real estate diversification benefits in the runup to the financial crisis, potential instability in the correlation highlights the import of examining diversification benefits in the post-financial crisis period.

The addition of REITs to stock indices began in late 2001, thereafter biases the estimation of the correlation between stocks and REITs upwards. Nonetheless, Huang and Zhong (2013), Westerheide (2006), and Cotter and Stevenson (2006) all find that rolling estimates of correlation generally increased from 1999 through 2005. Furthermore, Lee and Stevenson (2005) report real estate diversification benefits in the runup to the financial crisis. Case et al. (2012) report that the correlation with U.S. stocks rises towards, and finally reaches 0.59 by late 2008. In addition, Lu et al. (2013) report that the diversification benefits of REITs eroded during the financial crisis. Notwithstanding the early evidence of time-varying benefits, and the reported erosion of diversification benefits during the financial crisis, there is a dearth of evidence on the diversification benefits post-financial crisis.

Case et al. (2012) report that correlation between real estate and U.S. stocks reached its high of 0.59 by late 2008. This paper finds that for the post-financial crisis period the correlation remained near that high, with no evidence of a drift over time. For the post-financial crisis period this study finds correlations with U.S. stocks of 0.570 and 0.585, for the two real estate proxies, respectively. This study finds that subsequent to the aforementioned increase in the correlation during the runup to the financial crisis, the correlation between U.S. stocks and real estate did not return to its pre-financial crisis level but rather remained elevated thereafter. This finding that the post-financial crisis correlation did not return to its pre-financial crisis level is in conflict with the Conover et al. (2002) finding of falling correlation in the post-1987 market crash period. Furthermore, this analysis disaggregates real estate volatility and finds that over one-half of real estate volatility is idiosyncratic. Nonetheless, while higher correlation reduces diversification benefits, this study finds that real estate still offers diversification benefits post-financial crisis.

Data

This study analyzes the proportion of REIT volatility which is idiosyncratic, and it also analyzes the correlation between REITs and both U.S. equities, and the constructed Benchmark portfolio, respectively. The study further examines the time trend in the correlation between REITs and the Benchmark portfolio. The returns of two popular, accessible, and investable real estate vehicles are examined. These investment vehicles are broadly representative of investable real estate alternatives. Ticker VFINX is used for the U.S. equity market proxy. The balance of the developed-market equities is represented by the Morgan Stanley Capital Index, ticker EFA, representing the Exchange Traded Fund for Europe, Asia and Far East. The two real estate investment vehicles are the Vanguard real estate fund, ticker VGSIX, and the National Association of Real Estate Investment Trusts NAREIT all-REIT index. VGSIX is one of the largest low-cost, widely-diversified real estate funds, and is accessible to all investors. The NAREIT all-REIT index is a widely followed investable index of REIT returns. VFINX is one of the largest broadly-diversified funds, and EFA is a broadly-diversified fund investing in developed market equities excluding the United States. All of these funds are liquid, and available to the average investor at relatively low cost. These investments were chosen for the analysis due to the fact that they reflected the alternatives that the study is designed to examine and are investable.

Returns are examined over the April, 2009 to June, 2020 interval. Daily data is available for VFINX, EFA and VGSIX, but only monthly data is available for the NAREIT all-REIT index. Data is from Yahoo Finance.

Analysis

Potential diversification benefits of real estate are analyzed in the context of an otherwise internationally diversified developed market equity portfolio. A portfolio is constructed using two developed market benchmarks. The constructed benchmark portfolio facilitates the analysis of the extent, if any, to which real estate diversification benefits are realizable for the investor who otherwise holds a diversified developed market equity portfolio. During the interval examined herein the World Bank market capitalization statistics indicate that the market capitalization of U.S. firms comprised approximately 30% of world market capitalization.

The constructed benchmark portfolio is weighted 30% in VFINX and 70% in EFA, and the benchmark portfolio is denoted as BM. These percentages approximate the respective proportion of global market capitalization which each comprises.

Table 1 presents the results for the funds, and for the benchmark portfolio. The risk-return profile of the benchmark portfolio is compared to the profile of the augmented-benchmark portfolio formed with 90 percent invested in the benchmark and 10 percent invested in real estate.

The monthly volatilities range from 4.394 %, for VFINX, to 5.748 % for VGSIX. VFINX has the highest return at 0.935 %, followed by VGSIX at 0.931 %, NAREIT at 0.904 %, and EFA with the lowest at 0.532 %.

Table 1: Descriptive Statistics Monthly Returns n = 145

Investment	Mean	Std Dev	Min	Max	Sharpe
VFINX	0.935	4.394	-21.727	13.267	
EFA	0.532	4.885	-20.835	13.191	
VGSIX	0.931	5.748	-31.231	31.312	
NAREIT	0.904	5.042	-30.266	30.813	
Portfolio: BM	0.569	4.592	-19.622	10.931	0.085
Portfolio: BM+VGSIX	0.604	4.577	-20.783	12.969	0.091
Portfolio: BM+NAREIT	0.602	4.566	-20.682	12.635	0.092

Ticker VFINX is the U.S. equity market proxy. The Morgan Stanley Capital Index, ticker EFA, proxies for the developed-market equities outside the U.S., it is an Exchange Traded Fund for Europe, Asia and Far East. Vanguard real estate fund, ticker VGSIX, and the National Association of Real Estate Investment Trusts NAREIT all-REIT index proxy for real estate exposure, respectively. Vanguard Fund, ticker VGSIX is one of the largest low-cost, widely-diversified real estate funds. NAREIT is the all-REIT index is a widely followed investable index of REIT returns. BM is the constructed benchmark portfolio invested 30% in VFINX, and 70% in EFA. BM+VGSIX is constructed with 90 % in BM and 10% in VGSIX. BM+NAREIT is constructed 90 % in BM and 10% in NAREIT.

Table 2: Correlation coefficients using monthly returns

	VFINX	EFA	VGSIX	NAREIT	BM
VFINX	1.000	0.872	0.570	0.585	0.930
EFA		1.000	0.659	0.679	0.991
VGSIX			1.000	0.995	0.679
NAREIT				1.000	0.700
BM					1.000

Ticker VFINX proxies for the U.S. equity market. The Morgan Stanley Capital Index, ticker EFA, proxies for the developed-market equities outside the U.S., it is the Exchange Traded Fund for Europe, Asia and Far East. Vanguard real estate fund, ticker VGSIX, and the National Association of Real Estate Investment Trusts NAREIT all-REIT index proxy for real estate exposure, respectively. Vanguard Fund, ticker VGSIX is one of the largest low-cost, widely-diversified real estate funds. NAREIT is the all-REIT index is a widely followed investable index of REIT returns. BM is the constructed benchmark portfolio invested 30% in VFINX, and 70% in EFA.

Return correlations are presented in Table 2. VGSIX is positively correlated with VFINX at 0.570, with EFA at 0.659, and with the Benchmark portfolio, at 0.679. NAREIT is positively correlated with VFINX at 0.585, with EFA at 0.679, and with the Benchmark portfolio, at 0.700. NAREIT and VGSIX are positively correlated at 0.995. Therefore the correlation estimate for REITs and U.S. Equities which is most directly comparable with the correlation estimates reported in the extant literature for the post-1987 market crash period, are 0.570 and 0.585 for the post-financial crisis for VGSIX and NAREIT, respectively.

Real estate-augmented benchmark portfolio results are presented in Table 1. The benchmark portfolio is augmented with the two real estate funds, respectively, and referred to as the VGSIX-augmented portfolio, and the NAREIT-augmented portfolio, respectively. The risk-return characteristics of the benchmark portfolio is compared to that of the two augmented-benchmark portfolios. Augmenting the benchmark portfolio with real estate increases return by about 3 basis points, for both VGSIX and NAREIT, respectively. Augmenting the benchmark also reduces portfolio volatility by 2 basis points, for both VGSIX and NAREIT, respectively.

Sharpe Ratios

Risk-adjusted portfolio returns are examined using the Sharpe ratio (Sharpe, 1964) and results are reported in Table 1. Augmentation of the benchmark with REITs increases portfolio return and decreases portfolio risk, and impacts the Sharpe ratio accordingly. The Sharpe ratio is 0.085 for the Benchmark, 0.091 for the VGSIX-augmented benchmark, and 0.092 for the NAREIT-augmented benchmark. The Sharpe ratios for the VGSIX-augmented portfolio and the NAREIT-augmented portfolio are therefore respectively 7%, and 8%, greater than that of the benchmark portfolio.

Real Estate Volatility and World Markets

Real estate variance is decomposed with respect to U.S. returns and ex-U.S. developed market returns, respectively. Inclusion of real estate reduces portfolio risk if real estate return volatility is not driven by global factors, but is instead idiosyncratic.

To facilitate the decomposition of real estate returns, the returns of EFA are first orthagonalized with respect to the returns of VFINX, following Chen and Ho (2009). The orthagonalized EFA return is the residual from the regression of EFA returns on VFINX returns. The following model is estimated wherein VFINX is benchmark asset 1, and the orthagonalized EFA residual is benchmark asset 2:

$$R_{i,t}^{fund} = \alpha + \beta_1 R_{1,t} + \beta_2 R_2 + \varepsilon_t. \quad (1)$$

$R_{i,t}^{fund}$ is the return on real estate fund *I*, in period *t*, and $R_{1,t}$ and $\mathrm{R}_{2,\mathrm{t}}$ are the returns on benchmark asset 1, and the orthagonalized residual from the regression of benchmark asset 2 returns on benchmark asset 1 returns, respectively. Equation (1) disaggregates the variance of real estate into the proportions attributable to benchmark asset 1, benchmark asset 2, and the idiosyncratic proportion, respectively.

Table 3: Monthly Returns Variance Decomposition: percentage of real estate return variance explained

Fund	U.S. (VFINX)	World ex-U.S. (EFA)	Idiosyncratic
VGSIX	44.2 %	2.7 %	53.1 %
NAREIT	47.3 %	2.7 %	50.0 %

Continuing with equation (1), the variance of R_i^{fund} can be written as:

$$Variance\ R_i^{fund} = \beta_1^2 Variance\ (R_1) + \beta_2^2 Variance(R_2) + Var(\epsilon). \quad (2)$$

Using the parameters estimated in equation (1), the proportion of VGSIX's variance which is attributable to benchmark asset 1 is written $[\beta_1^2 Variance(R_1)/Variance\ (R_i^{fund})]$ and the proportion attributable to asset 2 is written $[\beta_2^2 Variance(R_2)/Variance\ (R_i^{fund})]$, while the idiosyncratic proportion is written $[Variance(\varepsilon)/Variance\ (R_i^{fund})]$. Results are reported in Table 3.

The variance of VGSIX returns are attributable 44.2% to VFINX, 2.7% to EFA. Idiosyncratic risk therefore explains 53.1 % of the variance of VGSIX's returns. The variance of NAREIT-all equity REIT returns are attributable 47.3 % to VFINX, and 2.7 % to EFA, while idiosyncratic risk explains 50.0 % of the variance of NAREIT-all equity REIT returns.

Time Trend in Market Integration

Market integration dynamics are examined to understand how returns on real estate and the returns on developed market benchmarks are related. Two proxies for market integration are examined using daily returns.

Following the approach of Berger et al. (2013), market integration dynamics are examined by estimating the time trend using 44-day rolling windows. The market integration proxies from each of the windows are regressed on a time trend variable to estimate a linear time trend. Stepwise autoregressive regressions are fit to this time series, including up to 44 lags, in which only significant parameters are retained.

The first proxy is an estimate of the fund volatility which is explained by the two developed market benchmarks: VFINX, and EFA. The first proxy for market integration is the proportion of the VGSIX fund return variance explained by VFINX and EFA. This proxy is an estimate of the proportion of the fund's variance explained by the developed market benchmarks estimated in equation (1). The proportion is written 1-$\left(Variance(\epsilon)/Variance\left(R_i^{fund}\right)\right)$ from the results of the variance decomposition described in equation (2). This proportion is the R-squared from the results of the estimation of the regression in equation (1), referred to as Model 1 in Table 4. The second proxy is the correlation between the fund's return and the return on the 30/70 developed market benchmark, referred to as Model 2 in Table 4. Each market integration variable is calculated over a 44-day rolling window (calculated across day *t-43* through day *t*, where *t* is a trend variable). The following model:

$$MarketIntegration_{j,t-43,t-0} = a_j + b_j t + e_{j,t} \quad (3)$$

Where $MarketIntegration_{j,t-43,t-0}$ is either one of the two market integration proxies, indexed on j, which is defined as above for the fund, and is calculated from day *t*-43 through day *t*. Model fitted values are fitted with stepwise autoregression, including up to 44-lags, with only significant autoregressive parameters retained. P-values for the test of the hypothesis that the coefficient is equal to zero appear in parenthesis. Higher values for either of the market integration proxies suggests a tighter relationship between real estate and the developed market equity benchmarks.

Table 4: Market integration trend: daily returns

	Model 1		Model 2	
	Intercept	Coefficient	Intercept	Coefficient
VGSIX	0.5267	-0.0002	0.7520	-0.0005
	(0.0003)	(0.6428)	(0.0001)	(0.0706)

Ticker VFINX proxies for the U.S. equity market. Vanguard real estate fund, ticker VGSIX. The Morgan Stanley Capital Index, ticker EFA, proxies for the developed-market equities outside the U.S., it is the Exchange Traded Fund for Europe, Asia and Far East. The dependent variable in Model 1 is the R-squared from the results of the estimation of the regression of VGSIX on VFINX and the orthagonalized EFA residual, as described in equation (1). The dependent variable in Model 2 is the correlation between the VGSIX and the return on the constructed benchmark invested 30% in VFINX and 70% in EFA.

Parameter estimates are reported in Table 4. The time-trend parameter for the VGSIX fund is (-) 0.0002 (p-value <0.6428) for the explained variance trend, and is (-)0.0005 (p-value < 0.0706) for the correlation trend analysis. Therefore, the results of the market integration time trend analyses suggests that, if anything, the relationship between real estate and the developed benchmarks is not tightening over time.

The results suggest that real estate returns have a high proportion of idiosyncratic risk, as presented in Table 3, and that it is to a large degree independent of developed market benchmarks.

The results of the variance decomposition are consistent with the findings of the risk-return analyses. Real estate can be a successful hedge in the current environment. At least 50 percent of total real estate volatility is driven by idiosyncratic risk. The proportion of volatility associated with idiosyncratic factors has not diminished over time.

The analysis of risk and return finds that the addition of real estate exposure marginally increases the return, and decreases the volatility, of an otherwise internationally-diversified developed market portfolio. Augmenting an internationally diversified developed market portfolio with real estate increases the portfolio's Sharpe ratio by about 7%. Results offer evidence to support the assertion that adding investable real estate to an otherwise-diversified international portfolio can improve portfolio performance.

Conclusion

Comparing the correlation of U.S. stocks and real estate in the post-financial crisis period to the results of the post-1987 U.S. stock market crash period yields contrasting results. The correlation between U.S. stocks and real estate in the runup to both the financial crisis and to 1987 U.S. stock market crash experienced an increase, as reported by Conover et al. (2002), and by Case et al. (2012), respectively. However, Conover et al. (2002) find that in the post-1987 U.S. stock market crash period the correlation between real estate and U.S. stocks declined to its earlier level. In contrast the study finds that the correlation remains at an elevated level in the post-financial crisis period and exhibits no evidence of a trend. Therefore the correlation trend in the post-financial is unlike that in the post-1987-crash period. Correlation has remained elevated post-financial crisis, which diminishes the diversification benefits to some extent, but not entirely, given the analysis herein.

The pursuit of improved portfolio performance motivates the consideration of the addition of real estate to a diversified portfolio to improve portfolio performance. Existing research on the inclusion of real estate in a diversified portfolio is incomplete, and analysis of returns in the post-financial crisis period is sparse. Furthermore, much of the literature analyzing the diversification effects of real estate exposure examines non-investable real estate returns. In contrast, diversification benefits of investable REIT returns are examined here, and results have been juxtaposed with the results reported in the extant literature.

Brueggeman and Fisher (1993) report that equity REITs offer diversification benefits, while Chandrashekaran (1999) reports that the addition of REITs to a diversified fixed income and equities portfolio to yield performance only similar to that of a domestic equity portfolio. Lee and Stevenson (2005) report evidence on the diversification benefits of adding all available REITs to a diversified U.S. equity portfolio. This paper analysis examines the impact of the inclusion of real estate on an otherwise-diversified developed market equity portfolio from 2009 through 2020.

While average REIT returns are similar to that of developed market benchmarks, REIT volatility is greater than either of the developed market benchmarks. The correlation between real estate and the diversified worldwide developed market equity portfolio is approximately 0.7. Given real estate's correlation, greater volatility than developed market benchmarks, and similar level of return make the portfolio performance impact of the addition of REITs to a developed market equity portfolio an empirical question. This study finds that real estate returns have a high proportion of idiosyncratic risk and so real estate is, to some extent, independent of developed market benchmarks. Approximately 50 percent of real estate volatility is idiosyncratic.

This paper's empirical results suggest that the augmentation of the diversified developed market portfolio with real estate, in the post-financial crisis period, is associated with improved portfolio performance. Furthermore, the results of the market integration time-trend analyses suggest that, if anything, the relationship between real estate and the developed benchmarks is not tightening over time—and so the portfolio benefits of real estate are not diminishing over time.

Augmenting the developed market benchmark with 10% invested in real estate improve portfolio return by about three basis points per month, while decreasing volatility less than one basis point. While the Sharpe ratio for the benchmark is 0.085, the Sharpe ratio for the real estate-augmented portfolio is about 7% greater. The REIT results for the post-financial crisis period examined here are similar to the results of Brueggeman and Fisher (1993), and Chandrashekaran (1999), and in conflict with the results of Lee and Stevenson (2005). Recommendations for future research include the examination of diversification benefits of various types of REITs, other than equity REITs, in the post-financial crisis period.

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Appraisal of Incentives Scheme on Diaspora Remittance

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Abstract

In recent years, developing countries are placing more weight on the role of remittance inflow on economic development and stability of the financial system from their citizens abroad. According to the International Monetary Fund (IMF), remittances are household income from foreign economies arising mainly from the temporary or permanent movement of people to those economies. Realizing the significance of remittances on foreign exchange receipts, stability of the local currency, stock of foreign reserves, and general well-being of the people, several countries are increasingly rolling out policies and diverse programs aimed at boosting the inflow of diaspora remittances into their economies. This paper investigates the impact of the Central Bank of Nigeria's (CBN) Incentive Schemes on the inflow of diaspora remittances into the country. The paper adopted a descriptive analysis to show how impactful the Naira-for-dollar policy has been since its announcement in March 2021.

The research finds that remittances inflow increased by an average of 11% and the correlation between remittances inflow and movement of foreign reserves figures across the selected countries is by far stronger during the post-incentive policy regimes (between 2019 and 2022) compared to the years before. Moreover, while the incentives provided under the "local currency rebate for diaspora remittances schemes" continue to lag far below the transaction costs incurred by the associated parties, attempts to increase the rebate packages yielded a far lower result in the stock of remittances inflow. Thus, how much the countries are capable of increasing the rebate rates or sustaining the incentive schemes remain uncertain due to the fragile balancing effects on set fiscal thresholds. The study concluded that the rebate on the remittances scheme is not only detrimental to the country's fiscal position but arguably one of the least drivers of motive for sustaining or increasing inflows from the senders.

Introduction

Foreign savings have historically been considered important for increasing a country's capital production ratio in development economics studies. These studies have taken into account elements including foreign direct investment (FDI), official development assistance (ODA), foreign commerce, the transfer of technology, and, more recently, remittances. Remittances typically refer to the portion of migrants' earnings that is remitted back to their country of origin, their family, and communities, either in cash or in kind. According to the International Monetary Fund (IMF), remittances are household income from foreign economies arising mainly from the temporary or permanent movement of people to those economies. Remittances include cash and non-cash items that flow through formal channels such as electronic wire, or informal channels, such as money or goods carried across borders. International migrant remittances have steadily grown to become an important source of external finance in developing countries.

Over the past twenty years, remittance flows have multiplied five-fold, acting as a countercyclical force during global economic downturns in recipient nations. This makes it important to continue to analyze the potential of migrants' remittances to contribute to development. Today, they represent the largest source of external finance for many developing countries, ahead of Official Development Assistance (ODA) and Foreign Direct Investment (FDI). While private capital mainly flows to emerging countries, remittances are particularly important in poorer countries where they represent over a third of the Gross Domestic Product (GDP). They also form an important contributor to resilience in the face of economic or humanitarian crises. Remittances have proven to be more dependable and consistent than other forms of external financing like foreign direct investment, public debt, or official development assistance, according to a UNCTAD (2011) report. Remittances to low- and middle-income countries (LMICs) withstood global headwinds in 2022, growing an estimated 5% to $626 billion (World Bank, 2022). According to the latest World Migration and Development Brief, remittances to Sub-Saharan Africa, the region most affected by the global crisis, were expected to increase by 3.9% to $53 billion in 2022, up from 16.4% in the previous year (due mainly to strong flows to Nigeria and Kenya).

Though Nigeria’s economy largely depends on earnings from the export of minerals (crude oil and gas) for her foreign reserve stock, other notable sources of inflow include autonomous sources such as foreign loans and advances, foreign capital importation (foreign direct investment and foreign portfolio investment), capital gains, and diaspora remittances. However, Diaspora remittances were among the nation's top sources of non-oil foreign exchange. The Central Bank of Nigeria in recent years has been making concerted efforts to boost its reserves by using a combination of demand and supply side measures to strengthen the foreign exchange market due to unpredictable movement in prices of crude oil, which has for many years been the main source foreign exchange for the country and any shock to that singular source of forest often cause a disproportional effect on foreign reserves as well as the exchange rate of the Naira to the US Dollar.

Consequently, in recognition of the strategic significance of the Nigerian diaspora, the Nigerians in Diaspora Commission Establishment Bill was passed into law by the Federal Government in July 2017. The Law created the Nigerians in Diaspora Commission (NiDCOM), which was formed to engage and utilize the human, capital, and material resources of this demography in the socioeconomic, cultural, and political growth of Nigeria (Pwc, 2019). According to data from the Central Bank of Nigeria, diaspora remittances first outpaced oil revenue in 2015 as $21.2 billion was sent home officially by Nigerians living abroad, surpassing the $19.6bn oil export proceeds for those 12 months. Nigerians abroad sent home $19.7 billion and $22 billion in 2016, respectively, more than the $10.4 billion and $13.4 billion earned from oil exports during the same period. The CBN's annual economic reports show that in 2018 the total revenue from oil was $18 billion while Nigerians abroad sent home $25.1bn, the highest in four years.

Given the foregoing, this study is set to provide insight into the CBN incentive schemes on diaspora remittances in Nigeria. The study also seeks to evaluate the impact of the CBN ara-4-dollar in boosting the nation's reserve. While enormous studies have been conducted at examining the effect of remittances inflow on Nigeria's economic growth and development, none of these studies deemed it fit to evaluate the role of CBN's diaspora remittances policies at either affecting the inflow of remittances in the country or boosting its reserve. This becomes an important gap that this study seeks to fill.

After the introduction, the rest of the study is structured as follows. Section 2 provides overviews of the nature of Nigeria's external reserve, remittances, and the CBN incentive schemes in the form of stylized facts. The review of the existing literature on remittance flows is carried out in section 3. In section 4, a critical evaluation of the impact of the CBN Incentive Schemes was done while recommendations and conclusions of the paper are presented in section 5 and final sections, respectively.

Literature Review

Theoretically, several studies have emerged to explain the causes behind the migrants' decisions to send funds (cash and goods) to their relations back home. The framework of the 'new economics of labor migration' or NELM provides a foundational basis for this discussion. According to the NELM theory imperfections in rural credit and risk markets generate incentives to participate in migration by sending family members to work in the city or abroad, who subsequently share part of their earnings with the rural household, through remittances. Migrants function as financial intermediaries, substituting for the missing rural bank or insurance institutions. Once they are set up at their destinations, migrants provide the family members at the origin with required capital through remittances, and with income insurance, or simply the promise to remit if the origin household suffers an adverse income shock. A seminal paper by Lucas and Stark (1985) provides three general explanations of migrants' decisions to remit: a) the altruistic motive, b) the self-interest motive, and c) the mutually-beneficial contractual agreement.

First, according to the altruism or livelihoods school of thought, migrants send money to their families in their home country to increase their families' income and thus, consumption. Remittances are dispatched owing to affection and responsibility towards the family. The altruistic model states that sending remittances brings satisfaction to the migrant out of concern for the welfare of his family. When motivated by altruism, remittances can vary based on the number of household members that migrate and the poverty status of the receiving household, although it has been shown that poorer households obtain a larger proportion of their total income from remittances.

Second, remittances might be motivated by self-interest, when the migrant aims to inherit part of the family's wealth or take advantage of convenient investment opportunities (i.e. higher returns) in the home country. In this case, the migrant is primarily stimulated by an economic and financial self-interest, when sending remittances to the home country. The `argument supporting this line of theory is that at every point in time, the successful migrant in a foreign country saves.

Finally, remittances could be viewed as an outcome of a "mutually-beneficial contractual agreement" between the migrant and his family members. One of the underlying incentives for such arrangements could be the household's intention to diversify risks by locating its members in different countries (cities, regions) which are less likely to be hit by an adverse shock simultaneously (Lucas and Stark 1985). If it is assumed that economic risks between the sending and foreign country are not positively correlated, then it becomes an opportune strategy for the family as a whole to send some of its members abroad (often the most educated) to diversify economic risks. The migrant can then help to support his family in worsening times at home. Conversely, for the migrant, having a family in the home is insurance as bad periods can also take place in the foreign country. In this model, migration is a co-insurance strategy with remittances playing the part of an insurance claim.

On the empirical front, much of the current literature on workers remittances has followed two broad strands. While some studies have concentrated on the determining factors of remittance inflows – (Aydas et al., 2005; Gupta, 2005; Alleyne, 2006; and Mazumder, 2018) others have shown curiosity about the macroeconomic impact of remittances on growth (Chami et al., 2005; Ang, 2006; Siddique, 2010; Barua and Rafiq, 2020; and Jijin, Mishra, and Nithin, 2022). Ratha (2003) for instance, pointed out that remittances augment the recipient individuals' incomes and increase the recipient country's foreign exchange reserves. If remittances are invested, they contribute to output growth, and if they are consumed, then they also generate positive multiplier effects. Remittances, according to Jijin, Mishra, and Mithin (2022), are a crucial factor that helps to reduce the current account deficit and has consistently been a stable component of the balance of payments.

By generating a steady stream of foreign exchange earnings, they can improve a country's creditworthiness for external borrowing and through innovative financing mechanisms (such as securitization), they can expand access to capital and lower borrowing costs. While large and sustained remittance inflows can contribute to currency appreciation and so affect the production of cost-sensitive trade goods, Stahl and Arnold (1986) argue that the use of remittances for consumption may have a positive effect on growth because of their possible multiplier effect. Moreover, remittances respond to investment opportunities in the home country as much as to charitable or insurance motives.

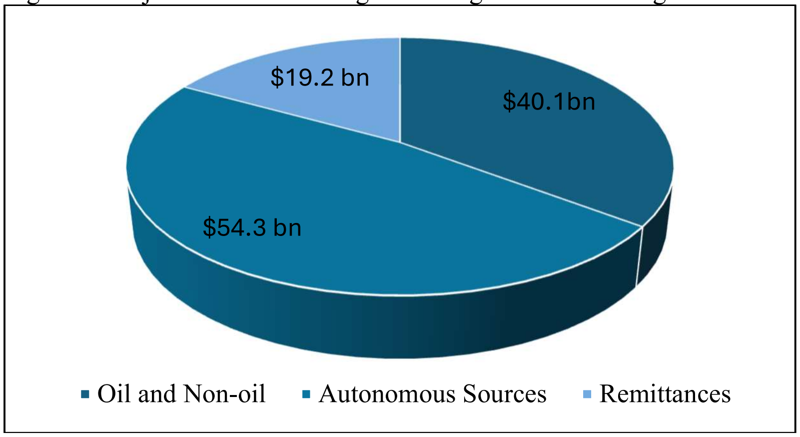
However, the net macroeconomic impact of remittances on receiving countries' economies is ambiguous. The literature investigating the economic impact of remittances on a host country's long-term economic growth still presents a considerable diversity of interpretations about the effects of workers' remittances even when the focus of the economic analysis shifts from the short to the long run, thus generating inconclusive results. While some studies emphasized the positive impact of remittances on economic growth in the country of origin of the expatriate workers, others strand of the literature reports an insignificant or even negative impact of remittances on the home country's long-term economic growth.

Stylized Facts on Foreign Reserves, Remittances, and Remittances incentives in Nigeria

Nigeria External Reserves: The foreign exchange receipts can be used to determine the sources of Nigeria's external reserves. The difference between foreign payments into and withdrawals from the economy, or the net of foreign exchange inflows and outflows, is what determines the value of reserves. The movement of the total amount of foreign exchange receipts, with appropriate consideration for what happens to the out payments, affects the reserve stock. Ceteris paribus, it is therefore anticipated that reserves will grow if inflow increases while outflow stays the same or when inflow remains constant while outflowing declines. Similar to how the stock of reserves will decrease when inflow decreases and outflow rises, or when inflow is stagnant and outflow rises.

Though the economy largely depends on earnings from the export of minerals (crude oil and gas) for her foreign reserve stock, other notable sources of inflow include non-oil export, diaspora remittances and autonomous sources such as foreign loans and advances, foreign capital importation (foreign direct investment and foreign portfolio investment) and capital gains. A cursory look at Nigeria's foreign earnings shows that proceeds from the export of oil and gas have dominated inflows through formal and official channels (Figure 1). While receipts from oil and gas account for 47.8% of total foreign exchange inflow into Nigeria in 2020, receipt from other autonomous sources was 35.3% and that of diaspora remittances was 16.9%. This composition explains why the movement in prices of crude oil in the international market does have a disproportional effect on foreign reserves in Nigeria.

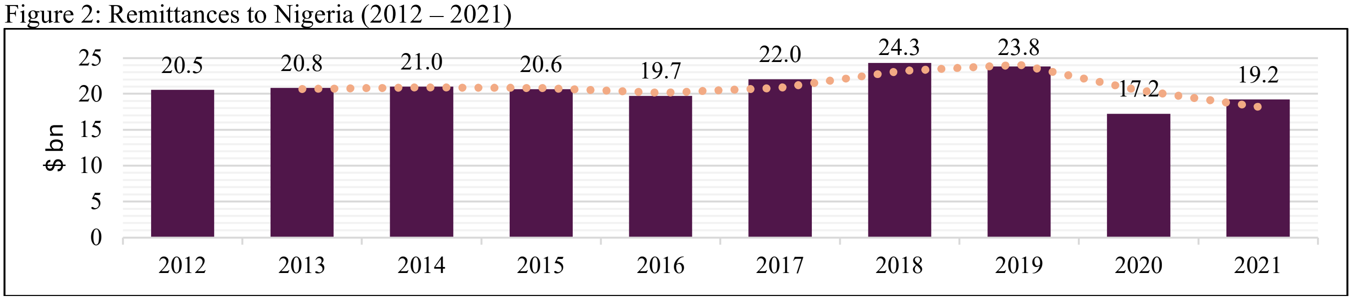
Figure 1: Major Sources of Foreign Exchange Inflow into Nigeria in 2020



Source: Central Bank of Nigeria (CBN) Foreign Statistical Bulletin, 2020

Remittances and Remittances Incentive Schemes in Nigeria: In addition to serving as a source of external financing for Nigeria, remittances received from expatriates make up a sizeable portion of the household's disposable income. A substantial portion of remittance transfers to Nigeria are reportedly used for domestic consumption, according to anecdotal data. A portion of these expenses is used to fund housing, healthcare, and educational projects, creating jobs locally in these vital service industries. An examination of the remittance's inflow into the country in Figure 2 shows that diaspora remittance to Nigeria increased from $20.54 billion in 2012 to $24.31 billion in 2018, and $23.81 billion in 2019 but dropped to $17.21 billion in 2020, perhaps due to the lockdown and restriction measures that were implemented by many countries in a bid to contain the spread of COVID-19 pandemic. However, with heavy dependence on money transfers, increasing attention is being paid to the size and impact of different channels. It is noted that remittances to Nigeria pass through the formal financial and payment channels (i.e, licensed International Money Transfer Operators (IMTO), banks, and recently fintech companies) and informal channels (i.e, cash sent through people who are traveling and non-cash items such as automobiles, electronics, clothes, etc.). Thus, diaspora remittance to Nigeria is likely much more than the figures reported as the CBN has limited control over informal inflows and how they are being utilized.

Figure 2: Remittances to Nigeria (2012 – 2021)

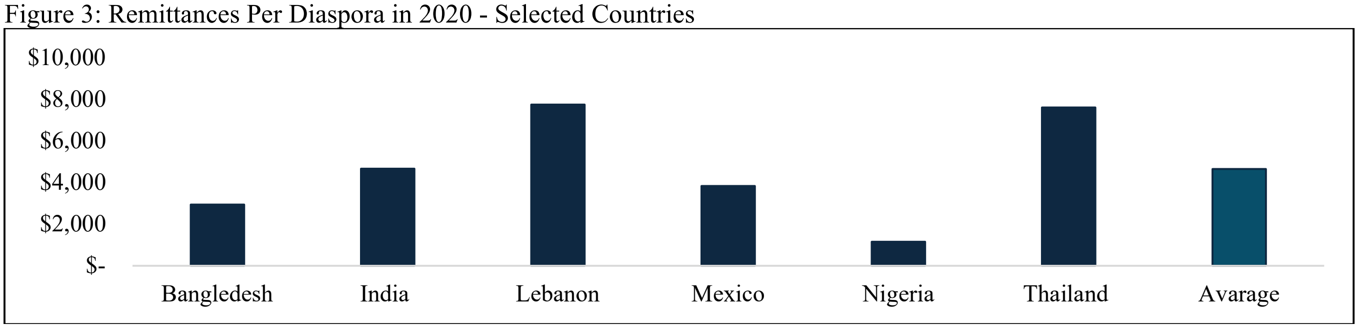


Source: Knowledge Partnership on Migration and Development (KNOMAD)

Despite the quantum of diaspora remittance inflows into the country, insights from other developing countries' data show that remittances to Nigeria remain largely underexploited against the backdrop of the number of Nigerians in the diaspora. For comparison, India with an estimated diaspora population of 17.90 million received $245.27 billion in remittances in the three years (between 2018 and 2020) while Nigeria with an estimated 15 million in the diaspora received about $64 billion during the same period (PwC, 2019). As evidenced in Figure 3 above, Nigeria's remittance inflow per diaspora in 2020 significantly lags behind other developing countries. This reality is arguably the outcome of the relatively lower earning profile of Nigerians abroad combined with the absence of coordinated programs directed at attracting remittances inflow into the country.

In a bid to stimulate diaspora remittances, in March 2021 the CBN launched the "Naira-4-Dollar Scheme" that gives beneficiaries of cash remittance access to their foreign exchange and an added ₦5.00 as a reward for every dollar remitted through the banking system. The policy was initially designed to run for only two months (March 8, 2021, to May 8, 2021) with the primary aim of incentivizing senders and recipients of International Money Transfers (IMT) and expanding Nigeria's foreign exchange sources. The scheme was extended indefinitely on May 5, 2021, in a bid to maximize CBN's objective of boosting forex supply and supporting the Bank's demand management strategy in the foreign exchange market as well as consolidate macroeconomic performance. This CBN initiative is in line with what was obtainable in other countries. These countries offer different incentives to increase remittance flows and channel them to productive uses. Table 1 provides a list of some countries, their incentive schemes, and the outcome of such initiatives.

Figure 3: Remittances Per Diaspora in 2020 - Selected Countries



Source: FMDQ Research; Data is from the United Nations Department of Economic and Social Affairs

Table 1: Evidence from Selected Countries that are Implementing Diaspora Inflow-Driven Incentive Schemes

Countries	Incentive Schemes	Year	Outcomes/Impacts
Bangladesh	Introduction of a 2.00% cash bonus to customers receiving remittances through banking channels abroad on TK$10,000.00 or more.	2020	Bangladesh's remittances rose 10.60% in 2020 from 2019.
	Increase the bonus from 2.00% to 2.50% on all remittances sent to Bangladesh by Bangladeshis in the diaspora.	2021	Inflow to Bangladesh increased by 12.20% in 2021.
Pakistan	Waiver of charges by banks and microfinance banks for customers who use online fund transfer services.	2020	Inward remittances to the State Bank's Roshan Pakistan digital accounts crossed $671.00 million in the six months leading up to March 2021.
	Cash incentives of PKR1 for each dollar remittance when exchange companies surrender 100.00% of their inward remittances in the interbank market.	2021	Increase foreign exchange reserves held by the Central Bank from $10.00 billion in 2020 to $13.41 billion in 2021.
Philippines	The Bangko Sentral ng Pilipinas (BSP), the central bank of the Philippines waived remittance fees for overseas Filipinos. MoneyGram waived money transfer fees from the United States, United Kingdom, France, Italy, and Spain to BPI Family Savings bank accounts.	2020	Cash remittance inflows in 2021 increased by 5.10% to $31.40 billion from $29.90 billion in 2020. It also exceeded the previous record of $30.10 billion in 2019 before the COVID-19 pandemic.

Source: International Organisation for Migration (IOM)

Evaluation of the Impact of CBN Remittance Incentive Schemes

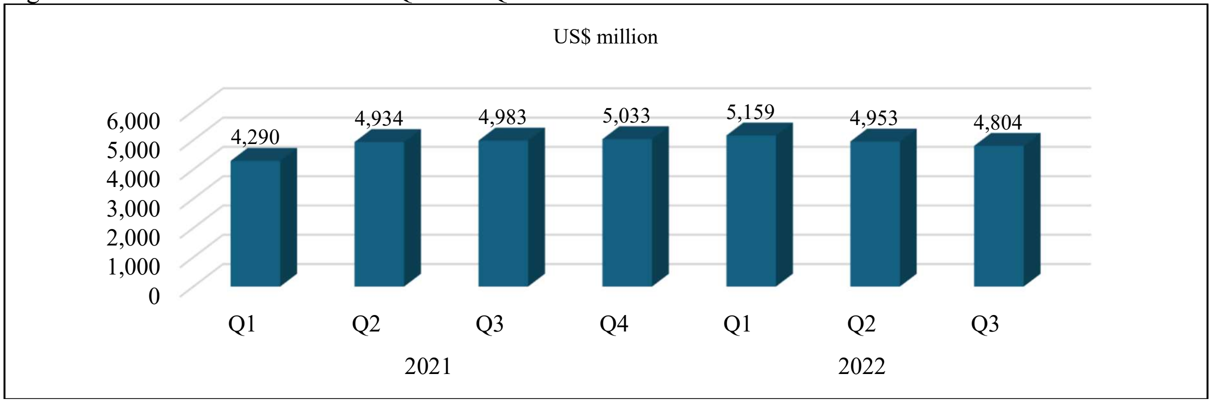
According to the World Bank Migration and Development Brief report (2022), remittance flows total US$794 billion globally in 2022, with Nigeria continuing to top Sub-Saharan Africa. However, the World Bank predicts that in 2023, growth in remittances to Sub-Saharan Africa will slow down to 3.9% as "unfavorable conditions in the global environment" remain. But the World Bank noted that for Nigeria, the continuation of the "Naira for Dollar" program is anticipated to support remittance inflows from Nigerians living abroad. The World Bank also noted that the introduction of the e-Naira, Nigeria's digital currency, could also help migrants and remittance service providers have easy access to bank accounts. The following details how the CBN incentive programs have affected remittances from overseas workers since they were implemented.

Leap in Remittances Inflows: Theoretically, diasporans are naturally empathetic to sending more money to their home countries during rising inflationary trends to cater to their relatives. This ordinarily is expected to lead to an increase in remittance flows into the country. A look at remittances inflows since the announcement of the Nara-4-dollar scheme shows that remittances through the banking system jumped from $4.94 billion in the second quarter of 2021 to $5.16 billion, at the end of the first quarter of 2022 (Figure 4).

This growth in remittance inflow can largely be attributed to the implementation of the scheme. The new policy was aimed at enlarging the scope and scale of foreign exchange inflows into the country to stabilize the exchange rate and support accretion to external reserves. However, Remittances inflow into the country declined to $4.80 billion in Q3, 2022 on the backdrop of the global economic crisis occasioned by the Russia-Ukraine war since the tail end of the first quarter of 2022. Accretion to Foreign Reserves: In terms of accretion to the stock of reserves, it is observed that in the third quarter of the year of implementing the Nara-4-dollar scheme, the accretion to foreign reserves increased to $41.57 billion from $32 recorded in quarter 2 of 2021. However, since the fourth quarter of 2021, the stock of foreign reserves has declined until it fell to around $37.39 billion in Q3 of 2022. Nigeria's foreign reserve stock is made up of inflow from the export of minerals (crude oil and gas) for her foreign reserve stock, non-oil export, diaspora remittances, and autonomous sources. Hence, it is difficult to firmly attribute the dynamics of foreign reserves figures to remittance inflow. However, as evidenced in Figure 5 below, this research finds that the correlation between remittances inflow and movement of foreign reserves figure is far stronger during the post-Naira-4-Dollar policy regime (between 2021 and 2022) compared to the years before.

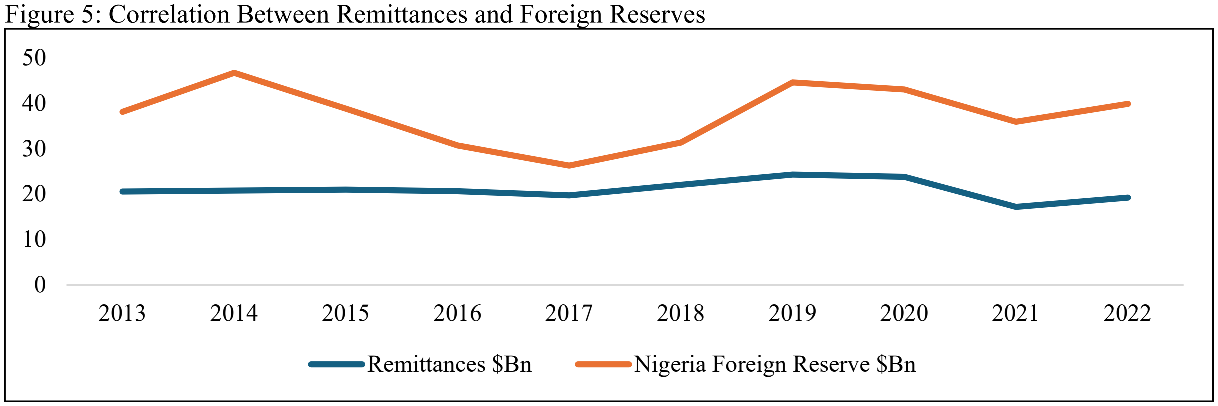
Other Impacts of the Scheme: The scheme is largely expected to boost forex supply and support the CBN's demand management strategy in the foreign exchange market as well as consolidate macroeconomic performance such as enhancing the stability of the Naira exchange rate, enhance liquidity, and stabilize the financial market. Broadly, there has been a quantum leap in the volume of transactions at the Investors' and Exporters' Foreign Exchange Window (I&E FX Window) of the foreign exchange market which rose sharply by 201.50% {year-on-year (YoY)}, to $14.53 billion between January to April 2022 from $4.82 billion in the corresponding period of 2021. However, the fundamental issue is that the overall supply from all sources of foreign exchange inflows remains far below the increasing demand for forex. Over the last year, the exchange rate has been hovering between ₦408.67/$1 and ₦419/$1 at the NAFEX I&E FX Window, which indicates some level of stability in that segment of the foreign exchange (FX) market. However, the parallel market continues to grapple with sustained volatility with rate movement from ₦486/$1 to about ₦600/$1 over the period. This research held that the effectiveness of the Scheme in stabilizing the exchange rate and enhancing liquidity in the forex market is empirically farfetched and largely aspirational, at least for now.

Figure 4: Remittances inflows: 2021Q1-2022Q3



Source: Source: CBN Statistical Bulletin, 2022

Figure 5: Correlation Between Remittances and Foreign Reserves



Source: FMDQ Research

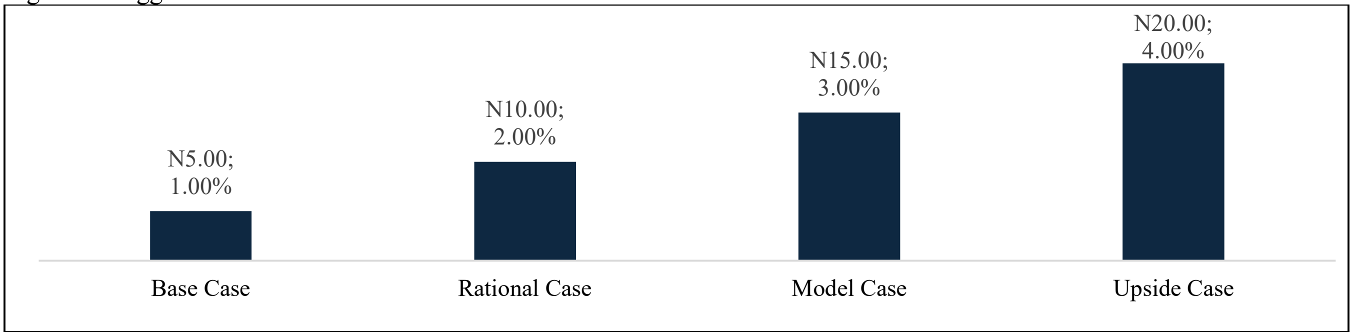
Recommendations

To ensure that the Naira-4-Dollar Scheme achieves its desired objective and ensures that remittances are utilized in ways that are beneficial to the economy, below are suggestions from stakeholders.

Make Naira Rebate Scheme More Attractive: The average transaction cost for sending $200.00 to Nigeria currently stands at 7.80% (amounting to N15.6). This largely outweighs the less than 1% reward (₦5.00 incentive for every dollar) for remittances through the banking system under the Naira-4-Dollar Scheme. Moreover, evidence from model countries such as Bangladesh shows that incentives for remittances are pegged at levels not less than 50.00% of the transaction costs at any point in time. For instance, the Bangladesh government introduced a 2.00% incentive on remittance exchange in the 2019 - 2020 fiscal year. While the average cost of remittances to Bangladesh is currently at 4.30%, the government of 2021 increased the bonus for money sent to Bangladesh by Bangladeshis abroad from 2.00% to 2.50%. To maximize the inflow of remittances under this Scheme, it is recommended to adjust the approved bonus rate from ₦5.00 to ₦10.00 per dollar (i.e., 25% of the transaction cost) and target pegging at ₦15.00 per dollar (i.e., 25% of the transaction cost) over the near term.

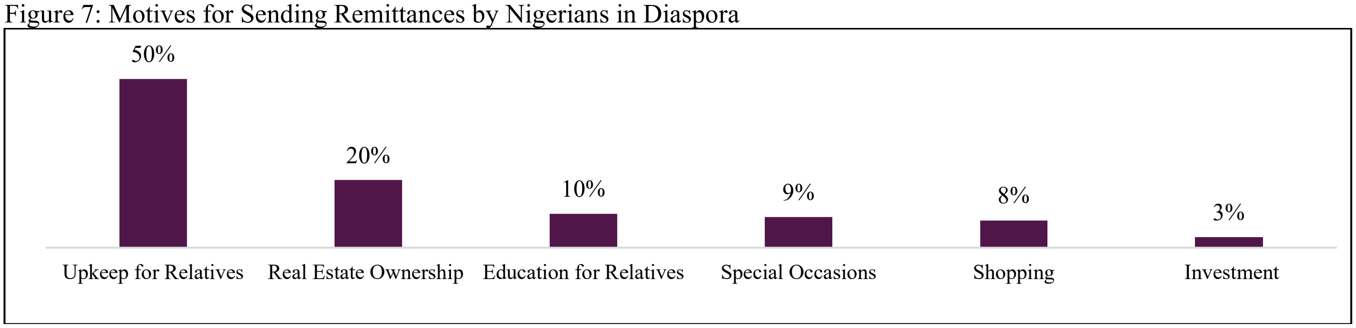
Expand Motives for Sending Remittances into the Country**:** According to WorldRemit, the immigrants from West Africa in the United States and United Kingdom listed daily expenses, gifts, medical expenses, and education of relatives in their home country as top reasons they sent money home in 2021. In the same vein, this research finds that only about 3% of remittances inflow in Nigeria are directed towards investment while the rest are spent on relatives, family celebrations, and real estate acquisition. (Figure 7) The major barriers to investing for those in the diaspora are the lack of credible real-time investment information, the absence of enabling investment products and platforms, and the minimum amount of funds set by investment firms.

Figure 6: Suggested Rebate on Remittances



Source: FMDQ Research
Note: The calculation assumed an official of N500/$ for this research

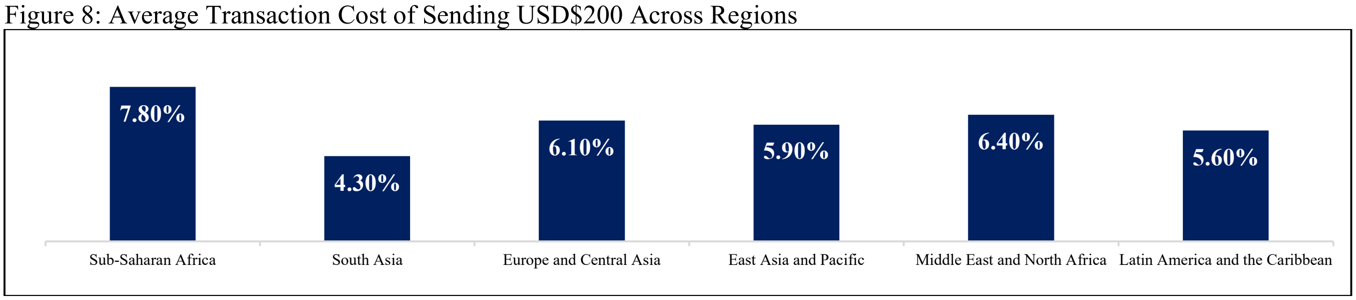
Figure 7: Motives for Sending Remittances by Nigerians in Diaspora



Source: Diaspora Survey, 2022 - FMDQ Research

Reduce Cost of Remittances: As part of the target of SDG 10 (Reduce inequality within and among countries) is to reduce transaction costs of migrant remittances to less than 3.00%. The global average cost of sending $200.00 was 6.00% in the fourth quarter of 2021, twice as high as the SDG 10 target. It is cheapest to send money to South Asia (4.30%) and most expensive to send to Sub-Saharan Africa (7.80%). Moderating transaction charges by IMTOs towards the level obtainable in South Asia can be explored by the CBN. It is believed that reducing transaction charges will not only boost the volume and value of remittances inflow into the country but ultimately improve liquidity in the foreign exchange market.

Figure 8: Average Transaction Cost of Sending USD$200 Across Regions



Source: Diaspora Survey, 2022 - FMDQ Research

Conclusion

This research has helped to evaluate how the Central Bank of Nigeria (CBN) is promoting diaspora remittances through its 'Naira 4 Dollar Scheme'. The policy, which was introduced in March 2021, entailed the payment of N5 for every $1 received as a remittance inflow to encourage Diaspora Nigerians to send their remittances through the banking system. Data from the Central Bank of Nigeria, CBN, shows that remittances from Nigerians working abroad have been on an upward trend since the third quarter of 2020, though with some slight reduction since the second quarter of 2022 due mainly to the global environment occasioned by the Russia-Ukraine crisis which has affected many European countries economy. As a policy prescription, it is advised that CBN prolong the Naira Rebate Scheme and enhance its appeal to promote an increase in remittance inflow. Policymakers should make sure that the costs of sending and receiving remittances from the diaspora are also minimized.

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Using Tableau to Visualize Data in Finance and Business Analytics Courses

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Abstract

This paper uses Tableau® to illustrate the use of data visualization in finance and business analytics courses. Specifically, while stock investments should not be made based on technical analysis alone, visualization of temporal stock data provides an educational opportunity to help students comprehend time series analysis. The paper discusses how to use Tableau® to calculate fixed averages and moving averages. While there are websites that provide moving average charts of stock data already, this exercise can help students understand how to use Tableau® to perform and visualize moving averages, which can be applied to other business problems as well.

JEL Codes: A20
Keywords: finance education, data analytics, data visualization

Introduction

There has been an explosion in the amount of data in the business world. Being able to turn vast amounts of data into useful information to make real time business decisions is a valuable skill. Students able to combine functional knowledge and technical skills to analyze big data are sought after in the market. This paper demonstrates how to take advantage of financial market data and data visualization software to create an authentic experiential educational class activity to develop financial, technical, and analytical skills. Students download real market data to be analyzed using data visualization software to apply knowledge in a finance setting.

With ubiquitous usage of personal computing devices such as smart phones and the frequent use of apps for social networking, banking, shopping, communication, and investment, there is an endless amount of data in the business world (Gupta and George, 2016). Given its importance in the business world, data analytics is becoming an increasingly important subject in business schools (Cegielski and Jones-Farmer, 2016).

In particular, this paper addresses how the teaching of foundational business courses can work synergistically by incorporating data visualization into finance courses. An important learning objective of business courses is to teach students to become proficient at dealing with real-world data. Even though made-up textbook data can be useful for educational purposes, students need to become familiar with real-life data. Further, using real-life data can be much more interesting to students. Knowing that the data has practical applications, students can be more motivated to study for a "boring" class.

Financial data is not only important but also relatable and interesting to students. One example of this would be stock market data. For many students, this data is interesting for both corporate finance and personal finance reasons.

There are many platforms and tools of data analytics, ranging from the most foundational tools such as Excel to the most cutting-edge artificial intelligence tools. This paper will specifically discuss a set of tools known as data visualization software. This will be motivated by the potential benefits of data visualization in the classroom, followed by an overview of data visualization software.

There are many pedagogical benefits of finance data visualization for data analytics courses. First, data visualization programs can efficiently handle large amounts of real-life data. Therefore, an instructor can easily incorporate real-life data in the course. While a small "textbook dataset" can be useful for learning foundational concepts, dealing with real-life data motivates students as they can see the relevance of learning in the business world. Further, since students will be dealing with real-life data after they graduate, practicing with real-life data can help students become proficient at handling such data. For example, students may have to apply various data filtering techniques to clean up data and focus on the specific data points that they need to analyze.

Further, data visualization can help students gain understandings of important concepts. An instructor can illustrate important concepts visually and interactively using data visualization programs. This paper will illustrate one such example – moving averages. Data visualization can potentially enhance student learning by being able to visualize such concepts interactively.

Literature Review

Various papers in the literature have explained the usefulness of data visualization, such as Tableau®, for teaching in business disciplines such as accounting (Hoelscher and Mortimer, 2018; Weirich, Tschakert, and Kozlowski, 2018; Laplante and Vernon, 2021). This paper provides new illustrations for finance education.

Tableau Introduction

Tableau® is an example of software programs known as data visualization software. Data visualization software does not merely create graphics or visualizations; its most important function is to allow the user to extract useful information from the data through a graphical user interface. Since it provides the analytical capability of making "intelligent" use of business data, data visualization software is also known as business intelligence tools (Albright and Winston, 2020). Compared to performing data analytics through writing programs in computer languages such as R or Python, having a graphical user interface allows the student to concentrate on learning concepts in business analytics.

Specifically, Tableau® is a software program made by the company Tableau Software LLC, which is a subsidiary of the company Salesforce.com, Inc. Other data visualization software programs include Microsoft Power BI®, Qlik®, Google Data Studio, and SAS Visual Analytics. This paper will use Tableau® as an example, although the principles can be applied to any data visualization program. Based on the authors' teaching experience, one advantage of Tableau® is that it can be run both on Mac computers and on PCs with Windows. As some students might have Mac computers while others have PCs, having a cross-platform software is conducive to teaching and learning. Also, Tableau® offers free academic license for educators and students, making it easy for faculty to adopt Tableau® in classroom teaching.

Tableau® has many capabilities. Tableau® allows its users to create tables and charts, and customize them. Charts include horizontal and vertical bar charts, pie charts, line charts, tree maps, scatter plots, and histograms. It is especially useful for customizing charts, which not only helps one to customize the visualization such as color and size, but also the axes and how you can portrait multiple pieces of information in one chart (an example is provided later in this paper).

A powerful feature of Tableau® is geographical analysis. Tableau® can examine data at various geographical levels: country, state, city, and zipcode. Further, it can easily create a map to show geographical distribution of data.

When Tableau® analyses data, it can aggregate data at certain levels. For example, if a company has multiple products, you can find the total revenue from all the products, or you can find the average among the products. It also allows a user to aggregate the data at multiple levels. For example, if multiple products fall into the same category, then you could find the aggregate revenue of each category instead of each individual product.

While Tableau® has built-in calculations such as sum and average, it also allows users to create their own formulas as a calculated field.

One very useful feature of Tableau® for data analytics is its ability to filter data in multiple ways to facilitate the data analysis. For example, a user can filter data by month or year of data; alternatively, a user can filter data by industry sector, and so on.

Yet another very powerful feature of Tableau® is to create dashboards. A dashboard is made of multiple sheets, and a user can interact with the sheets simultaneously. For example, a user can select an area on a map on one sheet, and Tableau® will use it to change the bar chart in another sheet.

In terms of the data that Tableau® can use, Tableau® allows a user to connect to data in various formats, such as Excel files, CSV (comma-separated values) files, text files, and Microsoft Access files. Further, it also allows a user to connect to a server such as Microsoft SQL Server, MySQL, and Oracle server.

The full conference version of this paper included a tutorial, which is omitted in this Proceedings version of the paper for brevity.

Data for Finance Courses

There are many financial datasets available. One example of data that is highly relevant in finance, and that has the advantage of being publicly available, is stock data such as price and volume on publicly traded stocks. This data is readily available for every trading day. Such data can be recorded in a time course data table. In a time course data table, each row represents one time point, such as a day, a week, or a month. For example, the Yahoo! Finance website allows you to search for individual stocks and then to find historical time course data on those individual stocks.

As an example of how to utilize this website, a student can go to http://finance.yahoo.com. The student can then search for the company Apple, Inc., which has the symbol AAPL. After navigating to the information for Apple (throughout this description, screen captures shown in the conference version of the paper are omitted for brevity), the student can click on

Historical Data, select Time Period, and choose "5Y," which selects a 5-year time window. For Frequency, the student can select "Daily," and then click "Apply." Next, the student can click Download to download the data. The downloaded file, AAPL.csv, is a CSV (comma separated values) file, which has the data in a tabular format separated by commas. If the student opens the file in Excel, a spreadsheet will appear in which each row represents one day. For each trading day, the student will see the opening price, high and low prices during that day's trading hours, closing price for the day, adjusted closing price (for stock splits and dividends), and daily trading volume.

Technical Analysis

Tableau® can be used for technical analysis involving stocks. Although trading should not be based on technical analysis alone, technical analysis may provide useful information regarding the stock. Comparing the stock price to a moving average of the stock price identifies trends and provides insights into stock price behavior. Identifying crossover points captures the point where the current price switches from being above the moving average to below, or vice versa. These crossover points may, in theory, provide information about momentum and support levels for stock prices.

Two different moving average approaches, simple moving average (SMA) and exponential moving average (EMA), are commonly used to compute moving averages. A simple moving average takes the average over a given period, with each data point within the period receiving the same weight. For example, in a 5-day simple moving average, the first data point is calculated as the mean for days 1 through 5, inclusive. The second data point is calculated as the mean for days 2 through 6, inclusive, and so on.

Obviously, the data points in various sequential averages will overlap. For instance, days 2 through 5, inclusive, are included as data points both in the first observation of the simple moving average and in the second. But, the time window "moves;" thus, this procedure calculates a moving average.

In Tableau®, a user can create a SMA by adding a Table Calculation, which will be explained in the next section.

In the exponential moving average (EMA) approach, more weight is given to the more recent days, providing quicker responses to current stock price trends. The shorter the period, the greater the responsiveness of the EMA to movements in the stock price (See https://corporatefinanceinstitute.com/resources/knowledge/trading-investing/exponential-moving-average-ema/).

Tableau® does not have a default Table calculation for SMA, but a user can add an EMA by adding a calculated field using the desired weighting scheme.

In either case, the moving average changes lag the stock price changes. Moving averages with shorter time periods are faster moving than are moving averages with longer time periods, due to the reduced lag. The trends are identified as downward trends or upward trends. Technical analysts contend that a downward trend provides a sell signal, and that an upward trend provides a buy signal. Moving averages are used to identify the direction of a trend and to smooth price and volume fluctuations, or "noise", that confuse interpretation.

Longer moving average time periods (50 to 200 days) are used to estimate long-term support levels for stock prices. The moving averages are used to look for overvalued or undervalued stocks. When a shorter-term average (e.g., a 15-day moving average) crosses below a longer-term average (e.g., a 50-day moving average), this is viewed as confirmation of downward momentum. Conversely, when a shorter-term average crosses above a longer-term average, this is viewed as confirmation of upward momentum. It should again be emphasized that technical analysis is only a piece of information and should not be used as the sole basis for making trades.

Simple moving averages lag as indicators. However, moving average convergence divergence (MACD) leads the simple moving approach. This measure considers both trends and momentum. MACD uses the 12-, 26-, and 9-day Exponential Moving Averages (EMAs). The MACD line is defined as the 12-day SMA minus the 26-day SMA (https://school.stockcharts.com/doku.php?id=technical_indicators:moving_average_convergence_divergence_macd).

The MACD is compared to the 9-day EMA of the MACD line, which is the signal line. When the MACD crosses through the 9-day EMA, a trading signal is provided. Specifically, when the MACD crosses below the signal line, this is viewed as a negative signal. On the other hand, when the MACD line crosses above the signal line, this is considered a positive signal.

As mentioned above, investment decisions should not be based on technical analysis alone. However, in a classroom setting interactive visualization of real stock prices can provide an engaging learning experience of financial data analytics and visualization. The paper then discusses a potential class assignment for financial data visualization.

Financial Data Visualization Assignment

The next step is to present a detailed class assignment, in which students are introduced to moving average analysis using Tableau®. The assignment begins as follows:

Summary: In this hands-on case, you will use Tableau®, a data visualization software application, to analyze the moving average of each stock's price movements over time.

Learning Objectives:

At the completion of this case, you should be able to:

1. Understand and explain basics of time course analysis.
2. Develop the ability to analyze real datasets from public sources.
3. Apply the concept of a moving average to analyze financial data.
4. Apply Tableau® to create visualizations of moving averages.

Case scenario:

You are an analyst at a financial investment firm, Long Term Success Investors. Your goal is to find stock investments that will go up in value over a long period of time. Specifically, you want to examine 5-year stock price data and understand the trends over time. Your task is to create visualizations that will help investors understand the long-term trend of a stock while not becoming confounded by short-term fluctuations.

For brevity, in this Proceedings version of the paper the authors summarize the remainder of the assignment as follows (and again omit screen captures).

The student is first instructed to go to Yahoo! Finance (as explained above), and to download 5 years of daily stock data for an individual firm, such as Apple. The student is next requested to go to Tableau® and connect to the data file. Then, the student is instructed to begin exploring the data in order to create analytics and visualizations.

This begins by asking the student to bring the "Date" from the list of fields on the left side of the screen into Columns. Next, the student is instructed to bring "Adj. Close" (adjusted close) from the list of fields on the left to the center of the worksheet where the "ABC's" are displayed.

The student is then asked to click "Show Me" on the right of the screen, and to select "Line charts." This begins the process of creating a graph. However, the graph as originally created is not particularly useful, because the line chart as initially created shows the sum of the data. So, the student is asked to click on the drop-down menu of SUM(Adj. Close) and change the measure from "Sum" to "Average."

When this is done, the visualization will show the average of "Adj. Close" for each year. To change to a different time scale, the student is asked to click on the drop-down menu of YEAR(Date) and choose a different time scale. It is important to point out to the student that the drop-down menu actually contains two sections of time scale. For instance, if selecting Quarter, the Quarter in the top section will combine the quarters from different years; the Quarter in the bottom section will separate the quarters from different years. In this instance, in order to show price progressions over time the student should select the time scale from the bottom section.

The student is then asked to address the following. First, the student is asked to try different time scales for the chart of "Adj. Close," and is asked, "What do you think are some issues with the charts? How can you improve the charts to help you better understand the temporal trends?"

The instructor can point out that for short periods of time the movements are quite sharp, having "rough edges." When one moves from daily numbers to weekly averages, the edges are not as rough. However, the transitions are sharp, because the averages are taken over separate (though adjoining) time intervals. For instance, the first data point displayed is the average adjusted closing price for the first week, the next data point is the average adjusted closing price for the second week, and so on. So, given the lack of overlap, the transitions will be sharp.

To address the issues of fixed interval averages, one solution is to take moving averages. This results in the overlap discussed above. So, a useful next step for the instructor is to acquaint the students with the Moving Average function in Tableau®. To calculate moving averages, the student will need to change the time scale back to "daily." The instructor can then walk the student through the process of creating a moving average (for instance, a 200-day moving average) based on the daily data.

The instructor can then point out yet another useful feature in Tableau®, which is the ability to create overlaying charts. For example, it is possible to overlay the 200-day moving average chart on the daily price chart. To do so, the student simply drags "Adj. Close" from the list of fields on the left over to the vertical axis in Sheet 1. The student should check to make sure that the time scale is Day, and should then click over the numerical labels on the vertical axis in order to have the daily data overlay the moving average data, rather than replace the moving average data.

The second part of the student assignment is to examine the daily chart overlaying the moving average chart, and identify the crossover points. The student is then asked whether there are different types of crossovers, and what those crossovers might represent.

The moving average chart allows us to see the overall time trend more clearly. The daily chart has many fluctuations, i.e., "rough edges," that represent short-term movements that may not be overall temporal trends. Further, the crossover points can be useful to investors (Mayo, 2021). When the daily chart crosses from above to below the moving average chart, it can mean that the stock price is trending down. On the other hand, when the daily chart crosses from below to above the moving average chart, it can mean that the stock price is trending up.

Third, the student can be asked to create the following: 12-day EMA, 26-day EMA, MACD line, signal line, and MACD histogram. Obviously, when the EMA formula is incorporated, it will only apply beginning with the second value; for the first value, the EMA will simply be the same as the daily price, since on the first day there is by definition no previous value that can be used in the calculation.

To incorporate a customized formula such as EMA in Tableau®, the student is instructed to go to Analysis, and then choose Create Calculated Field. The student enters 12-day EMA as the name of the calculated field, and then enters the appropriate formula as the calculation. In the current example, the formula looks like this: (2/(12+1))*(SUM([Adj Close])-PREVIOUS_VALUE(36.434811))+PREVIOUS_VALUE(36.434811). The first term represents the smoothing constant, SUM[Adj. Close] is the current price, and PREVIOUS_VALUE(36.434811) is the previous EMA.

Note that in the formula, SUM must be entered immediately prior to "Adj. Close." The reason for this is tricky, but important. Because Tableau® is always aggregating data, there needs to be an aggregate formula before Adj. Close. Since there is only one data point in one day, SUM[Adj. Close] is simply the adjusted close of that day. Another tricky part of the formula is that, as explained above, PREVIOUS_VALUE is not defined for the first day. So, the first value of Adj. Close simply has to be "typed in" to the formula. In this particular instance, that number was 36.434811.

As noted above, the student first calculates both the 12-day EMA series and the 26-day EMA series. Then, the MACD line is calculated as 12-day EMA – 26-day EMA. The signal line is calculated as the 9-day EMA.

Then, a histogram is created. To create a MACD chart, bring Date to Columns, change the timeframe to Day as before, and bring MACD Line, Signal Line, and MACD Histogram to Rows. To display the MACD histogram correctly, a few more steps are needed. First, click on Day(Date) again, and in the drop down menu, choose Discrete. This will allow the histogram to be displayed correctly. Next, in the Marks panel, for MACD Histogram, choose Bar in the drop-down menu. Now the charts are correct. To create an overlay chart, change the axis for Signal Line to Dual Axis:

In the next portion of the assignment, the student is requested to compare the charts of different companies and apply moving average analysis. For instance, in the current example the student so far has evaluated Apple (AAPL). In the next step, the student is asked to download the 5-year data for Apple (AAPL), JPMorgan Chase (JPM), and Goldman Sachs (GS). The student can, for instance, filter the date for a couple of years, such as 2021 and 2022. The student can then be instructed to create a line chart that has overlaid daily adjusted closes for all three of these stocks, and to examine the chart.

Next, the student is instructed as follows: "Duplicate the sheet above. Add Table Calculations to create 180-day moving averages of AAPL, JPM, and GS, respectively. As each stock has a different baseline price, add secondary Table Calculations so that the chart is shown as percentage of First. Compared with the daily chart above, in which one is it easier to visualize the long-term trend? Describe the long-term trends over the last 2 years."

When this is done, an interesting pattern emerges in the trends. It seems that JPM and GS have similar trends, which are different from that of AAPL; but, it is challenging to describe anything beyond that because of the daily fluctuations. Next, one can see if and how moving averages can help make it easier to describe the trends. So, the student is instructed to create a chart in which the moving averages of the three stock prices are overlaid.

Compared with the daily chart, with the moving average chart it is much easier to describe the long-term trend. First, AAPL slowly increases from January 2021 to September 2022; but, in the last three months of 2022 there is a downward trend. For JPM, the price rises from the beginning of 2021 to the beginning of 2022; then, there is a downward trend in 2022 until the final months of the year, where the price flattens out to be about the same place it started in 2021. Finally, for GS, the price has a much greater growth rate in 2021 than is seen for AAPL and JPM; then, the price declines in 2022, but bounces up in the last 2 months of the year. Overall, GS has a gain of almost 30% over the 2 years, while AAPL has a gain of less than 20%, and JPM has practically no gain.

Conclusion

This paper has provided a very limited sampling of the options that exist for using finance data visualization as a teaching tool. There are numerous additional features for both visualization and analytical purposes. For example, Tableau® allows a user to navigate a chart – zoom in, zoom out, etc. This can help us see different time points in different scales. Tableau® also can be used simply for purposes of preparing class presentations, even if the students are not being instructed in how to use Tableau® themselves. Finally, it should be noted that finance data visualization in general can be used for motivational and illustrative purposes, both in data analytics courses and in finance courses. In data analytics courses, the use of financial data may help to convince the students of the practical usefulness of the statistical methods they are being taught. And, in finance courses, the use of data visualization may create additional interest on the part of students who do not find finance to be inherently interesting.

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Is There Still a Day-of-the-week Effect in the Real Estate Sector?

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Abstract

This study contributes to the ongoing debate on the persistence of stock market anomalies in equity markets (McLean and Pontiff, 2016; Jacobs and Müller, 2020) and concentrates on the day-of-the-week effect in the European real estate sector. Interest payments and settlement effects were discussed as the main factors to explain this anomaly in the past. Today the persistence is highly questionable concerning the dynamically adjusting economic and institutional environment. While previous research indicated a significant Monday and Friday effect in other sectors, literature can only support a Friday effect for real estate. Furthermore, the real estate sector has a lower level of mispricing, which makes it more difficult for an anomaly to survive (Bampinas et al., 2016). The data are split in three ten-year periods to analyze the effect's existence over the long term. Applying OLS and GARCH models, results reveal an evolution in a significant Friday effect for cross-country indices (Europe and Global). From 1990 to 2000, the effect is weak but significant. It gains in importance during the period 2000 to 2009. In the final period until 2020, the anomaly weakens again but does not disappear for cross-country indices. From a country-specific perspective, there is no pattern in the significance of the day-of-the-week effect. In conclusion, there is a day-of-the-week effect in the real estate sector in Europe from a cross-country perspective.

JEL codes: G12, G13
Keywords: Day-of-the-week-effect, Real Estate, GARCH, Friday Effect

Introduction

Systematic seasonality leads to anomalies explained by reasonable compensation for settlement periods. The average return for days, weeks, and months has been studied comprehensively in previous literature. Seasonal patterns allow to develop of profitable investment strategies to generate profits. There are several variants of this seasonal pattern, the "weekend effect", the "January effect" or the "day-of-the-week effect" (Jaffe and Westerfield, 1985; Seyhun, 1988; Dubois and Louvet, 1996). The day-of-the-week effect states a significantly different return for one day of the week compared to the remaining days (Dubois and Louvet, 1996).

Several empirical studies, for example French (1980), Gibbons and Hess (1981), and Keim and Stambough (1984) reported that the daily return in the U.S. stock market is, on average, significantly higher on Friday while the Monday returns were significantly negative. The highest concentration of attention on this anomaly is around 2000 to 2010 (Friday and Higgins, 2000; Lenkkeri et al., 2006).

On one side, contrary to the predominant opinion of the existence of a day-of-the-week effect, Bampinas et al. (2016) cast doubt on the results. They do not obtain a day with a significantly higher or lower return when studying the European real estate sector from 1990 to 2010 and conclude that daily seasonality seems subject to data mining criticism (Bampinas et al., 2016). On the other side the ongoing debate on the persistence of stock market anomalies, as documented by Jacobs and Müller (2020), suggests that barriers to arbitrage trading can create segmented markets and that anomalies tend to represent mispricing instead of data mining.

Inconsistent findings make the day-of-the-week effect an often discussed anomaly. In addition to the discussion about the existence of the day-of-the-week effect, researchers document a weakening of this anomaly (Board and Sutcliffe, 1988; Steeley, 2001). By following this trend, the day-of-the-week effect in 2000 might have disappeared.

The focus lies on the real estate sector for multiple reasons to contribute to the discussion. First, Birz et al. (2022) argue that it is essential to analyze real estate separately because of institutional characteristics (Chan et al., 1990; Sa-Aadu et al., 2010; Devos et al., 2013). As a result, the real estate sector stands out because of a lower level of mispricing than other sectors, which makes it more difficult for an anomaly to survive (Bampinas et al., 2016). Second, its position as an outlier from the general stock market is also recognizable in the day-of-the-week literature. Results for the real estate sector differed from the rest in terms of significance for individual weekdays. Based on this inconsistency, an up-to-date perspective with modern statistical methods is lacking. Previous studies are mainly based on OLS regressions, and the period of the most recent analyses only goes until 2010. Taking all these aspects into consideration, it is unknown whether the day of the week effect still exists or not. Building on GARCH models and a more recent period, this study addresses the research question:

The paper proceeds as follows. Section 2 includes a comprehensive literature summary on seasonal patterns in various stock markets. Section 3 describes the dataset for the empirical analysis for the period from 1/3/1990 to 10/1/2020. The study searches for patterns on a country and cross-country level to include multiple perspectives on the European real estate sector. In the next section, 4 research methods are explained, which are used to derive empirical evidence. The datasets are estimated with three different GARCH models selected using the Schwarz Information Criterion. Section 5 provides a summary of the results, and section 6 concludes.

Review of Relevant Literature

One of the first studies analyzing the day-of-the-week effect in real estate is from Redman et al. (1997). They find that the day-of-the-week effect exists with a significantly higher return on Friday by focusing on the U.S. real estate segment with real estate investment trusts (REIT) from 1986 to 1993 (Redman et al., 1997). After 2000, many studies concentrated on seasonal patterns in global real estate markets. Friday and Higgins (2000) examine the daily returns of all publicly tradable REIT and all tradable mortgage REIT. From these two categories, a data set with equal weights is constructed for the period from 1970 to 1995. The results show that the return on Mondays is positive while the return on Fridays is also positive. If Friday's return is negative, Monday's return is also negative (Friday and Higgins, 2000). These findings do not hold for mortgage REITs. Friday and Higgins (2000) conclude that if investors plan to sell their REITs on Friday, they can increase their profits by extending the holding until Monday. To this end, their findings strengthen the argument that REITs can be viewed more like stocks on a day-to-day basis.

Connors et al. (2002) support the Friday effect by examining the average daily return of REITs for existing calendar anomalies from 1994 to 1999 and using an equal-weighted, value-weighted, and broad market CRSP data set. As a result, the highest average return on Friday is obtained.

Another perspective on the day-of-the-week effect is created by Hardin et al. (2005), who compare returns of REITs and the general stock market from 1994 to 2002. As a result, they show significant REITs seasonal patterns, but these patterns depend on the index selected for the REITs. Hardin et al. (2005) report that calendar anomalies are not detectable for an index weighted by market capitalization. In the next step, various European real estate stock market indices are examined by Lenkkeri et al. (2006) between the years 1990 to 2003. The authors find above-average returns for Friday for eight out of eleven European countries and conclude that there are profitable trading strategies to exploit seasonal patterns. These trading strategies are country-specific and not consistent from an overall perspective.

Lee and Ou (2010) examine MREITs from 2001 to 2007, estimating an OLS and a GARCH model to explore the day-of-the-week effect. The result for MREITs is positive for Tuesday and Friday and negative for Wednesday. In conclusion, Lee and Ou (2010) suggest that these findings affect the asset allocation and timing of securities issuance by listed firms.

One of the most recently published studies is by Bampinas et al. (2016), in which the authors examine twelve countries' global, European, and real estate indices. The period for this study is from 1990 to 2010, in which the day-of-the-week effect will be analyzed using different regressions. Bampinas et al. (2016) apply GARCH, EGARCH, and GJR-GARCH models to determine the best estimate. A significantly higher return for Friday is present in half of the countries and the two cross-regional indexes. A significant impact on Monday is seen in three countries. In addition to the GARCH models, they run a moving regression for which Friday has a significant influence of 15.37% when using the GED distribution. The authors cast doubt on the day-of-the-week effect in European real estate markets and argue that seasonal patterns could result from data mining. This interpretation conflicts with Jacobs and Müller (2020), who suggests that barriers to arbitrage trading can create segmented markets and that anomalies survive due to mispricing rather than data mining.

However, the day-of-the-week effect varies with indexes and statistic methods, which makes it necessary to have a closer look at the methodology in the literature. Table 1 "Literature overview of statistic methods and results," summarizes day-of-the-week literature and gives an overview over statistic methods, indexes and results. Remarkably, only three sources apply GARCH models. Most of the literature focuses on OLS regression, which can lead to biases in time series analysis. In this case, the day-of-the-week effect could result from data mining biases, as Bampinas et al. (2016) conclude. Nevertheless, studies based on GARCH models produce contradictory results by supporting or rejecting the day-of-the-week effect. Furthermore, the time range is until 2010, which creates a research gap of more than ten years until now.

The evidence provides an inconsistent picture and raises doubts about seasonal patterns. Especially in the 1970s and 1980s, most studies obtain clear proof of seasonal anomalies. It was not until the mid-1990s that financial economists began to examine real estate markets with a focus on REITs. The evidence is inconclusive. On one side, Bampinas et al. (2016) cast doubts about the day of the week effect due to errors in data mining. On the other side, Jacobs and Müller (2020) disagree and explain the anomaly with mispricing. The discussion has not ended, and most recent literature is inconsistent.

The literature research shows that it is time for a new perspective for two reasons. First, research methods are mainly based on OLS regressions, which can cause static biases like heteroscedasticity. And second, literature declares a weakening of the day of the week effect (Board and Sutcliffe, 1988; Steeley, 2001). Based on this time dependency, the anomaly could have changed or disappeared from the last research period until 2010. For this reason, a more recent data set and methodology is the starting point for the research.

Dataset

The dataset covers a time frame from 1/8/1990 to 10/1/2020, for which European indices are examined. Due to the time horizon of 30 years, the focus is additionally on different sub-periods according to global events, which have strongly influenced the world economy. The first data set begins on 1/8/1990 and ends 12/31/2000, a period until the dot-com bubble burst. Thus, eleven years are included in this data set, containing 2605 observations for the individual indices, except Finland for which there are 1805 observations.

Data set 2 covers the period from 1/1/2001 to 6/30/2009. The data contains the daily closing prices of the studied indices from 2477 days. The end date is set for the end of the world financial crisis and the stabilization of real estate prices. Furthermore, a concentration of research on the day-of-the-week effect takes place. Data set 3 refers to a period from 7/1/2009 to 10/1/2020, the end of the whole studied period. This dataset includes 2936 closing prices for all indices and covers a period until the Covid-19 pandemic shocked the stock market.

The individual data sets consist of the indices from the Financial Times Stock Exchange, European Public Real Estate Association, and National Association of Real Estate Investment Trusts (FTSE, EPRA, NAREIT). The data used for the analysis is retrieved from Thomson Reuters. Daily closing prices for the days Monday to Friday are the baseline for the empirical research. It should be noted that these are not synchronous for each country, especially for the European and American stock markets, because the trading hours and also holidays are different. This non-synchrony could lead to underestimating the correlations (Martens and Poon, 2001).

Nevertheless, the daily closing prices are used to prove the day-of-the-week effect. Following Bampinas et al. (2016) to create comparable results, the research is based on different dimensions. First, a country-specific perspective of European countries is analyzed. Second, there is a focus a cross-country index of Europe and third, a global index is the most generalized perspective. The analysis mainly focuses on the European countries and the cross-country index for Europe. Data from the following countries is analyzed: Belgium, Finland, France, Germany, Italy, Netherlands, Norway, Spain, Sweden, Switzerland, and the UK. In addition to the individual European countries, the FTSE EPRA NAREIT Developed Europe Total Return Index (European Index) is analyzed, which takes into account all countries entering the European Union. The individual countries are weighted in the European Index according to market capitalization. The composition, based on the size of the market capitalization of the European index, leads to a concentration on the countries Germany and Great Britain. Germany has a share of 31% and the United Kingdom 26% in this index. Sweden has the third-largest share, with 12% as of 10/1/2020.

In addition to the European index as a supra-regional index, the FTSE EPRA NAREIT Developed Total Return Index (referred to as the global index) is chosen. The largest share is held by American listed real estate stocks with 53%, followed by Japan with 11% and Germany with 6%. Europe has a total share of 18.59% in the global index as of 10/1/2020.

All indices are total return indices, quoted in euros and weighted according to the level of market capitalization. Since the index is examined for different countries, there are days without trading in these countries (e.g., variable public holidays). Following Savva et al. (2006), for these days without trading, the missing prices are replaced with the last price before the day without trading.

As many European countries' indices as possible are included to generate an overview of the European real estate market. However, this is impossible for some countries due to a lack of data availability. Thus, in addition to the European and global indices, eleven national data sets are examined with 8019 values, except Finland, where the data set is limited to 7240 observations.

Methodology

The analysis is based on logarithmic returns to make the different data sources comparable. The return is calculated as follows by using the closing prices P_t and P_{t-1}:

$$R_t = \ln \left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

R_t is the daily return in percent for each index, t describes the respective day for the return.

Basic Day-of-the-week Effect Model

To analyze the effect of each day, a model with five dummy variables representing each day is necessary. If the return is sought for any Monday, then the dummy variable for Monday takes the value one, and the other dummy variables take the value zero. Each variable represents one day of the week. The previous day's term is also called the autoregressive term AR (1). Following Lenkkeri et al. (2006), the model is described as follows:

$$R_t = \alpha_1 Mon_t + \alpha_2 Tue_t + \alpha_3 Wed_t + \alpha_4 Thu_t + \alpha_5 Fri_t + \alpha_6 R_{t-1} + \varepsilon_t \quad (2)$$

R_t is the calculated return in percent for the day, α_1-α_5 are the coefficients for each day estimated, Mon_t is the dummy variable for Monday, and so on. α_6 is the factor for the autoregressive term, R_{t-1} is the previous day's return, and ε_t is the random error term.

The purpose of this equation is to use OLS regression to detect existing anomalies, particularly for the day-of-the-week effect. For the detection of the day-of-the-week effect, the following hypothesis is to be investigated:

$$H_0\colon \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0$$

This hypothesis states that no day of the week has a significant influence, and thus the day-of-the-week effect does not exist. If the p-value is smaller than 0.05, the coefficient significantly influences the data set, and the hypothesis will be rejected. Thus, evidence for the day-of-the-week effect exists.

GARCH Model

Different GARCH models like the GARCH(1,1) model are examined. Bera and Higgins (1993) report that GARCH(1,1) models are sufficient for a good fit for time series in capital markets and that more extended models are rarely needed.

The equations below describe the GARCH(1,1) model, adapted from Bampinas et al. (2016). This model consists of two equations, the mean equation (3) and the variance equation (5). Equation (3) describes the equation of the day-of-the-week effect, for which the dummy variables and the coefficients are explained in the previous section. For the use of the mean equation in the GARCH model, variable AR terms are added, which are selected according to the best estimate when running the GARCH models. That is, whether no AR term AR(1) or AR(2) term is inserted depends on the selection according to the Schwarz Information criterion. The error term (4) is normal distributed.

$$R_t = \alpha_1 Mon_t + \alpha_2 Tue_t + \alpha_3 Wed_t + \alpha_4 Thu_t + \alpha_5 Fri_t + \sum \mu R_{t-1} + e_t \quad (3)$$

$$e_t \backslash \Omega_{t-1} \sim N(0, h_t) \quad (4)$$

$$h_t = \omega + \alpha e^2{}_{t-1} + \beta h_{t-1} \quad (5)$$

Equation (5) describes the variance equation of the GARCH(1,1) model, where h_t is the conditional variance, omega is the constant term, and $\beta h_{(t-1)}$ is the GARCH term, which represents the impact of new shocks on volatility. The ARCH term is present in the equation through $\alpha e^2{}_{(t-1)}$, which indicates how strongly volatility responds to market shocks.

The sum of alpha and beta must be less than 1 for the conditional variances to be stationary, for this omega must be greater than zero, alpha and beta greater than or equal to zero for the conditional variance to be strictly positive (Berra and Higgins, 1993).

EGARCH Model

The EGARCH model, unlike the GARCH model, is asymmetric. This asymmetric feature describes the response of volatility from the market to good or bad news. Following Nelson (1991), equation (5) is replaced by equation (6). The variance equation describes the simplest EGARCH model, the EGARCH(1,1) model. Through this new variance equation, the model forms an exponential GARCH process:

$$\log(h_t) = \omega + \alpha \left[\frac{|e_{t-1}|}{\sqrt{h_{t-1}}} \right] + \gamma \frac{e_{t-1}}{\sqrt{h_{t-1}}} + \beta \log(h_{t-1}) \quad (6)$$

where $\frac{|e_{t-1}|}{\sqrt{h_{t-1}}}$ represents the standardized shocks, gamma is the asymmetric term in the equation, and log(h_t) is the logarithm for the conditional variance. For β, the stability condition is $|\beta|$<1. Asymmetry exists when γ takes the non-zero value.
The EGARCH model is intended to reveal asymmetric effects affecting volatility in the estimation.

GJR-GARCH Model

The GJR-GARCH model differs from the GARCH model in that an asymmetric effect is included in this model. It consists of two equations, the mean equation, and the variance equation. The mean equation is described in equation (3) and is also used for this model. The variance equation is defined in equation (7) and includes an additional term compared to the variance equation of the GARCH model:

$$h_t = \omega + \alpha e^2{}_{t-1} + \gamma e^2{}_{t-1} I_{t-1} + \beta h_{t-1} \tag{7}$$

where $\gamma e^2{}_{t-1} I_{t-1}$ is the asymmetric term measuring the asymmetric influence. When γ is zero, the null hypothesis that bad news and good news have the same influence on the market can be accepted. For this case, the GJR-GARCH model is the standard GARCH model. If γ is non-zero, the null hypothesis must be rejected. Thus, two distinctions exist. If γ is greater than zero, bad news possesses a greater influence on the volatility of the market; if γ is less than zero, the positive news has greater impact on the volatility of the market.

Empirical Results

After the data is split into three periods, each period is analyzed separately. First, a descriptive analyze, and in the next step, OLS and GARCH models are trained. Data is presented index-specific.

Descriptive Analysis

Two of the 13 indices examined have negative average returns for the period from 1990 to the end of 2000. When reviewing the individual days of the week, five indices (Europe, Finland, Germany, Spain, United Kingdom) have a positive average return for Monday. Friday, the average return of nine indices (Europe, Global, France, Germany, Italy, Netherlands, Norway, Spain, and Sweden) is positive.

For data set 2, the descriptive statistics includes data from 2001 to mid-2009 with 2477 observations per index. The means for Europe and the global index in this period are the highest of the three data sets. The following section investigates whether the days are significant for the second data set. It can be seen for the individual indices that all indices have a negative average return for Monday and all indices have a positive average return for Friday.

The descriptive statistics for data set 3 summaries daily returns of 2937 days from mid-2009 to early October 2020. All countries except the Netherlands and Spain show a positive average return. In summary, of the 13 indices examined, two indices (Global and Finland) show a positive average return for Monday. For Friday, all indices reveal a positive average return. In detail, the European index has a negative average return for Monday and a positive average return for Friday. The global index shows a positive return for all days of the week except Thursday. For the countries with a negative average return across the data set, a positive average return is present for Spain only on Friday and the Netherlands on Tuesdays and Thursdays.

Finally, Table 1 presents the descriptive statistics for the whole period from January 1990 to the beginning of October 2020. The Netherlands, Norway, and Spain indices show a negative average daily return, while all other indices have a positive performance. For the European and global indexes, there are a negative average daily return for Mondays and a positive return for Fridays. Only Finland has a positive average daily return for Mondays, and all other countries have a negative return. Friday is a day with a positive average return for all countries. Descriptive statistics indicate a positive Friday effect but also show the inconsistency between countries and periods.

Table 1: Descriptive statistics

	Mean	Min	Max	Std Dev	Skew	Kurt	JB test	ADF	n
Europe	0.0001	0.040	-0.053	0.004	-0.738	15.763	55,153*	-18.15*	8019
Global	0.0001	0.036	-0.067	0.005	-0.844	16.157	58,794*	-18.99*	8019
Belgium	0.0001	0.045	-0.053	0.004	-0.139	15.725	54,125*	-20.13*	8019
Finland	0.0002	0.302	-0.244	0.013	1.849	133.208	243,024*	-13.73*	7240
France	0.0001	0.074	-0.069	0.005	-0.079	19.325	89,058*	-18.88*	8019
Germany	0.0001	0.060	-0.093	0.006	-0.609	19.759	94,344*	-18.62*	8019
Italy	0.00003	0.077	-0.097	0.008	-0.571	15.444	52,179*	-18.14*	8019
Netherlands	-0.00001	0.063	-0.097	0.006	-1.059	37.592	401,325*	-18.53*	8019
Norway	-0.0001	0.113	-0.130	0.010	-0.480	27.851	206,650*	-18.33*	8019
Spain	-0.0002	0.095	-0.146	0.009	-0.753	29.062	227,712*	-19.30*	8019
Sweden	0.0001	0.057	-0.083	0.007	-0.379	12.327	29,260*	-18.48*	8019
Switzerland	0.0001	0.050	-0.036	0.005	-0.007	9.617	14,629*	-17.93*	8019
UK	0.0001	0.047	-0.093	0.006	-0.787	19.183	88,371*	-18.58*	8019

***, **, * = $p < 1\%$, 5%, 10% significance level.

OLS Regression

A test for heteroscedasticity is performed to test whether the OLS regression is a robust estimator. If heteroscedasticity is present, OLS regression is no longer robust. The results show that for the global index, Germany, Italy, and Sweden there is no constant variance in error terms. Hetereoscedasticity is supported in multiple indices. In consequence, the focus lies on GARCH models to reduce the effect of hetereoscedasticity.

GARCH Models

In the next step, GARCH, EGARCH, and GJR GARCH models are trained. The best fitting model is used for the final result, according to the Schwarz Information criteria. Appendix Table 1 presents the results for estimating the preferred GARCH model in data set 1 (1990 to 2000). GARCH models with insignificant asymmetric terms have been replaced by EGARCH (marked with a "b"). For Monday, a negatively significant value is reported for the Netherlands. The countries France and Sweden and the global index have a positively significant coefficient for Friday, as do the European index and the countries Norway and Spain. In addition, Tuesday and Thursday are negative days with significant influence in Sweden. All added AR terms have a significant influence. For the variance equation of the GARCH, EGARCH, GJR-GARCH models, all factors in the equations have a significant influence.

For data set 2 (2000 to 2009), Appendix Table 2 describes the most significant GARCH models. Monday has a significant positive influence on the European index, Belgium, Finland, and Sweden. Finland, Germany, Italy, Spain, the Netherlands, Switzerland, the United Kingdom, and the European index have a positive significant variable for Friday. Tuesday significantly influences Belgium, France, Norway, and Switzerland positively. For Norway, the variable has a negative impact on Tuesday and a positive effect for France and Sweden. Wednesday is positively significant for the European index, France, Italy, the Netherlands, Spain, and Sweden. The dummy variable for Thursday is positively significant for the European index, and for France, Italy, Spain, and Switzerland, the coefficients from the variance equations all represent a significant influence.

Appendix Table 3 describes the results of the best-estimated GARCH models for data set 3 (2009 to 2020). The dummy variable for Monday for the United Kingdom is the only one of the 13 indices with a significant negative effect. All other coefficients for Monday show no significance. Friday has a significant positive influence on the European and global index and Germany, Spain, Belgium, France, and Sweden. Tuesday is positively significant for the European and global index, Belgium, Sweden, and Great Britain. Wednesday is positively significant for Switzerland, Norway, Sweden, and Great Britain. A positive significant influence for Thursday is found in Switzerland, Italy, and Spain. For the variance equation, all coefficients have a significant impact.

In Appendix Table 4, the results of the GARCH models for the entire period of the study are presented. Monday has no significant influence on any of the thirteen indices examined. In contrast to Monday, Friday significantly influences every index in the data. For the other days, the following can be read - Tuesday of the Norwegian index has a negatively significant influence. Besides Norway, Friday of the European index and for the countries France, Germany, and Switzerland are positive. Wednesday positively impacts the European index, Sweden, Italy, the Netherlands, and Great Britain. Wednesday is positively significant for the global index and the countries Belgium and Switzerland. Thursday is positively significant for Switzerland, Italy, and France. All coefficient terms for the variance equation significantly impact each GARCH model. In summary, for the data set of the whole period for the supra-regional indices and the eleven countries, Fridays have a significant effect on returns. For Monday, not a single term is significant.

Evidence for the Day-of-the-week Effect

The data sets are estimated with different GARCH models in the second part of the empirical investigation. The examination of data set 1 (1990 to 2000) shows a presence of the Friday effect for the European and the global index and four out of eleven countries. A significantly lower return is present in only one of the countries examined.

For data set 2 (2000 to 2009), the Friday effect is present for the European index and nine of the eleven countries studied, and thus a significantly higher return than for the other days of the week. Monday is significant for the European index and three countries. However, this day of the week is positive for the four indexes and thus shows no presence of the Monday effect. These seasonal patterns result in significantly lower returns for Monday.

Estimation of Dataset 3 (2009 to 2020) yields a Friday effect for the two cross-regional indexes and six of the eleven countries examined. The Monday effect is only present for the United Kingdom in this dataset. It is noticeable that when these whole period results are compared with the results of the sub-datasets from year to year, a weakening of the seasonal anomalies can be seen. In data set 2, significantly more indices exhibit seasonal patterns than in the last data set 3. Steeley (2001) already concludes this weakening of the day-of-the-week effect for the British stock market.

Table 2: Results of GARCH models for European real estate

	Variable	1990-2000b	2000-2009a	2010-2020a	1990-2020a
Mean Equation					
	Monday	0.0000	0.0002***	-0.0001	0.00003
		(0.97)	(0.06)	(0.47)	(0.63)
	Tuesday	0.0000	0.0002	0.0003**	0.0001***
		(0.85)	(0.22)	(0.03)	(0.06)
	Wednesday	0.0000	0.0002**	0.0002	0.0002**
		(0.44)	(0.04)	(0.11)	(0.02)
	Thursday	-0.0001	0.0002**	0.0001	0.00005
		(0.22)	(0.08)	(0.38)	(0.44)
	Friday	0.0002**	0.0006*	0.0003**	0.0004*
		(0.01)	(0.00)	(0.02)	(0.00)
	AR(1)	0.0910*	0.090*		0.083*
		(0.00)	(0.00)		(0.00)
	AR(2)	0.0650*			
		(0.00)			
Variance Equation					
	omega	-0.57*	0.000*	0.000*	0.000*
		(0.00)	(0.00)	(0.00)	(0.00)
	alpha	0.15*	0.0056*	0.055*	0.062*
		(0.00)	(0.00)	(0.00)	(0.00)
	beta	0.968*	0.911*	0.856*	0.899*
		(0.00)	(0.00)	(0.00)	(0.00)
	gamma	-0.18**	0.049*	0.055*	0.061*
		(0.01)	(0.00)	(0.00)	(0.00)
	SE of reg	0.002	0.005	0.005	0.004
	Adj. R^2	0.002	0.0003	0.00007	0.006
	n	2605	2477	2937	8019

***,**,* = p < 1%, 5%, 10% significant level. Numbers in parentheses indicate p-values. a and b denote GJR-GARCH and EGARCH model, respectively.

The empirical investigation for the whole period shows that Friday is significant for all thirteen indices examined. Furthermore, Friday has a highly significant effect on most of the indices. The Friday effect is present for the two supra-national indices and six out of eleven countries.

Especially Monday has no significant impact on any index. Thus, the Monday effect is not present for any index. A weakening of the day-of-the-week effect with increasing duration of the examined time, which can be seen in the estimation of the OLS regression, is not recognizable with the GARCH models.

To summarize, the results do not show a country-specific pattern for the day-of-week effect in the sample. These results are also demonstrated by Bampinas et al. (2016); the Monday effect is not detectable for the European real estate sector, and the Friday effect is not observable for all countries from 1990 to 2010. Lenkerri et al. (2006) show the same seasonal patterns with their study that a day-of-the-week effect is detectable only for respective countries and not all countries. The evident existence of the day-of-the-week effect, as proven and documented by many authors before 2000, cannot be established with this empirical study. Instead, study results are similar to those found in more recent studies after 2000.

As a part of the day-of-the-week effect only the Friday effect stays significant in all investigations regarding the European and global indices. The evaluation of this effect shows similarities to the concentration of research. Likewise, there is a maximum during the period 2000 to 2009. In the most recent period until 2020, the effect weakens but does not disappear. Table 6 summarizes the day-of-the-week effect in the European index for all data sets.

Influence of Countries on the European Real Estate Index

The European index highly depends on real estate companies from Germany and the United Kingdom. These two countries are weighted with more than 50 % in the examined European index (as of 10/1/2020).

The estimation with the GARCH models for the whole period shows that Germany and Great Britain have a Friday effect, as does the European index. Consequently, the two largest countries can initiate this Friday effect in the index since not every country has a positive Friday effect.

In data set 1, the European index has a positive significant effect for Friday, which only the European index has and not for the countries of Germany and Great Britain.

For data set 2, the estimation for the European index and the countries Germany and Great Britain results in a positive Friday effect. However, it should be noted that Germany and Great Britain are not the only countries with a positive Friday effect. Therefore, it can also be assumed that the Friday effect of the European index depends on several countries and not only on Germany and Great Britain.

The estimation of the GARCH models in data set 3 show a Friday effect for the European index and Germany but not Great Britain. Beyond Germany, there are other countries where Friday shows significant return patterns. Therefore, it cannot be assumed that only Great Britain and Germany influenced the European index. According to the literature, the day-of-the-week effect is not constant over countries and periods.

Conclusion

This paper contributes to the ongoing debate on the persistence of stock market anomalies in global equity markets (McLean and Pontiff, 2016; Jacobs and Müller, 2020) and concentrates on the day-of-the-week effect in the real estate market over the period 1990 to 2020. The empirical investigation is carried out for the whole period and three sub-datasets. The data sets are estimated by OLS regressions, and GARCH models to exclude heteroscedasticity.

For the entire period, the GARCH models support the consistent evidence for the Friday effect. However, there is an evolution of the day-of-the-week effect over time. In different periods, different weekdays are significant. Only the Friday effect stays significant in all periods. After a maximum in the period 2000 to 2009, there is a weakening in the last period. The impact of the day-of-the-week anomaly seems connected to public attention. After the research hype decreased, the effect also weakened but did not disappear in the period from 2009 to 2020. The Friday effect still influences the real estate sector. The significance of other weekdays is changing during the periods without a pattern.

Furthermore, the day-of-the-week effect in different European countries is unsystematic and changes between periods. The real estate sector has a local business character. Therefore, country-specific effects are possible, but a pattern is not found.

To summarize, the day-of-the-week effect is mainly represented through the Friday effect, which is constantly significant in cross-country indices for the European and global real estate stock market. As an avenue for future research, the question of farther weakening of the day-of-the-week effect can be posed. In addition, research can focus on the analysis of country-specific differences. Furthermore, there is a possible causality between research concentration and the impact of the Friday effect. However, this paper indicates that the day-of-the-week anomaly still exists in the European real estate sector from a cross-country perspective.

Appendix

The appendix presents the comprehensive results of the GARCH, EGARCH and GJR GARCH calculation. For each index, the best fitting model is chosen according to the Schwarz Criteria. The use of GJR GARCH models for an index is marked by an “a”. EGARCH is marked with a "b". Redundant variables are omitted from the models.

Appendix Table 1: GARCH, EGARCH, GJR-GARCH Output for Data Set 1 1990 to 2000

	Europe[b]	Global[a]	Belgium[a]	Finland	France	Germany[a]	Italy	Netherlands	Norway	Spain[a]	Sweden[b]	Switzerland[b]	UK
Mean Equation													
Monday	0.000003	-0.0002	-0.0002	-0.0004	-0.0001	-0.00002	-0.0001	-0.0002**	0.0002	0.00007	-0.0002	-0.0001	0.0003
	(0.97)	(0.16)	(0.17)	(0.64)	(0.23)	(0.90)	(0.66)	(0.01)	(0.51)	(0.79)	(0.40)	(0.54)	(0.10)
Tuesday	0.00001	0.00002	0.00005	-0.0002	0.0001	0.00008	-0.0004	-0.000005	0.0001	0.000003	-0.0004**	0.0002	0.0001
	(0.85)	(0.87)	(0.72)	(0.89)	(0.44)	(0.66)	(0.13)	(0.95)	(0.72)	(0.99)	(0.04)	(0.40)	(0.33)
Wednesday	0.00007	0.0002	-0.0001	-0.0007	-0.0000	0.00009	-0.0004	0.0002	-0.0004	-0.0002	0.0003	-0.0002	0.0002
	(0.44)	(0.23)	(0.40)	(0.42)	(0.81)	(0.64)	(0.85)	(0.10)	(0.16)	(0.45)	(0.27)	(0.50)	(0.15)
Thursday	-0.000121	-0.00004	0.00003	-0.0003	0.0001	-0.00006	0.0003	-0.00006	-0.0002	-0.0004	-0.0006**	0.0001	-0.00007
	(0.22)	(0.82)	(0.88)	(0.67)	(0.43)	(0.78)	(0.25)	(0.56)	(0.63)	(0.20)	(0.01)	(0.62)	(0.68)
Friday	0.0002**	0.0005*	0.0002	0.0005	0.0006*	0.0002	0.0001	0.0004	0.0007**	0.0007**	0.0008*	0.0004	-0.0002
	(0.01)	(0.00)	(0.12)	(0.53)	(0.00)	(0.26)	(0.95)	(0.66)	(0.03)	(0.03)	(0.00)	(0.12)	(0.31)
AR(1)	0.091*	0.153*	-0.143*	-0.041**				0.108*		0.116*	0.106*	-0.206*	
	(0.00)	(0.00)	(0.00)	(0.03)				(0.00)		(0.00)	(0.00)	(0.00)	
AR(2)	0.065*			-0.103*									
	(0.00)			(0.00)									
Variance Equation													
omega	-0.57*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	-0.232*	-0.152*	0.000*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
alpha	0.15*	0.045*	0.084*	0.023*	0.17*	0.023*	0.16*	0.26*	0.03*	0.043*	0.189*	0.073*	0.064*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
beta	0.968*	0.83*	0.837*	0.976*	0.63*	0.95*	0.80*	0.30*	0.973*	0.86*	0.99*	0.99*	0.918*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
gamma	-0.18**	0.093*	0.104*			0.049*				0.069*	-0.03*	-0.025*	
	(0.01)	(0.00)	(0.00)			(0.00)				(0.00)	(0.00)	(0.00)	
SE of reg	0.002	0.004	0.005	0.022	0.003	0.005	0.006	0.003	0.009	0.007	0.008	0.006	0.004
Adj. R^2	0.002	0.02	0.03	0.011	0.004	-0.002	-0.0008	0.001	0.002	0.009	0.006	0.035	-0.0004
n	2,605	2,605	2,605	1,826	2,605	2,605	2,605	2,605	2,605	2,605	2,605	2,605	2,605

***,**,* = p < 1%, 5%, and 10% significant level, numbers in parentheses indicate p-values. A and b denote GJR-GARCH and EGARCH model, respectively.

Appendix Table 2: GARCH, EGARCH, GJR-GARCH Output for Data Set 2 2000 to 2009

	Europe[a]	Global[a]	Belgium[a]	Finland	France[b]	Germany[b]	Italy[b]	Netherlands[a]	Norway[a]	Spain	Sweden[a]	Switzerland	Uk[a]
Mean Equation													
Monday	0.0002***	0.0001	0.0003**	0.0005***	0.0002	-0.0003	0.0002	0.0001	-0.00003	0.00009	0.0004***	-0.000004	0.0003
	(0.06)	(0.36)	(0.02)	(0.06)	(0.17)	(0.21)	(0.41)	(0.24)	(0.95)	(0.72)	(0.05)	(0.97)	(0.20)
Tuesday	0.0002	0.00006	0.0004*	0.0002	0.0003***	0.0002	-0.0002	0.0002	-0.0007***	0.0004	0.0002	0.0002***	0.0000002
	(0.22)	(0.68)	(0.00)	(0.48)	(0.08)	(0.20)	(0.45)	(0.11)	(0.07)	(0.25)	(0.37)	(0.09)	(0.99)
Wednesday	0.0002**	0.0001	0.0002	0.00006	0.0005*	-0.00002	0.0007*	0.0003**	0.00009	0.0006**	0.0005**	0.0001	0.0002
	(0.04)	(0.47)	(0.12)	(0.83)	(0.00)	(0.90)	(0.00)	(0.04)	(0.84)	(0.02)	(0.01)	(0.27)	(0.21)
Thursday	0.0002**	0.0001	0.0001	-0.00007	0.0004**	0.0003	0.0006**	0.0001	0.0001	0.0007**	0.00003	0.0003***	0.00005
	(0.08)	(0.38)	(0.24)	(0.77)	(0.01)	(0.12)	(0.01)	(0.31)	(0.73)	(0.04)	(0.88)	(0.05)	(0.79)
Friday	0.0006*	0.0003	0.00003	0.0009*	0.0007*	0.0006*	0.0004***	0.0005*	0.0001	0.0006***	0.0007*	0.0004*	0.0006*
	(0.00)	(0.12)	(0.80)	(0.00)	(0.00)	(0.00)	(0.05)	(0.00)	(0.63)	(0.08)	(0.00)	(0.00)	(0.00)
AR(1)	0.090*	0.102*	-0.122*	-0.091*				0.086*	-0.033	0.103*	-0.003	-0.119*	
	(0.00)	(0.00)	(0.00)	(0.00)				(0.00)	(0.18)	(0.00)	(0.87)	(0.00)	
AR(2)												-0.072*	
												(0.00)	
Variance Equation													
omega	0.000*	0.000*	0.000*	0.000*	0.000*	-0.364*	-0.483*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
alpha	0.0056*	0.018*	0.034*	0.058*	0.194*	0.242*	0.287*	0.070*	0.041*	0.236*	0.052*	0.102*	0.035*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
beta	0.911*	0.919*	0.946*	0.933*	0.979*	0.982*	0.972*	0.891*	0.916*	0.668*	0.909*	0.877*	0.930*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
gamma	0.049*	0.087*	0.038*		-0.054*	-0.053*	-0.022**	0.061*	0.076*		0.058*		0.053*
	(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.01)	(0.00)	(0.00)		(0.00)		(0.00)
SE of reg	0.005	0.005	0.005	0.008	0.006	0.008	0.008	0.005	0.011	0.01	0.007	0.004	0.007
Adj. R^2	0.0003	-0.003	-0.004	-0.004	-0.003	-0.001	-0.004	0.0009	-0.005	-0.008	-0.002	0.006	-0.003
n	2,477	2,477	2,477	2,477	2,477	2,477	2,477	2,477	2,477	2,477	2,477	2,477	2,477

***,**,* = p < 1%, 5%, and 10% significant level, numbers in parentheses indicate p-values. A and b denote GJR-GARCH and EGARCH model, respectively.

Appendix Table 3: GARCH, EGARCH, GJR-GARCH Output for Data Set 3 2009 to 2020

	Europe[a]	Global[a]	Belgium[a]	Finland	France[b]	Germany[b]	Italy[b]	Netherlands[a]	Norway[a]	Spain	Sweden[a]	Switzerland	Uk[a]
Mean Equation													
Monday	0.0002***	0.0001	0.0003**	0.0005***	0.0002	-0.0003	0.0002	0.0001	-0.00003	0.00009	0.0004***	-0.000004	0.0003
	(0.06)	(0.36)	(0.02)	(0.06)	(0.17)	(0.21)	(0.41)	(0.24)	(0.95)	(0.72)	(0.05)	(0.97)	(0.20)
Tuesday	0.0002	0.00006	0.0004*	0.0002	0.0003***	0.0002	-0.0002	0.0002	-0.0007***	0.0004	0.0002	0.0002***	0.0000002
	(0.22)	(0.68)	(0.00)	(0.48)	(0.08)	(0.20)	(0.45)	(0.11)	(0.07)	(0.25)	(0.37)	(0.09)	(0.99)
Wednesday	0.0002**	0.0001	0.0002	0.00006	0.0005*	-0.00002	0.0007*	0.0003**	0.00009	0.0006**	0.0005**	0.0001	0.0002
	(0.04)	(0.47)	(0.12)	(0.83)	(0.00)	(0.90)	(0.00)	(0.04)	(0.84)	(0.02)	(0.01)	(0.27)	(0.21)
Thursday	0.0002**	0.0001	0.0001	-0.00007	0.0004**	0.0003	0.0006**	0.0001	0.0001	0.0007**	0.00003	0.0003***	0.00005
	(0.08)	(0.38)	(0.24)	(0.77)	(0.01)	(0.12)	(0.01)	(0.31)	(0.73)	(0.04)	(0.88)	(0.05)	(0.79)
Friday	0.0006*	0.0003	0.00003	0.0009*	0.0007*	0.0006*	0.0004***	0.0005*	0.0001	0.0006***	0.0007*	0.0004*	0.0006*
	(0.00)	(0.12)	(0.80)	(0.00)	(0.00)	(0.00)	(0.05)	(0.00)	(0.63)	(0.08)	(0.00)	(0.00)	(0.00)
AR(1)	0.090*	0.102*	-0.122*	-0.091*				0.086*	-0.033	0.103*	-0.003	-0.119*	
	(0.00)	(0.00)	(0.00)	(0.00)				(0.00)	(0.18)	(0.00)	(0.87)	(0.00)	
AR(2)												-0.072*	
												(0.00)	
Variance Equation													
omega	0.000*	0.000*	0.000*	0.000*	0.000*	-0.364*	-0.483*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
alpha	0.0056*	0.018*	0.034*	0.058*	0.194*	0.242*	0.287*	0.070*	0.041*	0.236*	0.052*	0.102*	0.035*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
beta	0.911*	0.919*	0.946*	0.933*	0.979*	0.982*	0.972*	0.891*	0.916*	0.668*	0.909*	0.877*	0.930*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
gamma	0.049*	0.087*	0.038*		-0.054*	-0.053*	-0.022**	0.061*	0.076*		0.058*		0.053*
	(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.01)	(0.00)	(0.00)		(0.00)		(0.00)
SE of reg	0.005	0.005	0.005	0.008	0.006	0.008	0.008	0.005	0.011	0.01	0.007	0.004	0.007
Adj. R^2	0.0003	-0.003	-0.004	-0.004	-0.003	-0.001	-0.004	0.0009	-0.005	-0.008	-0.002	0.006	-0.003
n	2,477	2,477	2,477	2,477	2,477	2,477	2,477	2,477	2,477	2,477	2,477	2,477	2,477

***,**,* = p < 1%, 5%, and 10% significant level, numbers in parentheses indicate p-values. A and b denote GJR-GARCH and EGARCH model, respectively

Appendix Table 4: GARCH, EGARCH, GJR-GARCH output for whole period

	Europe[a]	Global[a]	Belgium[a]	Finland[a]	France[a]	Germany[a]	Italy[b]	Netherlands[a]	Norway[a]	Spain[a]	Sweden[a]	Switzerland	Uk[a]
Mean Equation													
Monday	0.00003	0.000001	0.0001	0.0002	-0.00006	-0.00004	0.00004	-0.0001	-0.00009	-0.000	0.0001	-0.00002	0.00006
	(0.63)	(0.99)	(0.13)	(0.30)	(0.45)	(0.71)	(0.74)	(0.15)	(0.64)	(0.54)	(0.35)	(0.78)	(0.58)
Tuesday	0.0001***	0.0001	0.0003*	0.00002	0.0002***	0.0002***	-0.0001	0.00009	-0.0003***	0.0002	0.0001	0.0002***	0.0002
	(0.06)	(0.18)	(0.00)	(0.92)	(0.05)	(0.05)	(0.30)	(0.21)	(0.09)	(0.18)	(0.40)	(0.06)	(0.12)
Wednesday	0.0002**	0.0002***	0.0001***	0.00001	0.0001	0.0001	0.0002**	0.0002**	0.00004	0.0002	0.0004*	0.0002***	0.0003**
	(0.02)	(0.06)	(0.08)	(0.61)	(0.26)	(0.28)	(0.04)	(0.03)	(0.83)	(0.14)	(0.00)	(0.06)	(0.01)
Thursday	0.00005	0.00006	0.00009	-0.00007	0.0002**	0.00004	0.0006*	-0.0000004	-0.0001	0.0001	-0.0002	0.0003*	0.00002
	(0.44)	(0.43)	(0.25)	(0.69)	(0.02)	(0.68)	(0.00)	(0.99)	(0.46)	(0.48)	(0.12)	(0.00)	(0.86)
Friday	0.0004*	0.0004*	0.0002**	0.0004**	0.0006*	0.0004*	0.0003**	0.0002*	0.0003***	0.0006*	0.0006*	0.0004*	0.0002***
	(0.00)	(0.00)	(0.01)	(0.03)	(0.00)	(0.00)	(0.02)	(0.00)	(0.05)	(0.00)	(0.00)	(0.00)	(0.08)
AR(1)	0.083*	0.13*	-0.10*	-0.039*				0.078*		0.095*	0.054*	-0.116*	
	(0.00)	(0.00)	(0.00)	(0.00)				(0.00)		(0.00)	(0.00)	(0.00)	
AR(2)												-0.053*	
												(0.00)	
Variance Equation													
omega	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*	-0.574*	0.000*	0.000*	0.000*	0.000*	0.000*	0.000*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
alpha	0.062*	0.042*	0.045*	0.009*	0.067*	0.042*	0.255*	0.063*	0.021*	0.145*	0.058*	0.104*	0.066*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
beta	0.899*	0.885*	0.906*	0.98*	0.877*	0.908*	0.961*	0.89*	0.946*	0.80*	0.907*	0.88*	0.894*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
gamma	0.061*	0.087*	0.083*	0.011*	0.074*	0.080*	-0.036*	0.074*	0.047*	0.049*	0.055*		0.056*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)		(0.00)
SE of reg	0.004	0.005	0.004	0.013	0.005	0.006	0.008	0.006	0.010	0.009	0.007	0.005	0.006
Adj. R^2	0.006	0.006	0.005	0.00004	0.0009	0.0004	-0.0006	0.003	0.0006	0.0009	0.006	0.015	0.00008
n	8019	8019	8019	7240	8019	8019	8019	8019	8019	8019	8019	8019	8019

***,**,* = p < 1%, 5%, and 10% significant level, numbers in parentheses indicate p-values. A and b denote GJR-GARCH and EGARCH model. respectively.

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