A Probabilistic Model for Measuring Stock Returns and the Returns from Playing Lottery Tickets: The Basis of an Instructional Activity

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ABSTRACT

Many individuals indicate that playing the lottery is a legitimate way to generate wealth, including retirement savings. We offer a methodology designed to create interactive learning activities comparing the results of playing the lottery versus investing in stocks to create wealth. To emphasize the stochastic nature of investment and lottery returns, we employ a Monte Carlo simulation that draws from probability distributions created from lottery payoffs and historical stock returns. The model results demonstrate, visually and numerically that stock investments generally outperform the lottery in generating wealth.

Introduction

Financial planning for retirement is becoming increasingly complex as employer-sponsored defined benefit retirement programs are replaced with employee-directed defined contribution programs and calls for Social Security reform include its partial privatization. In this evolving environment, individuals are increasingly required to make their own financial decisions for retirement. Whether these decisions generate a sufficient retirement accumulation will be determined by both market performance and allocation decisions. The move to these self-directed programs is based on the presumption that the individual investor possesses a sufficient level of financial literacy. However, there is evidence to suggest that a significant portion of individuals do not have a minimum understanding of investment basics required to make sound decisions. In order to educate our students, we have developed a simulation exercise of the probabilistic outcomes associated with alternative strategies of wealth accumulations.

This project was motivated by students' comments stating they believe buying lottery tickets to be the surest strategy for accumulating wealth for retirement. The generality of our experience is supported by a Consumer Federation of America - Financial Planning Association (2006) survey which found 21% of surveyed Americans think that winning the lottery represents the most practical way for them to accumulate several hundred thousand dollars. The proportion increased to 38% for families with annual earnings less than $25,000. A similar question asked in an Australian survey found that 15% of those surveyed plan on lottery winnings to help them achieve their financial goals. The percentage increased to 18% when the results were limited to people with annual income less than $70,000 Australian, approximately $52,000 U.S.[2] ("Financial plan …," 2006).

The beliefs reported by these surveys are behaviorally supported by the purchase of lottery tickets. In 2002, Americans spent an average of $184 on lottery games, including video terminal games (Hansen and

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[2] Interbank exchange rate - July 1, 2006

Prante, 2006). In 2003, Americans spent around $45 billion to play lotteries, more than on any other single form of entertainment (Hansen, 2004). There is a clear need to educate college students about the financial consequence of using the lottery in an attempt to generate wealth and inform them of alternative long-term investment strategies. These students, as indicated above, hold similar beliefs to those found in the referenced studies. Our methodology could be used at any level of the college experience from freshman to senior. It would be appropriate for any number of courses, from life planning skills courses to upper-level financial planning courses.

The fundamental message our methodology is designed to convey is that using stock investments is preferable to playing the lottery for long-term planning purposes. While investing in the stock market or other alternatives involves risk, when compared to lottery wagering, stock investment is significantly more likely to provide a larger terminal value. Our approach joins two streams of financial planning literature: studies of savings accumulation and stochastic modeling of retirement withdrawals.

First, previous presentations of savings accumulation have applied a constant or "guaranteed" rate of return to the investment alternative. These treatments include Hansen and Prante (2006) who suggest that lottery playing diverts money from retirement saving. They demonstrate that if the money spent on playing the lottery is invested in a retirement account, at the end of a forty-year period, the dollar return from the investment would be 811% greater than the dollar return from playing the lottery; this assumes a constant return of 9% (2% inflation and 7% historic S&P 500 return). In response to U.S. survey respondents who deemed an accumulation of $200,000 as unattainable without playing the lottery, Slaughter (2006) demonstrates the ease of accumulating this sum with the money spent on lottery tickets and a simple investment strategy. He proposed an initial investment of $1,000 and a monthly deposit of $100 in stock. Assuming a 7% annual growth rate and a 4% constant dividend, the investment fund would grow to $276,706 in 30 years. Note that these simple treatments do not calculate the expected returns of playing the lottery and apply a simple average annual rate of return to calculate the ending value of the alternative investments, ignoring the stochastic nature of stock returns.

A second stream of research analyzes the allocation and withdrawal rates in retirement planning (Cooley, Hubbard and Walz, 2003; Booth, 2004; Boinske, 2003). Cooley, Hubbard and Walz (2003) use both Monte Carlo simulation and overlapping period calculations for determining returns. The returns for the simulations were drawn randomly from the distributions for the appropriate asset class. For example, the return for an investment in stocks was assigned according the log-normal distribution fit to the S&P 500 returns over the 1946 – 2001 period. A similar method is used by Cooley, Hubbard and Walz for bond returns. Booth (2004) also applies a log-normal distribution for generating stock returns for his analysis. Boinske (2003) uses Monte Carlo simulation where rates of return are randomly assigned based on the expected value and range of returns over a 75-year period. The S&P 500 serves as the basis for the stock return mean and range in Boinske's simulations.

The basis for the study is further discussed in the next section. The methodology, the data and the results are then provided. The last section of the paper discusses the implications of the results and identifies directions for future development of the methodology and its pedagogical uses.

Study Purpose

As the preceding discussion notes, a large portion of the general population believes that playing the lottery is a preferred, or at least a legitimate, way to generate wealth to attain financial goals. It is obvious that these individuals do not understand that the lottery is designed to have a negative return. The lottery is actually less risky, but on average, much less favorable for generating higher average account balances. The lottery yields a very narrow distribution of account values, while the market exhibits a much flatter distribution of account values. It will likely be surprising to students that the lottery exhibits less risk than investing in the market. However, even with the increased risk, it should become clear that investing in the stock market will, with a high degree of probability, result in a higher terminal account value than the lottery. This result is consistent with the traditional positive risk-return relationship.

Our construct extends the existing treatments by Hansen and Prante (2006) and Slaughter (2006) who approach the lottery versus long-term investment strategy by calculating future expected mean values based on *fixed* annually compounded returns. We extend the existing treatments by incorporating a methodology that considers the stochastic nature of returns for both the lottery and long-term investment strategies. We think, that in contrast to using fixed average returns, the distribution of outcomes associated with various investment alternatives can be more effectively communicated by a probabilistic methodology.

The probabilistic methodology offered here can serve as a foundation for creating a variety of interactive learning activities about using the lottery and other strategies to create wealth. We employ a Monte Carlo simulation that draws outcomes from probability distributions created from lottery payoffs and historical stock returns. The stock return methodology most closely resembles that applied by Cooley, Hubbard and Walz (2003). The Monte Carlo simulations are conducted to probabilistically determine the net value of lottery winnings and the ending value of a stock fund over students' approximate remaining working life.[3] The two outcomes are then compared to determine the difference in the realized returns. Although there is a very small probability of hitting the lottery jackpot, in almost all instances the return to stock investments dominates the return to playing the lottery.

We provide the details of our methodology and the development of our model in the following section. However, when used in the classroom, students would be told that they have a choice to invest in the lottery or in the stock market, with the goal of saving for retirement. They would be informed that the lottery has the exact odds as those in the multi-state lottery game "Powerball" and that the stock market investments are made in securities traded on major stock exchanges. The results of students' investment decisions would then be determined through the model and discussed in class. The probabilistic nature of the model allows for varying results, including the possibility of hitting the Powerball jackpot. Yet, even with a jackpot win, the stock investment has a higher mean value as evidenced by higher accumulations at the end of the simulation period. Students would see the variation in the results, but understand the idea that the stock investment is more of a "sure thing."

Methodology and Data

The methodology developed in this study uses stochastic returns to mimic the variability found in both equity market returns and lottery returns. While it is informative to look at the positive expected return on one dollar per day invested in the stock market versus the negative expected return on one dollar per day invested in the lottery, using a probabilistic methodology focusing on a distribution of possible returns given "real life" trials adds an element that transcends the abstract nature of a single expected rate of return and more accurately represents return distributions over time.

The methodology employs @Risk software, a simulation engine which works within Excel. This software is commonly used in the financial services sector and in corporate financial decision making. The methodology simulates 7,560 days of activity in an equity market ("the retirement fund"), in which $1 is contributed daily and returns are randomly generated by using a probability distribution fit to historical returns of the New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and NASDAQ.[4] Simultaneously, 7,560 days of activity in a multi-state lottery game, Powerball, are also simulated. Contributions of $1 are also made daily and returns in the lottery portion of the model ("the lottery fund") are based on the odds of matching 12 possible winning and non-winning outcomes generated using the parameters of Powerball. The final outcome of one trial of the simulation is an aggregate account balance based on the wagers and stochastic returns of the retirement fund as well as the aggregate account balance based on the contributions and stochastic returns of the lottery fund. The balance for each alternative is generated using randomly assigned returns or payoffs. The process of building ending balances of retirement and lottery funds repeats until convergence is reached. Using @Risk automates the iterative process of generating multiple trials. Further, it presents results showing detailed information on the experience for each individual investor (one trial) as well as probability distributions of ending balances for the retirement fund and the lottery fund across multiple investors (all trials). These results are displayed in a

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[3] We use a 30 year time horizon, consistent with a popular notion among students that they will be in a position to retire by age 50.
[4] The use of 7,560 trading days is based on an average of 252 trading days per year for 30 years.

visually appealing graphical format, which facilitates discussion and interpretation of stochastic results with students.

Because returns are stochastically assigned for the investment fund, the timing of contributions as well as the long-term patterns in positive and negative returns are more accurately reflected in the fund performance. Many retirement planning tools use a fixed return based on the historic performance of the particular investments considered. For example, 5% might be used for bond fund investments because this is the historic average for these types of funds. However, as noted in Cooley, Hubbard and Walz (2003), Booth (2004) and Boinske (2003) and as this paper's results demonstrate, timing is an issue. When average return is used, the resulting variation in realized returns is ignored. The timing of contributions dramatically affects the long-term growth.

Any combination of lottery and financial indices could be considered. However, because this model will eventually serve as the basis for interactive learning activities, this application simulates the performance of the three primary U.S. equity markets and the performance of the Powerball lottery game over a 7,560 day period. Using just one index keeps the simulation less complex and allows the focus to be on the return provided by a daily investment versus a daily lottery ticket purchase.

Experimental Design – Lottery

For the lottery, the experimental design is relatively straight forward. In this application, the multi-state Powerball game is modeled. A Powerball drawing consists of a random draw without replacement of five numbers from a pool of numbers ranging in value from 1 to 55. These are referred to as the "white numbers." In addition, one "Powerball" number is chosen from a separate pool of numbers ranging in value from 1 to 42. Matching all five "white" numbers and the "Powerball" number results in the player winning the jackpot, which has a lower bound of $15 million and no upper bound. The jackpot is progressive in nature, meaning that it continues to grow until someone wins. To make the problem tractable, it is assumed that the jackpot prize is fixed at $100 million. Further, matching fewer than all six numbers may result in a fixed prize, lower than that of the jackpot. The amounts of these prizes are based on both the number of white numbers matched and whether or not the Powerball number is matched. There are eight of these lesser prizes. Finally, in the most likely outcome the player will not match enough numbers to win any prize in three of the possible outcomes. [5] These twelve unique outcomes are presented in Table 1.

Table 1: Combinations Of Powerball Lottery Payoffs

White Balls Matched	Powerball Matched	Prize	Odds for Winning Combinations
5 of 5	1 of 1	$100,000,000	1:146,107,962
5 of 5	0 of 1	$200,000	1:3,563,609
4 of 5	1 of 1	$10,000	1:584,432
4 of 5	0 of 1	$100	1:14,254
3 of 5	1 of 1	$100	1:11,927
3 of 5	0 of 1	$7	1:291
2 of 5	1 of 1	$7	1:745
2 of 5	0 of 1	$0	N/A
1 of 5	1 of 1	$4	1:127
1 of 5	0 of 1	$0	N/A
0 of 5	1 of 1	$3	1:69
0 of 5	0 of 1	$0	N/A

[5] It is important to note that the odds of winning, for example, $3 are 1:69 rather than 1:42 because the $3 prize is contingent upon matching exactly the Powerball number and exactly zero "white" numbers. Because of the possibility of winning more valuable prizes, the odds of winning exactly $3 are worse than the odds of winning at least $3.

Each trial represents the result of a student's investment choice in the lottery. To model the outcome of the lottery, a 7,560 row x 1 column vector is created in Excel. Excel's random number generator is used to create a random number from the uniform distribution with a value ranging between one and the odds of winning that prize, inclusive. For example, the odds of winning $200,000 are 1:3,563,609, so the random number generated will be between the values of 1 and 3,563,609, inclusive. This is repeated for each of the 9 outcomes with a positive outcome, resulting in a 7,560x9 matrix representing the results of one trial of 7,560 days. Should Excel's random number generator return a value of "1" in any cell in row x, the maximum prize value is recorded from the 1x9 vector in row x (winning combinations are mutually exclusive), net of the $1 required to play. Should no prize cell return a value of "1", -$1 is recorded as the net winnings for that day. We must make an assumption concerning the treatment of lottery winnings. While anecdotal evidence suggests that at least a portion of lottery winnings are parlayed by purchasing more lottery tickets, we conservatively assume that all prize winnings are placed into a riskless account with daily compounding earning 5% APR for the amount of time remaining between the date of the win and the end of the 30 year period.[6] For each trial, the balance in the lottery fund is the maximum prize per day summed across all 7,560 days. See Figure 1 for an example of a row with a winning outcome and a row with a non-winning outcome. In Panel 1a, the $4.00 prize is won because the randomly drawn number is "1," resulting in the $4.00 maximum prize for that trial. In Panel 1b, no prize is won because none of the randomly drawn numbers is "1," resulting in a maximum prize of -$1.00 for the trial.

Figure 1
Excel Output for Lottery Trial (Excerpt)

1a. Winning Outcome

Prize	$3	***4***	$7	$7	$100	$100	$10,000	$200,000	$100,000,000
Random Number	61		272	143	7385	12806	545593	3263145	19106053
Max Prize	***$4.00***								

1b. Non-winning Outcome

Prize	$3	4	$7	$7	$100	$100	$10,000	$200,000	$100,000,000
Random Number	17	2	371	86	3932	3766	481322	76975	83663507
Max Prize	-$1.00								

Experimental Design – Stock Market

Stock market returns are assumed to be independently and identically distributed (i.i.d.) like the lottery. However, the uniform distribution used for drawing lottery numbers is not appropriate when generating random stock market returns. In order to choose the distribution that best fits the historic stock market returns, we rely on historical daily return data. This method is similar to that used in Cooley, Hubbard and Walz (2003). Using data from the Center for Research in Security Prices (CRSP) between 1965 and 2005, the @Risk distribution fitting tool is used to choose from among several different continuous probability distributions to find the one that most closely matches the daily returns on the CRSP value-weighted

[6] Since the number of periods in the matrix represents trading days, the investment returns on lottery winnings are adjusted to reflect true daily compounding, not "trading day compounding."

index[7]. The distribution exhibiting the best fit to the daily return data (using a chi-squared test) was the logistic distribution[8] with a mean of 5.3014×10^{-4} and standard deviation of 4.5827×10^{-3}. The stock market returns for the sample period are then generated by making 7,560 random draws from the logistic distribution with the mean and standard deviation shown above. Starting in day one, $1 is added to the investment account. The value of the retirement fund is then determined according to the Equation 1.

(1) $V_{t+1} = [V_t \times (1 + k_t)] + 1$ where
V_t = the value of the fund, in dollars, on day t
k_t = the randomly selected return for day t

The value of these contributions at the end of 7,560 days, $V_{7,560}$, constitutes the value of the retirement fund for one trial.

Simulation and Results

Using @Risk, the 7,560 day experiment is run until convergence is reached. Rather than simply looking at the fixed expected value of $1 invested every day in the market (or the lottery), the model's randomness creates a distribution of outcomes simulated over the course of 7,560 days. In other words, using simulation emphasizes the effects of risk and generates a unique value for each trial, whereas using fixed expected returns may be viewed as riskless by students, since the expected value never changes. Following is a discussion of the results of one typical simulation. Summary statistics are reported in Table 2.

Table 2
Summary Statistics for a Typical Simulation Run[9]

	Equity Market (Simulation)	**Powerball Lottery**
Mean	$98,678.45	-$5,180.10
Min	$13,704.77	-$6,231.21
Max	$707,749.80	$275,539.80
Standard Deviation	$63,906.00	$5,873.62
5th Percentile	$33,249.68	-$6,024.83
95th Percentile	$217,771.40	-$4,989.83

[7] The CRSP value-weighted index is a portfolio constructed from all securities traded on the American Stock Exchange (AMEX), New York Stock Exchange (NYSE) and the National Association of Securities Dealers Automated Quotation system (NASDAQ) with the exception of American Depository Receipts (ADRs). The amount of each firm held in the index is proportional to its market value of equity. In other words, larger firms comprise a larger proportion of the index's holdings than smaller firms.
[8] The choice of the logistic distribution, although the best of the distributions tested, is not perfect. Data distributed logistically exhibit zero skewness. There is clear evidence to suggest that stock returns are positively skewed. However, ignoring positive skewness is not an issue in demonstrating the value of equity investment over lottery expenditures, since ignoring positive skewness will bias the results in favor of using lottery tickets as the dominant investment strategy.
[9] The expected value of the retirement account based on the fitted mean return of the equity markets is $101,804.21.

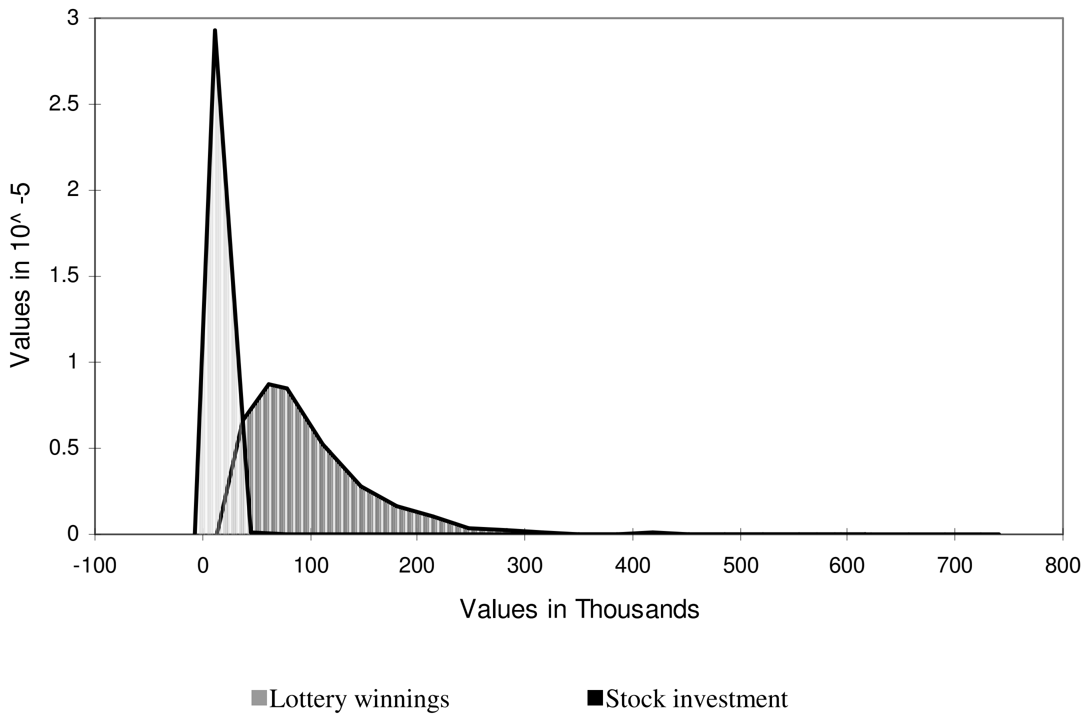
As noted in the Table 2, simply calculating an expected value of the retirement account based on the fitted mean return of the equity markets yielded a mean retirement account value of $101,804.21. The Monte Carlo simulation process yielded a mean retirement account balance of $98,678.45 in a typical sample trial. Further, the standard deviation as well as the limits at the upper and lower tails emphasize returns are indeed random. Five percent of the time, the final retirement account balance is expected to be less than $33,249.68. In addition to showing risk inherent to equity markets, these results are in sharp contrast to those expected from the lottery. With the lottery, a positive outcome is quite rare. In fact, only five out of one hundred people could expect to do better than a $4,990 net *loss* in their purchase of 7,260 tickets. Table 3 outlines the probabilities of building account balances in excess of several thresholds of interest for both the lottery and the stock market. Probability distribution functions are graphed for both the lottery and the retirement fund in Figure 2.

Table 3: Probabilities of Earning More Than X Dollars Under Stock Market Vs. Lottery

Stock Market	
x	**P(ending balance > x)**
$700,000.00	0.0000204
$600,000.00	0.0002828
$500,000.00	0.0005453
$400,000.00	0.0032880
$300,000.00	0.0189523
$200,000.00	0.0665758
$100,000.00	0.3680878
$0.00	1.0000000

Lottery	
x	**P(ending balance > x)**
$40,000.00	0.0006061
$35,000.00	0.0018182
$30,000.00	0.0030303
$25,000.00	0.0048485
$20,000.00	0.0066666
$15,000.00	0.0084849
$10,000.00	0.0109091
$0.00	0.0145454
($5,655.51)	0.5000000

Figure 2: Probability Distributions Functions for Lottery vs. Investments



As noted above, the students need not understand the methodology to benefit from seeing results such as those in Figure 2. They need only to make their investment decisions. The probabilistic results of those decisions would then be provided to the students and discussed from the perspective of which alternative actually involved more risk and which provided higher dollar returns.

Conclusions and Implications

The methodology and processes in this study represent the first step in developing an interactive learning activity to help educate college students about the realities of playing the lottery as a means to generate wealth. As the results indicate, it is very unlikely that the lottery will provide a retirement fund of any positive value. However, investing just $1 per day over a thirty-year period will provide an average of around $100,000 in retirement funds.

Our next step is to use the methodology to create instructional activities that help students develop an understanding of the problems associated with playing the lottery rather than starting an investment program. Through its use, students can better appreciate that the lottery is not a viable means for building wealth. It will also help students determine the benefits of planning for retirement and making other investment decisions based on financial factors including rates of return, investment time horizons and the power of compound interest on asset growth. The instructional activities can be designed for any level of college course from freshman to senior. They can also be used across various curricula from a life skills course to an upper-level financial planning course.

The methodology used in this study can also be used for further research. As discussed in this paper and others, the timing of contributions and the random walks of returns evidenced in the financial markets affects investment performance. The probabilistic-based simulation can help explore these issues in the accumulation period prior to retirement and help better understand how to address the effects of returns and timing in retirement and investment planning.

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