JOURNAL OF

ECONOMICS and FINANCE EDUCATION

JEFE

editor

E. F. Stephenson

MANAGING EDITOR

AEF

ACADEMY of ECONOMICS and FINANCE

Volume 21 FALL 2022 Number 2

Academy of Economics and Finance

Teachers' Knowledge and Attitudes about the Fed

Evgeniya A. Duzhak, K. Jody Hoff, and Jane S. Lopus[1]

ABSTRACT

A survey of teachers in the Fed's twelfth district investigates knowledge about how the central bank influences the economy. Focusing on teachers from different content areas, we find that teacher knowledge, for the most part, varies as expected given different teaching disciplines. Teachers are more aware of the Fed's goal of price stability than full employment and score relatively high on questions relating to current events. As well, teachers with more knowledge were more likely to have positive attitudes about the job the Fed was doing. We recommend improving Fed communications to improve economic education for teachers and the public.

Introduction

In what is frequently viewed as a post-truth era (Lewandowky et al. 2017), teachers' objectivity and factual knowledge is critical for countering rumors, false assumptions, and misinformation. This is perhaps especially true with respect to teaching economics, where social studies teachers often have only minimal coursework in the subject matter (Bosshardt and Watts 2005). Because knowledge of economics is essential for people to be informed consumers, producers, savers, investors, and, most importantly, citizens and voters, economics is a critical part of a person's overall education. In order to effectively teach the subject, a good understanding of economics is not only important for economics teachers, but for some other social studies teachers as well. For students, the high school economics class is often the only formal training in economics that these future voters receive.

As of 2020, 23 out of 50 U.S. states required students to complete a course in economics for high school graduation, another two states required that a high school economics course be offered as an elective, and all 50 states included economics in the K-12 standards (Council for Economic Education 2020). Economics is also taught in some other social studies classes in the context of history, government, and current events. A central topic in a typical U.S. high school economics curriculum is understanding the role that the Federal Reserve System (the Fed), the central bank of the United States, plays in influencing the economy (Council for Economic Education 2010). It is therefore interesting to review results from a 2017 survey of 766 high school teachers about their knowledge of monetary policy and the role of the Fed.

In analyzing our survey data, we investigate factors related to teacher knowledge and attitudes about the Fed with a focus on the subjects taught by the teachers and expectations about what different types of teachers should reasonably know. Following a review of the literature, we describe our survey and the resulting data and examine responses to individual questions in our assessment instrument. We next look at correlations between knowledge about the Fed and attitudes about how the Fed was doing at the time of the survey. We conclude by discussing the role of Federal Reserve communication and how improved communication might improve economic education.

Literature Review

Teaching economics in high schools has been a topic of several survey articles and much focused research since the 1960s (Walstad and Watts 2015). Within this research, a topic of interest has been how teacher

[1] Duzhak: Federal Reserve Bank of San Francisco; Hoff: Boise State University; Lopus (corresponding author): California State University East Bay; jane.lopus@csueastbay.edu. This work was supported by the Federal Reserve Bank of San Francisco. Opinions expressed in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of San Francisco or the Board of Governors of the Federal Reserve System.

characteristics affect student learning of economics, with part of the focus on teacher preparation. Overall, teachers are often found to be ill-prepared to teach economics. A 2005 study that reviewed teachers' college transcripts found that teachers credentialed to teach high school social sciences generally do not take more than one course in economics, and about a third take no undergraduate economics courses (Bosshardt and Watts 2005). This is problematic because research has shown that teachers need several courses in economics before student knowledge improves (Lynch 1990), and that a three-year master's degree program in economics for teachers is effective in improving both teacher and student knowledge (Allgood and Walstad 1999). However, a 2014 study found that teachers with undergraduate degrees in economics were associated with lower student scores on an essay test compared to teachers without economics degrees, perhaps due to an inability of the teachers to communicate their advanced knowledge to students (Valletta et al. 2014).

Perhaps surprisingly, there is not a consensus about the effects of statewide or district course mandates on student learning of economics. Some studies show positive effects (Gill and Gratton-Lavoie 2011) while others do not (Grimes et al. 2018). Theoretically, mandates could improve economic knowledge because when more students are required to take economics, more resources may be provided for teaching economics and teachers could be incentivized to increase their economics knowledge and teaching skills. On the other hand, many teachers in states with mandates may be ill-prepared to teach economics, leading to negative effects on student learning. A 2016 study on effects of mandates concludes that most research findings show that current mandates do not increase economic literacy among high school students (Perumal et al. 2016).

Much of the economic education literature involving teacher attitudes looks at attitudes toward teaching economics and the impact this has on student learning. Research has shown that teachers who have negative attitudes about teaching economics, perhaps tied to state mandates, have lower achieving students (Marlin 1991). However Dills and Placone (2008) found that South Carolina teachers' attitudes toward economics did not significantly affect student test scores, but that teachers who volunteered to teach economics rather than being assigned to teach it had higher achieving students.

At least two recent studies investigate what the general public knows about economic policies. Blinder and Krueger (2004) use telephone survey data to determine factors that affect public opinion about economic issues including the public debt, social security, and the minimum wage, arguing that this is important because public opinion influences politicians and thus political decisions. They find that knowledge about these issues, measured by nine factual questions, influences opinions more than self-interest, but less than ideology. The authors conclude that the level of knowledge shown by the general public in their study was "reasonable," which was on average about 50 percent correct for those who attended college. Van der Cruijsen (2015) looks specifically at knowledge of monetary policy as measured by a survey of Dutch households' ability to correctly identify objectives of the European Central Bank, given a list of 11 possibilities. Only three of the 11 objectives are correctly identified by a majority of respondents. The author argues that knowledge of the central bank is important due to the role of households in forming inflationary expectations.

Other recent studies look specifically at knowledge about central banks and attitudes toward central banks. Using 2001-2017 survey data from 2,000 respondents in England to construct a knowledge index about monetary policy, Haldane and McMahon (2018) relate this knowledge to satisfaction with the central bank's actions. Viewing satisfaction as a proxy for trust in the central bank, they find that trust in general fell during and following the financial crisis of 2008. They further find that satisfaction with central banks' actions is positively correlated with the level of understanding about central banks. Similarly, a New Zealand survey of business firms found that managers were generally uninformed about central bank strategies, and that lack of knowledge was associated with inflation expectations largely unrelated to the Reserve Bank of New Zealand's widely publicized inflation target (Kumar et al. 2015). Similar results have been found for households in the United States (Binder 2017).

This article contributes to the economic education literature by investigating survey responses from a cohort of high school teachers from different disciplines about their knowledge of the Fed and monetary policy, and how this level of knowledge may relate to their attitudes about the Fed. Our teachers fall into three groups: those who have taught economics, those who have taught other social studies but not economics, and those who have never taught any of the social studies. Because we would not expect non-social studies teachers to know more economics than the public in general, we include this group as representative of the college-educated general public.

Survey and Data Description

In April and May 2017, the Education and Outreach Department of the Federal Reserve Bank of San Francisco (SF Fed) conducted a survey of high school teachers located within the Fed's 12th District.[2] The mission of this department was to help students, educators and the public learn about the Fed and the Fed's role in the U.S. economy. The 12th District consists of the nine western states of Alaska, Arizona, California, Hawaii, Idaho, Nevada, Oregon, Utah, and Washington, and includes close to 21 percent of the U.S. population (Federal Reserve Bank of San Francisco 2020). The survey was conducted by a market research firm that initially contacted 9,500 high school teachers via stratified sampling. The resulting sample consisted of responses from 766 teachers,[3] however many responses were incomplete due to respondents dropping out of the survey.

The survey consisted of 25 questions addressing knowledge, demographics, teaching experience, attitudes about the Fed and teaching, knowledge of Fed resources and what resources teachers find most useful. In addition to our current research study, responses to these questions establish benchmarks to help assess and improve future Fed education programs. Nine knowledge questions[4] address teachers' basic understanding of the objectives of the Fed, monetary policy and economic conditions. Eight of the knowledge questions were written by the SF Fed's Education and Outreach Department, and the other was based on a question from the 2015 Pew Research Center News IQ Quiz (Pew Research Center 2015).

Table 1 provides descriptive statistics for responses to background questions from the survey.[5] Overall, 52 percent of the respondents identify as male, and over half are aged 41 to 60. All respondents in the dataset were currently teaching, with 97 percent teaching full time. The teachers are from all states in the Fed's 12th District, with the largest number (45 percent) from California. Education is the most-reported undergraduate college major, followed by social sciences other than economics and majors related to math and science.[6] About five percent of respondents report majoring in economics. Most teachers have master's degrees (and although not shown in the table, most of these are in education). Almost 50 percent of teachers participated in more than 20 hours of professional development in the past two years.

The teachers are experienced, with half having taught for 15 years or longer. About 28 percent were teaching or had taught economics, 30 percent were teaching or had taught social studies but not economics, and 42 percent had never taught social studies. In the spring of 2017, at the time of the survey, approximately 77 percent of respondents believed the Fed was doing a good job. The majority of teachers, 62 percent, had never heard of the teaching resources available through the Fed. Although not shown in the table, when asked to choose the most useful resources from a list of six possibilities, teachers preferred (in ranked order) lesson plans, interactive tools and videos over games, tests, and podcasts.

Economics Knowledge and Type of Teacher

To examine the knowledge of the teachers about the Fed and monetary policy, we first look at the percent of correct responses to each of the nine knowledge questions and sort for whether the teachers have taught economics, have taught social studies but not economics, or are not social studies teachers. Dividing teachers into these categories provides insights into basic economics knowledge of economics teachers, who would logically teach about the Fed, and other social studies teachers, who may do so in the context of topics such as history or government. However, these other social studies teachers also may teach in disciplines such as psychology or sociology that may not touch on content related to the Fed. Knowledge of non-social studies

[2] The California State University, East Bay Institutional Review Board approved the research protocol for this study.

[3] A total of 770 people responded to at least one question in the survey. However we are unable to determine the subjects taught by four respondents. Because subjects taught are a central topic of this paper, we do not include these four in the study.

[4] A tenth knowledge question asked for the definition of inflation used in class, and provided an option for "do not cover the topic of inflation in any of my classes." We do not include this question in our analysis because we cannot determine the knowledge of the teachers who do not teach about inflation.

[5] Survey questions not included in Table 1 are the name of the school district and questions specific to economics teachers, including qualifications and attitudes toward teaching economics and monetary policy.

[6] College majors total more than 766 and over 100 percent due to double majors.

Table 1: Descriptive Statistics (excluding content questions)

Survey Question		Frequencies
Gender	Male	291 (52%)
	Female	247 (44%)
	Prefer not to disclose	23 (4%)
Age (years)	≤ 29	57 (10%)
	30 – 40	120 (21%)
	41 – 50	166 (30%)
	51 – 60	140 (25%)
	> 60	78 (14%)
Currently teaching this academic year	Yes, full time	746 (97%)
	Yes, part time	20 (3%)
State where teaching	AZ	103 (13%)
	CA	343 (45%)
	NV	48 (6%)
	OR	54 (7%)
	UT	71 (9%)
	WA	60 (8%)
	Other (AK, HI, or ID)	87 (11%)
Undergraduate college major (Includes double majors)	Business	45 (6%)
	Economics	40 (5%)
	Education	415 (55%)
	Other social science	352 (46%)
	Math-Science	289 (38%)
	Other	100 (13%)
Master's degree	Yes	503 (66%)
	No	263 (34%)
Professional development hours in past two years	0 – 1	25 (3%)
	2 – 10	126 (17%)
	11 – 20	235 (31%)
	> 20	373 (49%)
Teaching experience (years)	< 5	127 (17%)
	5 – 10	123 (16%)
	11 – 15	128 (17%)
	> 15	385 (50%)
Teaching experience (subjects)	Economics	214 (28%)
	Social studies; never economics	227 (30%)
	Other subjects; never social studies	325 (42%)
Number social science classes taught in past academic year	0	350 (46%)
	1 – 2	117 (15%)
	3 – 5	172 (22%)
	> 5	90 (12%)
Fed doing a good job	Yes	530 (77%)
	No	161 (23%)
Heard of and use of Fed educational resources	Never heard of; never use	376 (62%)
	Heard of; never use	77 (13%)
	Heard of; use occasionally	127 (21%)
	Heard of; use a great deal	23 (4%)

N: range 561 to 766, due to missing data

teachers, who are unlikely to teach about the Fed, may be representative of the knowledge of the college-educated general public. Because of their teaching assignments, which are related to their college majors, we expect that economics teachers will score highest and should have a good command of the content.

We expect that other social studies teachers will score next highest, due to some not teaching about the Fed and monetary policy. We expect that non-social studies teacher will score lowest of the three groups, but we expect that the knowledge they exhibit will indicate awareness of current events and issues related to the Fed's objectives and policies. Looking at incorrect responses to the knowledge questions as well as correct responses allows us to see where there is confusion with respect to the content.

Table 2 shows each of the nine questions and the percentage of correct and incorrect responses divided by teacher type. Questions 1 through 5 are true or false questions asking teachers to identify the Fed's main objectives.[7] Questions 2 and 4 are true, and Questions 1, 3, and 5 are false. Question 1, printing money, is correctly identified as false by about 71 percent of economics teachers, 59 percent of other social studies teachers, and 62 percent of non-social studies teachers. We consider these scores to be reasonable, given that thinking that the Fed prints money is a common misperception about the role of the Fed (Lopus and McDaniel 2015). Also, reputable sources in the popular news media do at times refer to the Fed printing money, perhaps because the specifics of open market operations are difficult to explain. (See Irwin 2020.)

Moving to the other incorrect options for the Fed's main objectives, over three quarters of economics teachers and over half of other teachers know that stock market stability is not a main objective of the Fed (Question 3). Knowledge that the Fed does not balance the (implicitly government) budget (Question 5) is even stronger, with over 96 percent of economics teachers, over 85 percent of other social studies teachers and almost 80 percent of non-social studies teachers correctly identifying this option as false. While stock market stability and balancing the budget are perhaps not as abstract concepts as printing money, we are reassured with this level of knowledge for the three types of teachers.

Questions 2 and 4 represent the Fed's dual mandate of price stability and full employment respectively. Teachers overwhelmingly identify price stability as a main objective of the Fed in Question 2. Close to 88 percent of economics teachers, 78 percent of other social studies teachers, and 66 percent of non-social studies teachers are aware that the Fed seeks to maintain price stability. Because of the importance of the goal of price stability as part of the Fed's dual mandate, we find the knowledge of the teachers to be encouraging. However, we are surprised at the relative lack of knowledge about the Fed's objective of maintaining full employment. Although the Fed has had a dual mandate from Congress since 1977 to promote both price stability and maximum employment, fewer than 41 percent of economics teachers and only 13 to 14 percent of other teachers correctly identify full employment as a main objective of the Fed.

Question 6 asks what the Fed should do if facing negative GDP growth and a one percent inflation rate, a situation that could be indicative of a recession. Designed as a multiple-choice question with four options and one correct response, the correct answer, "b. Buy government securities," was the one most frequently chosen by economics teachers. But perhaps surprisingly, only 44 percent of economics teachers responded correctly to the question and only 16 to 17 percent of other teachers correctly responded. However, while we would expect that those who teach macroeconomics would know details of the tools of open market operations, it is less likely that other social studies teachers or non-social studies teachers would know this. Non-economics teachers most often chose "a. Target a higher federal funds rate and increase the reserve requirement." Both of these monetary policy tools would be contractionary and would serve to exacerbate negative GDP growth. The second most commonly chosen answer by all teachers, including economics teachers, was "c. Increase the money supply by achieving a higher federal funds rate." While increasing the money supply would correctly address the negative GDP growth, which may have made this answer appealing, the response is incorrect because this is not achieved by raising the federal funds rate.

Question 7 differs from the other four questions in that it is not directly about the Fed or monetary policy, but rather asks teachers to identify what national statistic is displayed in a graph ranging from (implicitly) 2001 – 2017. This question also differs in that it is the one adapted from the Pew Research Center's 2015 News Quiz. Question 3 is similar to Pew's question except that the Fed survey substituted "GDP growth" for the Pew option "The high school dropout rate." Most teachers chose the correct answer, unemployment, with 85 percent of economics teachers, 69 percent of other social studies teachers, and 74 percent of non-social studies teachers responding correctly. These relatively high scores for all teachers are encouraging because

[7] On the survey, Questions 1 – 5 were written as one question asking respondents to identify the main objectives of the Fed from the list of five options, checking all that apply. We break this down into five true or false questions following van der Cruijsen (2015) and to better determine teacher knowledge of individual objectives.

Table 2: Knowledge Questions (% Response by Type of Teacher)

	Economics	Other Social Studies	Non-Social Studies
*Questions 1 – 5: Identify the Fed's main objectives (True – False)**			
1. Printing money (False)	**70.6%**	**58.6%**	**62.2%**
2: Price stability (True)	**87.9%**	**78.0%**	**66.2%**
3: Stock market stability (False)	**75.7%**	**57.3%**	**55.1%**
4. Full employment (True)	**40.7%**	**13.7%**	**13.5%**
5. Balance the budget (false)	**96.3%**	**85.5%**	**79.4%**
*Percent choosing "true" only for responses 2 and 4	**27.6%**	**4.4%**	**3.7%**
N (Total number of respondents=766)	*214*	*227*	*325*
Question 6: Which scenario would, in your opinion, be the one the Fed should follow when facing negative GDP growth accompanied by a 1% inflation rate?			
a. Target higher federal funds rate & increase reserve requirement	15.5%	32.9%	36.0%
b. Buy government securities	**43.3%**	**17.4%**	**16.3%**
c. Increase money supply by achieving a higher federal funds rate	25.1%	26.1%	26.2%
d. Keep things unchanged	16.0%	23.6%	21.5%
N (Total number of respondents=520)	*187*	*161*	*172*
Question 7: This graph shows the trend in what national statistic? 			
a. Inflation rate	8.6%	11.2%	11.0%
b. Corporate tax rate	0.5%	2.5%	1.7%
c. Unemployment rate	**85.0%**	**68.9%**	**74.0%**
d. GDP growth	5.9%	17.4%	13.3%
N (Total number of respondents=527)	*187*	*161*	*179*
Question 8: What can the Fed do to reduce high inflation?			
a. Increase government spending	5.5%	7.2%	5.8%
b. Increase the federal funds rate	**55.2%**	**27.6%**	**16.9%**
c. Decrease government spending	4.4%	11.2%	19.2%
d. Decrease the federal funds rate	11.0%	19.1%	16.3%
e. Increase the federal funds rate & decrease government spending	23.8%	34.9%	41.9%
N (Total number of respondents=505)	*181*	*152*	*172*
Question 9: Who is the current Chair of the Federal Reserve Board?			
a. Ben Bernanke	2.7%	7.6%	14.0%
b. Alan Greenspan	2.7%	8.9%	18.6%
c. Paul Volcker	1.1%	5.7%	11.0%
d. Janet Yellen (as of 2017)	**93.5%**	**77.7%**	**56.4%**
N (Total number of respondents=514)	*185*	*157*	*172*
Average % Correct on 9 questions	**72.0%**	**53.9%**	**48.9%**

Note: Correct answers in **bold**

they show knowledge of current events related to economics, whether or not the teachers teach about unemployment. The non-social studies teachers may have outperformed the other (non-economics) social studies teachers because many teach math and may be more familiar with interpreting graphs. The second most common answer was "d. GDP growth" for non-economics teachers and "a. Inflation rate" for economics teachers. Like Question 6 (and Question 9) respondents could choose one correct answer out of four options. Although the groups of respondents are not directly comparable, it is interesting that 81 percent of the Pew respondents who were college graduates responded correctly (Pew Research Center 2015), compared to an average of 74 percent of teachers in our survey. This may be due to the different response options.

While Question 6 asks about appropriate monetary policy for a recession (without using the term), Question 8 asks for the appropriate monetary policy during an inflationary period. Both questions required knowledge about details of open market operations, so we would expect that many economics teachers would know the answers as well as some other social studies teachers, but we expect that fewer non-social studies teachers would have knowledge of the specifics of fighting inflation with respect to the federal funds rate. There were five possible responses to Question 8, with "b. Increase the federal funds rate" as the correct response. As expected, most economics teachers (55 percent) responded correctly, compared to fewer other social studies teachers and non-social studies teachers (28 percent and 17 percent respectively). The non-economics teachers were more likely to choose option e (increase the federal funds rate and decrease government spending) than economics teachers, apparently not recognizing that control of government spending is fiscal policy and not conducted by the Fed.

Although Questions 6 and 8 are not directly comparable in part because they have different numbers of possible responses, it is interesting that social studies teachers (economics and others) more often responded correctly to Question 8 (how the Fed can address inflation) than to Question 6 (how the Fed should address a recession). This is despite Question 8 having a lower probability of guessing correctly (20 percent versus 25 percent for Question 6). More accurate knowledge about how to respond to inflation than to a recession is consistent with the responses to Questions 2 and 4, where more teachers identified price stability as an objective of the Fed compared to full employment. However, this result contradicts a finding about the knowledge that players exhibit in playing *Chair the Fed, A Monetary Policy Game* conducted by the Federal Reserve Bank of San Francisco. A 2021 study reports that players in the simulation were more likely to correctly address problems of unemployment than inflation (Duzhak et al. 2021).

Question 9 asked respondents to identify the Chair of the Federal Reserve Board. As of 2017, the time of the survey, the correct answer was "d. Janet Yellen." Like Question 7, as a current events question, responses may indicate, in part, whether teachers are following current economic news. We would expect that more economics teachers and other social studies teachers would do so, compared to non-social studies teachers. Most teachers in each category got the question right, including close to 94 percent of economics teachers. Seventy-eight percent of other social studies teachers knew that Janet Yellen was head of the Fed, as did 56 percent of those who had never taught social studies.

Looking at average overall percentage scores for different types of teachers on the nine questions, we see that economics teachers scored 72 percent compared to about 54 percent for other social studies teachers and about 49 percent for non-social studies teachers. These scores were brought down by low scores on Questions 4, 6, and 8. While lack of knowledge about the goal of full employment for the Fed is puzzling (Question 4), a lack of knowledge about technicalities of open market operations (Questions 6 and 8) is not surprising. Many high school economics teachers may teach generalities of monetary policy for addressing economic fluctuations (e.g. increase or decrease the money supply) without specifics related to open market purchases and the federal funds rate. This is more often the approach to explaining monetary policy taken in the mainstream news media and may be in keeping with relatively strong teacher knowledge of current events as evidenced in Questions 7 and 9, where teachers received the highest scores.

Looking at the number of responses to the nine questions in Table 2, we see that many teachers dropped out of the survey after the first question, and that they did not do so randomly. While the full sample of 766 teachers responded to Questions 1 – 5, only 505 – 527 responded to the following four questions. Although they may have skipped questions along the way, we see that 86 percent (185/214) of economics teachers remained in the survey to respond to Question 9 compared to 69 percent (157/227) of other social studies teachers and 53 percent (172/325) of non-social studies teachers. Because identifying the objectives of the

Fed was difficult,[8] those dropping out may have assumed they would be unable to answer any of the additional questions correctly, and were therefore reluctant to try. And because non-economics teachers were more likely to drop out than economics teachers, their true knowledge is likely lower than reported, taking attrition into account.

Taken together and as expected, we observe that economics teachers scored higher on each question compared to other social studies teachers, and that those who do not teach social studies scored the lowest except on Questions 1 and 7, where they scored higher than non-economics social studies teachers. Teachers in general appear to know more about the Fed's role pertaining to inflation than unemployment, and also score relatively high on questions related to current-events or recent economic data (Questions 7 and 9). With the exception of the lack of knowledge of the Fed's full employment objective, and given the technical nature of two of the questions regarding monetary policy, we believe that the teachers' knowledge is well within expectations for their respective teaching disciplines.

Attitudes about the Fed and Knowledge Scores

We next turn to a discussion of teacher attitudes toward the job the Fed was doing at the time of the survey, and how this correlates to the knowledge scores. As referenced in Table 1, a survey question asked:

Which of these statements comes closer to your own view, even if neither is exactly right?

A. The Fed is doing a good job regulating the economy.

B. The Fed is doing a bad job regulating the economy.

Table 3 shows the breakdown in the responses to this question for economics teachers, other social-studies teachers, and non-social studies teachers. The overall Pearson correlation coefficient of the knowledge score and opinion about the job the Fed was doing was positive and significant. We see that economics teachers are more likely to say that the Fed was doing a good job (87 percent) compared to other social studies teachers (77 percent) and non-social studies teachers (69 percent). Because economics teachers scored higher on the knowledge test, followed by other social science teachers and with non-social studies teachers scoring the lowest, the positive correlation between knowledge and positive attitudes is reflective of the type of teacher.[9]

Table 3: Attitude about Whether the Fed is Doing a God Job (% Response by Type of Teacher)

	Economics Teachers	Other Social Studies Teachers	Non-Social Studies Teachers
Fed is Doing a Good Job	86.6%	77.0%	69.3%
Fed is Doing a Bad Job	13.4%	23.0%	30.7%
N (Total number of respondents=691)	*202*	*209*	*280*
Pearson correlation coefficient for knowledge score and thinking the Fed is doing a good job = .235; p<.01			

To put this into perspective we provide information about the state of the U.S. economy around April and May 2017, at the time of our survey. Table 4 summarizes inflation and unemployment data from the U.S. Bureau of Labor Statistics from March through June 2017, which relate directly to the Fed's dual mandate of stable prices and maximum employment. First, we see that the inflation rate at the time of the survey compared to the prior year and throughout 2017 was very close to the Fed's target rate of two percent. While the Fed does not target an unemployment rate, there is widespread agreement that the U.S. economy was at or near full employment in spring 2017, with an unemployment rate of about 4.4 percent. For example, in a speech delivered to the Bank of Japan on May 25, 2017, Charles Evans, President of the Federal Reserve Bank of Chicago and a Fed Governor, stated "Today we have essentially returned to full employment in the U.S." (Evans 2017).

[8] As shown in Table 2, only about 28 percent of economics teachers and about four percent of other teachers correctly identified both price stability and full employment as objectives of the Fed, while choosing no other options.

[9] As a test of robustness, we also regressed teacher knowledge on attitude about the Fed while controlling for teacher education, experience, sex, and statewide mandates for teaching economics. We found the negative relationship between knowledge and attitudes to be significant and robust given the controls.

Table 4: Inflation and Unemployment Rates, March – June 2017

	March 2017	April 2017	May 2017	June 2017	2017 Average Annual Rate
Inflation Rate (from prior year)	2.4%	2.2%	1.9%	1.6%	2.07%
Unemployment Rate (monthly)	4.4%	4.4%	4.4%	4.3%	4.34%

Source: U.S. Bureau of Labor Statistics; retrieved December 15, 2020

Because there has been an overall decrease in trust in central banks since the financial crisis (Haldane and McMahon 2018), for some teachers saying that the Fed was doing a bad job in 2017 may be reflective of a general lack of trust in the institution of the central bank rather than their not knowing that the Fed was achieving its dual mandate. There are of course other reasons why teachers may perceive that the Fed was doing a bad job, including ideology and knowledge of perceived past failures in the Fed's policies, including during the Great Depression and the high inflation during the 1970s and early 1980s. Also, the survey question only allowed respondents to indicate if they thought the Fed was doing a good job or a bad job, and did not provide options for the degree of agreement with the statement or a "no opinion" or "do not know" option. Therefore survey respondents who know little about the Fed and its policies likely would not feel qualified to judge the job that the Fed was doing, and some may have defaulted to "doing a bad job." But given these caveats, it is also likely that teachers who do not thoroughly understand the Fed's dual mandate and other aspects of monetary policy may be less likely to know that the Fed was achieving its dual mandate. From this perspective, it is reassuring that almost 70 percent of non-social studies teachers indicated that the Fed was doing a good job (along with over 86 percent of economics teachers and 77 percent of non-economics social studies teachers), which may be reflective of knowledge of the college-educated general public on the state of the economy in the spring of 2017.

Conclusions

While we believe that the knowledge and attitudes of the teachers are for the most part reflective of expectations given their instructional disciplines, we suggest that overall increased knowledge of the Fed and its policies could have positive effects for the economy through, for example, the formation of expectations (van der Cruijsen 2015) or influencing public policy (Blinder and Krueger 2004). A policy implication may lie in calling for more and better economic education, both for teachers and the general public. Deciding how this increase in economic education would be carried out is a challenge. Given the mixed outcomes for state mandates requiring high school economics courses, more research is necessary to determine what works in high school economics classes before we can unequivocally recommend universal high school economics requirements.

It may be that the Fed itself could communicate better with the general public about its purpose and policies, including with teachers. As Haldane and McMahon (2018) point out, central bank communications are often directed at experts and pitched at too high a level for understanding by most of the population, but this could change. More effective communication from central banks could build both knowledge and trust, and could benefit the mainstream media as well, which is an important means of transmitting information. Most Federal Reserve Banks have education divisions dedicated to improving economic education, and they could serve as conduits to increasing communication accessible and understandable to teachers and to the general public. That our survey indicates that most teachers have never heard of Fed educational resources indicates that the Fed could do a better job of communicating directly to teachers.

Whether increased and clearer communication from central banks will be successful in informing the general public about Fed policies is a matter of debate (Blinder 2018). However in a post-truth era, it is critical to provide data and information using comprehensible, non-technical language, and improving quality education for all should be everyone's priority, including that of central banks. Teachers and everyone else need to be able to objectively evaluate economic events and policy effects to make rational assessments about the validity of economic policies. With more education and unbiased information, we may have more informed citizens and voters, and a better functioning economy.

References

Allgood, Sam, and William B. Walstad. 1999. "The Longitudinal Effects of Economic Education on Teachers and Their Students." *Journal of Economic Education* 30: 99-111.

Binder, Carola. 2017. "Fed Speak on Main Street: Central Bank Communication and Household Expectations." *Journal of Macroeconomics* 52: 238-251.

Blinder, Alan S. 2018. "Through a Crystal Ball Darkly: The Future of Monetary Policy Communication." *The American Economic Review Papers and Proceedings* 108: 567-571.

Blinder, Alan S., and Alan B. Krueger. 2004. "What Does the Public Know about Economic Policy, and How Does it Know It?" *Brookings Papers on Economic Activity* 1: 327-387.

Bosshardt, William, and Michael Watts. 2005. "Teachers' Undergraduate Coursework in Economics in the *Baccalaureate and Beyond* Longitudinal Study." *Journal of Economic Education* 36: 400-406.

Council for Economic Education. 2020. *Survey of the States.* (*https://www.councilforeconed.org/wp-content/uploads/2020/02/2020-Survey-of-the-States.pdf*)

Council for Economic Education. 2010. *Voluntary National Content Standards in Economics.* (*https://www.councilforeconed.org/wp-content/uploads/2012/03/voluntary-national-content-standards-2010.pdf*)

Dills, Angela K., and Dennis Placone. 2008. "Teacher Attitudes and Teacher Knowledge in Economic Education." *Social Science Research Network* 1090639.

Duzhak, Evgeniya, K. Jody Hoff, and Jane S. Lopus. 2021. "Chair the Fed: Insights from Game Usage Data." *Journal of Economic Education* 52: 89-101.

Evans, Charles. "*Lessons learned and challenges ahead. A speech delivered on May 25, 2017, before the Bank of Japan – Institute for Monetary and Economic Studies in Tokyo, Japan*." (*https://www.chicagofed.org/publications/speeches/2017/05-25-lessons-learned-and-challenges-ahead-bank-of-japan*)

Federal Reserve Bank of San Francisco. 2020. "*San Francisco Fed Key Facts*." (*https://www.frbsf.org/our-district/about/key-facts/*)

Gill, Andrew M., and Chiara Gratton-Lavoie. 2011. "Retention of High School Economics Knowledge and the Effect of the California State Mandate." *Journal of Economic Education* 42: 319-337.

Grimes, Paul W., Meghan Millea, and M. Kathleen Thomas. 2008. "District Level Mandates and High School Students' Understanding of Economics." *Journal of Economics and Economic Education Research* 9: 3-16.

Haldane, Andrew, and Michael McMahon. 2018. "Central Bank Communications and the General Public." *American Economic Review Papers and Proceedings* 108: 578-583.

Irwin, Neil. "The Fed's Message: The Money-Printing Presses are Fired Up and Ready to Go." *The New York Times*. March 23, 2020. (*https://www.nytimes.com/2020/03/23/upshot/coronavirus-fed-extraordinary-response.html*)

Kumar, Saten, Hassan Afrouzi, Olivier Coibion, and Yuriy Gorodnichenko. 2015. "Inflation Targeting Does Not Anchor Inflation Expectations: Evidence from Firms in New Zealand." *Brookings Papers on Economic Activity* 2: 151-225.

Lewandowsky, Stephan, Ullrich K.H. Ecker, and John Cook. 2017. "Beyond Misinformation: Understanding and Coping with the 'Post-Truth' Era." *Journal of Applied Research in Memory and Cognition* 6: 353-369.

Lopus, Jane, and Jody Hoff. 2009. "An Empirical Analysis of Alternative Assessment Strategies in the High School Economics Class." *The American Economist* 54: 38-51.

Lopus, Jane S., and Kristen McDaniel. 2015. "Five Fundamental Misunderstandings Students Have About Money and the Fed and What To Do About It." *Social Education* 79: 78-81.

Lynch, Gerald J. 1990. "The Effect of Teacher Course Work on Student Learning: Evidence from the TEL." *Journal of Economic Education* 21: 287–296.

Marlin, James W. Jr. 1991. "State-Mandated Economic Education, Teacher Attitudes, and Student Learning." *Journal of Economic Education* 22: 5–14.

Perumal, Andrew, Roger B. Butters, and Carlos J. Asarta. 2016. "State Educational Mandates in Economics: Do We Really Need the Treat(ment)?" *Perspectives on Economic Education Research* 10: 1-20.

Pew Research Center. 2015. *What the Public Knows – In Pictures, Words, Maps and Graphs*. (*https://www.pewresearch.org/politics/2015/04/28/what-the-public-knows-in-pictures-words-maps-and-graphs/*)

Valletta, Robert G., K. Jody Hoff, and Jane S. Lopus. 2014. "Lost in Translation? Teacher Training and Outcomes in High School Economics Classes." *Contemporary Economic Policy* 32: 695-709.

van der Cruijsen, Carin. 2015. "How Much do People Know about Monetary Policy?" *World Economic Forum.* (*https://www.weforum.org/agenda/2015/08/how-much-do-people-know-about-monetary-policy/*)

Walstad, William. B., and Michael Watts. 2015. "Perspectives on Economics in the School Curriculum: Coursework, Content, and Research." *Journal of Economic Education* 46: 324–339.

Are Banks "Ripping Off" the Consumer When It Comes to Mortgages?

John S. Walker, Jonathan K. Kramer, and Ju Zhou[1]

ABSTRACT

In this pedagogical paper, we answer two questions: If someone has a fixed-rate mortgage and pays it off after only a few years, is the bank "ripping off" the consumer because of the low equity; and when is the interest portion of a mortgage payment *exactly* equal to the principal portion? The mathematics inherent to a fixed-rate mortgage dictate that a greater portion of interest is paid in the early years. To find the exact point in time when the interest and principal payments are exactly equal requires examining this question using continuous function mathematics.

Introduction

The genesis of the first question answered in this paper came during a collegial banter between two colleagues. Dr. Smith, a professor in the Department of Business Administration at Kutztown University at the time, asked us a blunt question about our lecture on mortgages.[2] He wanted to confirm the notion he had that banks, as he phrased it, are "ripping off" consumers who pay off their mortgage after only a few years. Dr. Smith, the quintessential professor who does what he can to prepare students for not only career success but also success in life, went on to say that he tells his students that mortgage loans are nothing more than a rip-off for those who obtain one soon after graduation to buy a small house or condo, only to move or step up to a larger house a few years later when they pay off their mortgages and find that they have accumulated very little equity in their homes.

While Dr. Smith initially framed his discussion as a question, as the conversation continued, it became more of an assertion than a question. He felt confident—and he conveys this to his students while teaching—that banks are without question ripping off their customers when it comes to fixed-rate mortgages. We explained that a mortgage is a financial contract with transparent terms. The interest rate paid on a mortgage is fully communicated to the customer. Competition between banks and mortgage companies drives mortgage rates to a market-determined level. However, mortgage loans can be confusing to consumers, including young graduates. Bucks and Pence (2008, p. 232) discuss confusion surrounding adjustable-rate mortgages and conclude that "neither lenders nor savvy households have an incentive to design or advocate for products that are more transparent."

A careful examination of the mathematics inherent to a mortgage reveals that to have a level payment and to cover the interest payments on outstanding indebtedness, early mortgage payments have a higher portion of interest. Over time, though, as the mortgage approaches maturity, the portion of the mortgage payment going to principal paydown rises while the portion going to interest falls, with the buildup of equity accelerating. The changes in these portions are not linear with time. Despite this fact, research shows that individuals often make decisions as if the relationship were linear and thus, without intervention, make poor borrowing decisions (see Foltice 2017). One way a consumer can shift the split between interest and principal payments to a more favorable balance at the beginning of the loan is to opt for a 15-year mortgage rather than a 30-year mortgage. Typically, 15-year mortgages come with lower interest rates than 30-year mortgages. The combination of the shorter term on the mortgage and lower interest rate creates a more favorable split

[1] John S. Walker (walker@kutztown.edu) and Jonathan K. Kramer (jkramer@kutztown.edu) are both Professors of Finance, Department of Business Administration, Kutztown University of Pennsylvania. Ju Zhou (zhou@kutztown.edu) is Professor of Math, Department of Math, Kutztown University of Pennsylvania.

[2] The name of the professor has been changed to "Dr. Smith" to protect his identity. Our discussion debunks his notion that banks are ripping off their mortgage clients.

between interest expense and principal paydown in the early years. Also characteristic of shorter-term mortgages, the 15-year mortgage requires larger overall payments, which might exceed the consumer's budget. Goff and Cox (1998) look at the advantage of a 30-year mortgage versus a 15-year mortgage.

The main objective of this paper is to contribute to financial education and literacy, specifically to provide students (and colleagues like Dr. Smith) with a deeper understanding of the mathematics behind a mortgage and thus debunk the notion that banks are ripping off their mortgage clients when they pay off their fixed-rate mortgages after only a few years. Furthermore, some business schools include a course on calculus in their curriculum. Consistent with the objective of this paper, we provide an example of how calculus can be used to analyze a mortgage. This paper is organized as follows. The next section of this paper presents a graphical and tabular way to show the imbalance between the equity build in the early years compared to the latter years of a mortgage. The third section and fourth section then use discrete mathematics and continuous function mathematics to show how to find the approximate (discrete) and exact (continuous function) moment when the portion paid to interest equals the portion paid to principal. In the fifth section, we extend the analysis using calculus to show the sensitivity of cash flow portions to interest rates. Finally, a concluding section summarizes the relevancy of this analysis to the consumer.

Analysis of a Typical Fixed-Rate Mortgage

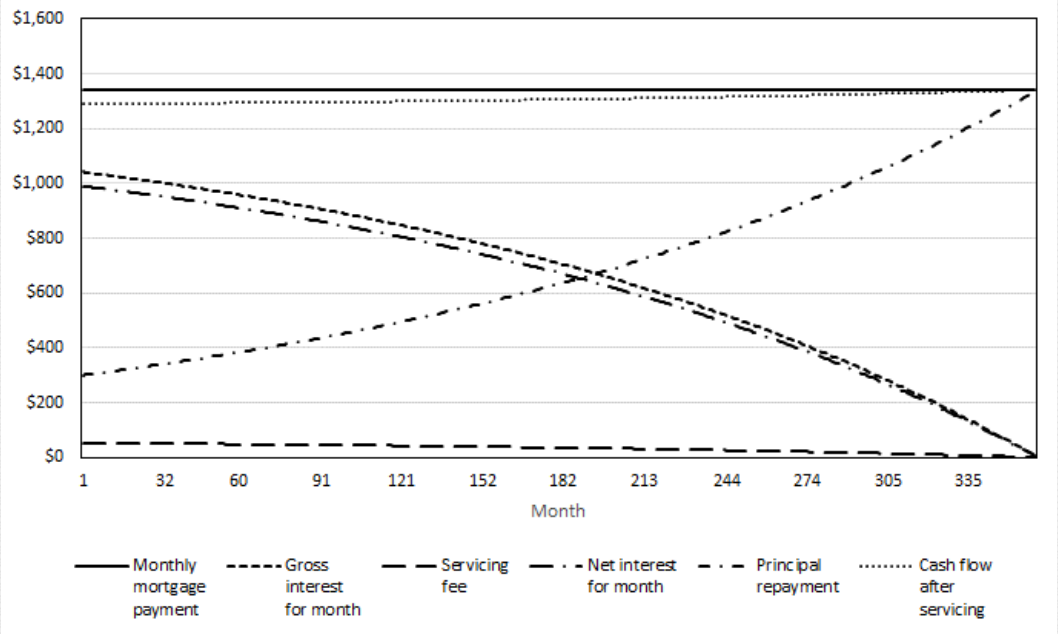
In the Appendix, we provide the numbers for the first 36 months of a 30-year, fixed-rate $250,000 mortgage at 5% interest. The three variables that determine the monthly payment are (1) the length of the mortgage (30 years or 360 months), (2) the interest rate (5%), and (3) the loan amount ($250,000). By using a financial calculator or Excel, the mortgage payment can be found easily. It can also be found using the following equation (Fabozzi 1993, p. 330–331):

$$MP = MB_0 \left[\frac{i(1+i)^n}{(1+i)^n - 1}\right], \tag{1}$$

where MP is the mortgage payment, MB_0 is the original mortgage balance, i is the periodic interest rate (annual nominal interest rate in decimal form divided by 12 in the case of a monthly mortgage) and n is the number of periods for the mortgage, e.g., the number of months for a monthly mortgage. The fixed-rate mortgage payment based on the above variables is $1,342.05. The fixed-rate mortgage offers consumers peace of mind that their monthly payment is invariant to changes in interest rates.

Figure 1 shows the trajectory of the mortgage cash flows from beginning to end. Note that the monthly payment is horizontal, consistent with a fixed payment for the life of the mortgage.

Figure 1: Graphical Representation of Mortgage Cash Flows (for a 30-year, fixed-rate mortgage at 5% interest rate)



Inherent to the fixed-rate mortgage, the monthly amount going to principal paydown starts low and trends upward, while the monthly amount going to pay interest starts high and trends downward.[3] As the interest rate on the mortgage rises, the difference or gap between the y-intercepts for the interest and principal payment lines increases.[4] Loan servicing for a mortgage is typically a fee ranging from 25 basis points to 50 basis points of the outstanding loan amount. As the outstanding balance falls, so does the loan servicing fee. The total cash flow on a fixed-rate mortgage can be divided into three pieces: (1) the servicing fee, (2) the net interest after removal of the servicing fee, and (3) the principal payment. The cash flow after servicing represents the net cash flow going to interest and principal payments.

To address Dr. Smith's question about banks ripping off their consumers, Figure 2 shows the amount of equity on an outstanding loan over time, starting at 0% at the beginning and ending at 100% at the end of the loan. Note that the profile for a hypothetical 0% interest rate mortgage is a perfectly straight line. In other words, the buildup of equity is linear over time. Using this profile as a frame of reference, the convexity on the profiles for 5%, 10%, and 50% interest rates increases monotonically with an increasing interest rate. The 0% and 50% interest rates are used purely for illustrative purposes, as no one would expect to pay 0% or 50% interest on a mortgage. The equity profile for the 50% interest-rate scenario accentuates the point that the equity buildup on a mortgage starts slowly and then accelerates exponentially. Furthermore, for the 50% interest rate scenario, there is little equity buildup until the 250-month mark when the curve turns sharply upward.

Figure 2: Equity Profile over the Life of the Mortgage for Four Different Interest Rate Scenarios

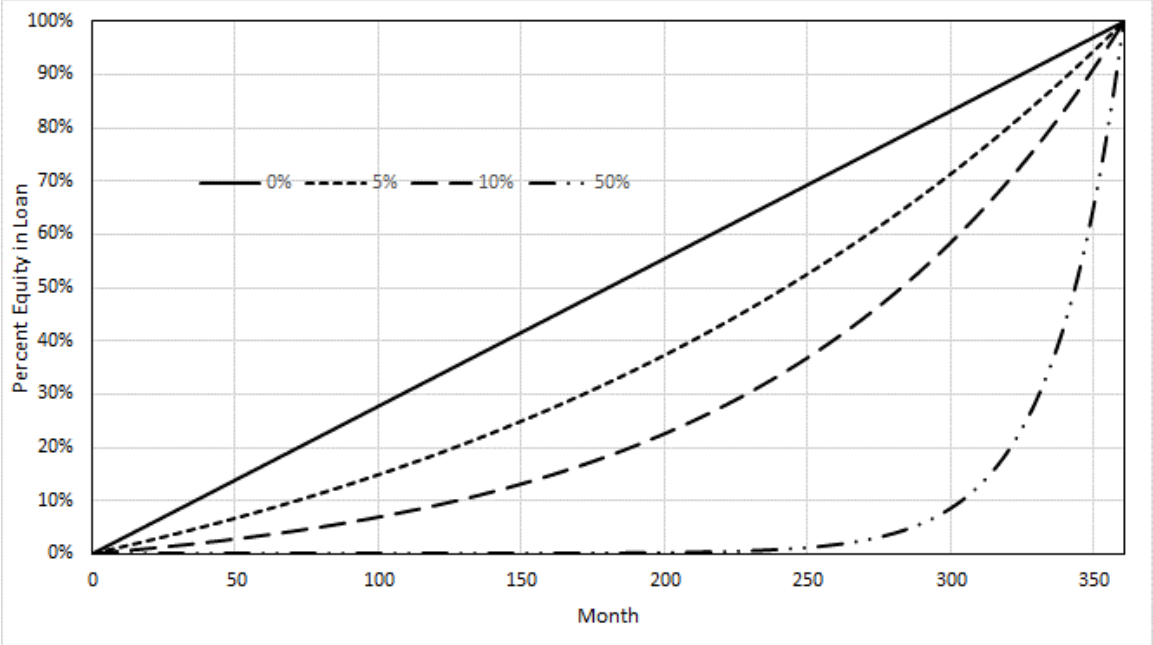


Figure 3 shows the history of the average 30-year fixed-rate mortgage rate of interest in the United States since the early 1970s. Since early 1971, the average mortgage rate is 7.97%, so the current rate of 3.33% is quite low in historical terms. Indeed, the maximum rate over the nearly 50-year period is 18.63%, while the minimum rate is 3.29%. During the last decade, rates have been the lowest seen in 50 years, so in terms of building equity, the recent generation of homebuyers has benefited from low rates.

Continuing with our analysis of a 30-year, fixed-rate mortgage at 5%, suppose that a homebuyer moves or steps up to a larger home after just three years or 36 months. You can imagine that a young professional is looking to move into a larger home and start a family. Table 1 shows the percent of equity accumulated in

[3] Historically, with most fixed-rate mortgages, the interest portion exceeds the principal paydown portion at the beginning of the loan. However, if the interest rate is very low, the portion going to pay down principal will exceed the interest payment.

[4] Technically, there are no y-intercepts shown in Figure 1 because the x-axis starts at 1, and a y-intercept is, by definition, where x = 0. For a mortgage, the first payment coincides with x = 1.

his loan during the first three years and, for comparison, the final three years. This comparison shows the imbalance between equity build in the early years versus the later years. Based on a rather low mortgage rate—in historical terms—of 5%, a homeowner has just 4.66% equity in his loan at the end of three years. This is less than half the equity build shown for the 0% loan. Then, when we arrive at the backend of the 5% mortgage, almost four times as much, or 17.91%, of the loan is repaid during the final three years.

Figure 3: 30-Year Fixed-Rate Mortgage Average in the United States over the Last 50 Years

Table 1: Equity Buildup as a Function of Interest Rate for the First and Last Three Years of the Mortgage

	Interest Rate			
	0%	**5%**	**10%**	**50%**
First Three Years	10.00%	4.66%	1.85%	0.00%
Last Three Years	10.00%	17.91%	27.20%	77.00%
PRIN/PMT$_{start}$	1.00	0.24	0.06	0.00
PRIN/PMT$_{end}$	1.00	0.93	0.86	0.51

Source: St. Louis Federal Reserve

To serve as a reference point, the linear equity build under the hypothetical 0% interest rate scenario shows that the portion of equity build during the first three years of the loan period (i.e., 10% of the life of the loan) is exactly equal to the portion of equity build during the remaining three years. At 0% interest for the 30-year mortgage, $\frac{1}{360}th$ of the principal is paid back each month. For any nonzero interest rate, there is an imbalance between the portion of interest paid versus the portion of principal paid, and this imbalance favoring the last three years monotonically rises with the interest rate on the mortgage.

Another way to look at this unfavorable split between interest paid by the consumer on his mortgage and principal paydown is to frame this in dollar terms. At the end of year three at 5% interest, the cumulative payments made are $48,313.95, with $11,641.01 going to principal and $29,680.60 going to interest. That is, just $0.24 of every $1 paid goes to pay down principal (also shown in Table 1), while the remaining $0.76 has gone to pay interest. In the latter three years of the mortgage, the cumulative payments made are again $48,313.95 for the fixed-rate mortgage, with $44,778.57 going to principal and $3,535.38 going to interest. This lopsided balance between high interest paid and low principal paydown as well as the slow equity buildup during the early years of a mortgage are why some might feel that a bank is ripping off its customers, as Dr. Smith asserted. As the interest rate increases on the mortgage, as shown in Table 1, the imbalance becomes more pronounced. But if a customer wants a fixed payment month after month with a fixed interest rate, there is no other way to proportion the interest and principal components of the cash flow over the life of the mortgage.[5] Indeed, this is inherent to mortgage mathematics. While a consumer might not like a slow equity buildup, he should blame the mathematics and not the bank.[6]

[5] See Choi and Spero (2010) for a mathematical examination of the fixed-rate vs. variable-rate mortgage decision.

[6] As for the bank, the internal rate of return (IRR) on the mortgage is the same regardless of when the loan is prepaid assuming there is no prepayment penalty. However, if there *is* a prepayment penalty, then the IRR is greater than the interest rate on the mortgage. While a higher IRR corresponds to a higher effective interest rate paid by the borrower, this, arguably, should not be viewed as the

Ultimately, whether the bank is ripping off its mortgage customers should not be measured by how much or how quickly equity builds up over time. For example, consider what's termed an "interest-only" loan. When a borrower opts for an interest-only mortgage, this gives him the option—in some cases—to pay interest only on the outstanding balance for up to 10 years. During the interest-only period of the mortgage, there is *no* equity buildup from principal payments. However, there could be equity in the home from the initial down payment and price appreciation.[7]

The Crossover Point between the Mortgage Interest and Principal Payments

The second question we examine was asked by Jessica, a student enrolled in our introductory course on financial management taken by all our B.S.B.A. business students. She works in real estate as a property manager, so she is familiar with the workings of a mortgage. When we discussed the falling interest portion and rising principal portion of a loan, as shown earlier in Figure 1 above, she asked, "When is the interest portion *exactly equal* to the principal portion?" Now the quick answer is, "Never." Mortgage mathematics is discrete math, not continuous math; thus, while the interest and principal lines intersect, as we see in Figure 1 above, technically, the two payments are never *exactly equal*.

If we return to our ongoing analysis of the fixed-rate, 5% mortgage for $250,000, we show in Table 2 the cash flow for the first three months and then jump to months 192 through 197. Notice that in the 194th month the interest paid exceeds the principal repayment ($671.86 > $670.20) by $1.66. Then in the next month (the 195th month), the interest paid is *less than* the principal repayment ($669.07 < $672.99) by $3.92. So, in the world of discrete math, there is *no* month when the gross interest and principal repayment amounts are exactly equal. But if we think in terms of the continuous world, the time to parity occurs somewhere between the 194th month and 195th month. This, of course, can be given in units of years, as the crossover point occurs between 16.1667 years and 16.2500 years, an interval of 0.0833 years.

Table 2: Cash Flow Showing the Crossover Point between Interest Paid and Principal Repayment

Month	Beginning mortgage balance	Monthly mortgage payment	Gross interest for month	Servicing fee	Net interest for month	Principal repayment	Cash flow after servicing	Ending mortgage balance
1	$250,000.00	$1,342.05	$1,041.67	$52.08	$989.58	$300.39	$1,289.97	$249,699.61
2	$249,699.61	$1,342.05	$1,040.42	$52.02	$988.39	$301.64	$1,290.03	$249,397.97
3	$249,397.97	$1,342.05	$1,039.16	$51.96	$987.20	$302.90	$1,290.10	$249,095.08
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
192	$162,578.20	$1,342.05	$677.41	$33.87	$643.54	$664.64	$1,308.18	$161,913.56
193	$161,913.56	$1,342.05	$674.64	$33.73	$640.91	$667.41	$1,308.32	$161,246.14
194	$161,246.14	$1,342.05	$671.86	$33.59	$638.27	$670.20	$1,308.46	$160,575.95
195	$160,575.95	$1,342.05	$669.07	$33.45	$635.61	$672.99	$1,308.60	$159,902.96
196	$159,902.96	$1,342.05	$666.26	$33.31	$632.95	$675.79	$1,308.74	$159,227.17
197	$159,227.17	$1,342.05	$663.45	$33.17	$630.27	$678.61	$1,308.88	$158,548.56

From a pedagogical perspective, we suppose we could leave the answer there, but we can take this a couple of steps further. We repeated the above analysis, but instead of assuming monthly mortgage payments, we assume *weekly* payments. Rather than divide the annual nominal mortgage interest rate by 12 (monthly), we divide it by 52 (weekly) to calculate the revised cash flows. Under this scenario, the time to parity occurs between the 839th and 840th week, in that the gross interest paid in the 839th week is greater than the principal paydown ($154.86 > $154.63), and then in the 840th week, the interest paid is less than the principal paydown ($154.71 < $154.78). As done with the monthly mortgage scenario, the weeks can be converted to years; in years, the time to parity occurs between 16.1346 and 16.1538 years, an interval of 0.0192 years. By changing from a monthly to a weekly payment schedule, the consumer can reach the time to parity between interest

bank ripping off the customer. It's the customer's decision whether to prepay the mortgage, and the additional interest expense should be considered the cost of exercising the prepayment option.

[7] We thank an anonymous referee for suggesting that we consider the case of an interest-only loan, where the customer simply pays the opportunity cost of money with no principal paydown.

paid and principal paydown just a little bit faster. Likewise, the effective interest rate increases from 5.1162% (5% nominal rate with 12 monthly payments per year) to 5.1246% (5% nominal rate with 52 weekly payments per year).

Using the effective interest rate for the monthly mortgage of 5.1162%, we impute the nominal rate on a weekly mortgage that would provide the same effective rate and find it to be 4.9920%. When we use that nominal rate for the weekly mortgage, we expect to find the same crossover interval as found for the monthly mortgage, but that is not the case. Rather, we find an interval of 16.1154 years to 16.1346 years, which is one week earlier than the interval found for the weekly mortgage, using 5% as the nominal rate.

This analysis can be extended, in theory, indefinitely by increasing the number of payment periods for the mortgage. The next logical period to use is daily. Table 3 shows the intervals when the amount of principal paid goes from greater than to less than the amount of interest paid. The crossover intervals are first shown using the same units used for the mortgage, i.e., year, month, week, and day. Then these intervals are all converted to a common unit—i.e., years. In all cases, the crossover point occurs between the 16th and 17th year.

Table 3: Intervals for Crossover of Interest and Principal Payments (30-year mortgage, 5% interest)

	n	Interval			Unit	In Years			Range
Annual	30	16	-	17	Year	16.0000	-	17.0000	1.0000
Monthly	360	194	-	195	Month	16.1667	-	16.2500	0.0833
Weekly	1,560	839	-	840	Week	16.1346	-	16.1538	0.0192
Daily	10,950	5,890	-	5,891	Day	16.1370	-	16.1397	0.0027
						These Are Discrete Intervals			

Finding the Time to Parity between Interest and Principal

To pin down the answer to Jessica's question as to the *exact* crossover point, we need to arrive at equations for the gross interest per month line and the principal repayment line. When we introduce continuous functions into the analysis, we shift from the discrete world, in terms of talking about a crossover point, to the continuous world.[8] Thus, we can now think of the time to parity as the time when the interest payment is exactly equal to the principal payment. Keep in mind, however, that the time to parity in the continuous world is somewhat an abstraction because the U.S. dollar is not a continuous currency; rather, it is a discrete currency. Its smallest denomination is the penny or 1 cent (1¢). In a true continuous world, the currency would be infinitesimally divisible.

To arrive at the precise crossover time, we can make use of the mathematics presented by Cherry and Gorbett (2008, pp. 287–289) for amortizing loans. Suppose that (1) a mortgage has n payments in total, each payment amount is normalized to \$1, and the interest rate is i per period, and (2) for any integer value of t, where $0 \leq t \leq n$, $B(t)$, $I(t)$ and $P(t)$ are functions of t, representing the balance at t, interest of the t^{th} payment, and principal of the t^{th} payment. The balance at time t is given by:

$$B(t) = \frac{1-v^{(n-t)}}{i}, \tag{2}$$

where $v = \frac{1}{1+i}$. Because the formula is based on the normalized payment of \$1, the balance at time t for any fixed-rate mortgage is the product of $B(t)$ and the fixed-rate mortgage payment, i.e., $B(t) \times MB_0 \left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$. Recall that MB_0 was defined above as the original mortgage balance. Next, the interest portion over the interval $[t-1, t]$ is given by:

$$I(t) = B(t-1)i = \frac{1-v^{(n-(t-1))}}{i} i = 1 - v^{(n-(t-1))}. \tag{3}$$

[8] This is similar to what is implicit to simple payback period calculations. When we find the payback period to be equal to an integer plus a fractional year, this implicitly implies that the cash flow occurs on a continuous basis rather than a series of lump sums. Another example of using continuous math in finance is the noninteger *N* annuity calculation found in Walker and Kramer (2018).

As done with the computation of the balance on the loan above, the dollar amount of interest paid is $I(t) \times MP$, where MP is the periodic mortgage payment. Finally, the principal portion of the payment over the interval $[t-1, t]$ is given by:

$$P(t) = 1 - I(t) = 1 - B(t-1)i = v^{(n-(t-1))}. \quad (4)$$

Correspondingly, the dollar amount of principal paid is $P(t) \times MP$.

The above interest and principal functions are defined on $\{1, 2, 3, \ldots, n\}$, and their sum is 1 at any integer moment. But more generally, we can assume both functions are defined for the interval $[1, n]$ and the sum of these two functions is 1 at any moment over the interval. Since we are interested in the exact moment when these two functions are equal, we are looking for the moment where $P(t) = \frac{1}{2}$. By definition, given that the sum of the two functions is 1, when $P(t) = \frac{1}{2}$, $I(t) = \frac{1}{2}$. We can find that exact time by using our formula for $P(t)$ shown in Equation 4 and setting $P(t) = \frac{1}{2}$ and solving for t:

$$\begin{gathered} P(t) = v^{(n-(t-1))} = \tfrac{1}{2} \\ (1+i)^{(n-(t-1))} = 2 \\ \ln(1+i)^{(n-(t-1))} = \ln(2) \\ \big(n-(t-1)\big)\ln(1+\mathrm{i}) = \ln(2) \\ \big(n-(t-1)\big) = \tfrac{\ln(2)}{\ln(1+i)} \\ t = n + 1 - \tfrac{\ln(2)}{\ln(1+i)} \end{gathered} \quad (5)$$

Equation 5 can now be used to answer Jessica's question in *exact* terms, as the variable t can take on any real number (not just an integer value). The results are shown in Table 4, which is identical to Table 3 above, except now the last column (t in units of years) lists the exact times when the interest payments are exactly equal to the principal payments. For example, for a 30-year fixed-rate mortgage, with a 5% interest rate, at 16.1915 years, the interest payment and principal payment are equal or at parity.

Table 4: Time to Parity for Interest and Principal Payments (for a 30-year mortgage at 5% interest)

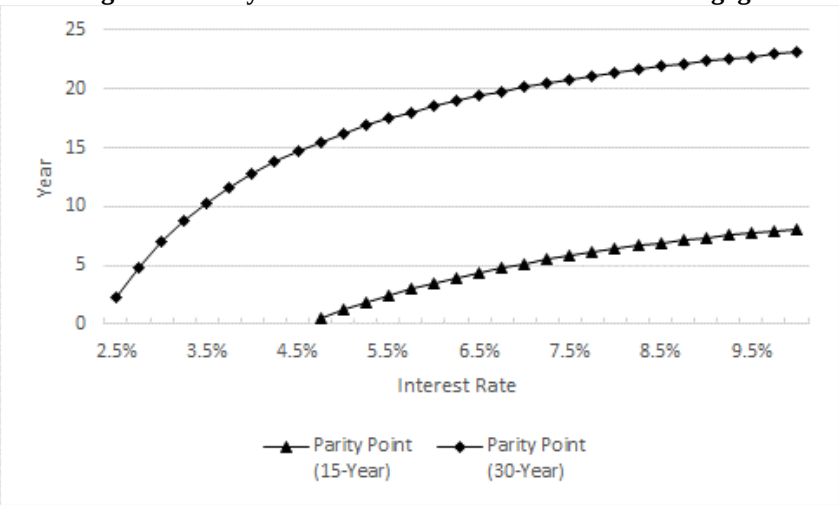
	n	Interval			Unit	In Years			Range	t (years)
Annual	30	16	-	17	Year	16.0000	-	17.0000	1.0000	16.7933
Monthly	360	194	-	195	Month	16.1667	-	16.2500	0.0833	16.1915
Weekly	1,560	839	-	840	Week	16.1346	-	16.1538	0.0192	16.1496
Daily	10,950	5,890	-	5,891	Day	16.1370	-	16.1397	0.0027	16.1388
						These Are Discrete Intervals				↑
										Exact Crossover

For 15-year (where n is 180) and 30-year (where n is 360) monthly mortgages, Equation 5 can be used to find the time to parity for a range of interest rates, shown in Figure 4. The lines are nonlinear with a positive first derivative and a negative second derivative, meaning that the curves become less steep as the interest rate increases. In laymen's terms, the higher the interest rate, the longer the homeowner has to wait until he reaches parity. The other point that should be of interest to a consumer weighing the advantages and disadvantages of a 15-year versus a 30-year mortgage is that he will reach parity much faster with a 15-year mortgage than a 30-year mortgage. For example, for the 15-year mortgage at 5% interest, the time to parity is 1.1915 years while for the 30-year mortgage, the time to parity is 16.1915 years—exactly 15 years longer. Psychologically, we expect that many consumers will want to move up the date when the portion of their mortgage payment going to pay down principal exceeds the portion going to interest. Note, to simplify the analysis, this comparison assumes an equal interest rate for the 15-year and 30-year mortgages. Typically, a consumer will receive a slightly lower rate on a 15-year mortgage, so the time to parity would be more than 15 years sooner.

Finally, both curves in Figure 4 have an x-intercept, and the x-intercept has an economic interpretation. For example, the x-intercept for the 30-year mortgage is 2.31%. What does that mean? If the fixed mortgage rate is less than 2.31%, the portion of the mortgage payment going towards the principal is more than 50%

from the start. As we know from mathematics, this portion going to principal grows with time, so there is no crossover point. Looking back at Figure 1, if the 30-year monthly mortgage has an interest rate of 2.31%, the y-intercept of the interest payment and principal payment curves are equal—or, in numerical terms, the y-intercept is equal to half the fixed mortgage payment.

Figure 4: Parity Points for a 15-Year and 30-Year Mortgage



The Elasticity of Parity

When we teach microeconomics, we introduce the term "elasticity" in the context of price elasticity of supply and demand. This concept can be applied to the time to parity on a mortgage, as we can foresee that a consumer would like to know how his time to reach parity is affected by the interest rate on the mortgage agreement. Consumers are usually offered different interest rates based on how many points they are willing to pay upfront on their mortgage agreement. If they opt for a lower interest rate by paying points, how does that change the time to parity? We can use calculus as a tool to estimate the interest-rate point elasticity of parity (ε_i) by taking the partial derivative of Equation 5 to find $\frac{\partial t}{\partial i}$ and multiplying that by $\frac{i}{t}$:

$$\varepsilon_i = \frac{\partial t}{\partial i} \cdot \frac{i}{t} = \frac{\ln(2)}{(1+i)\ln(1+i)^2} \cdot \frac{i}{t} \tag{6}$$

Importantly, the sign of the derivative is positive, meaning that as the interest rate on the mortgage increases, the time to parity extends and vice versa. There's a direct relationship between i and t, which is common sense; but we see that the math is also consistent with our expectation. For example, if a consumer looking at a 30-year, monthly-pay mortgage is considering moving from a 5% to a 4½% interest rate, the point $\varepsilon_i = 0.8562$ while the arc $\varepsilon_i = 0.9489$, which is a reminder to students that the point elasticity is never equal to the arc elasticity unless the relationship is represented by a straight line. The gap between the point and arc elasticities is accentuated as the interval widens. If we reduce the interval to very small, for example from 5.000% to 4.999%, the point $\varepsilon_i = 0.8562$ while the arc $\varepsilon_i = 0.8563$. We expect that students understanding calculus will find this derivation of the point elasticity and comparison to the arc elasticity interesting, while students who have not taken calculus will struggle with this part of the lesson.

From a practical standpoint, how is this analysis useful? The elasticity is an estimate of the percentage change in the parity point for a given percentage change in the interest rate. If a consumer has the option to pay points to reduce the interest rate on his mortgage from 5% to 4½% (a 10% reduction to the interest rate), the parity point will fall by approximately 8.6% (10% × 0.8562), based on the point elasticity, and approximately 9.5% (10% × 0.9489), based on the arc elasticity. We know from our earlier analysis that the parity point for a 30-year mortgage with a 5% interest rate is 16.2 years. Thus, if the time to parity between interest and principal payments is 8.6% to 9.5% sooner, this would correspond to between 1.4 and 1.5 years sooner. A consumer can then factor this into his decision as to whether paying points upfront is worth the investment. Each consumer is unique, meaning that he or she will weigh the importance of reaching the parity point differently. Psychologically, we expect that some consumers will be anxious to move up the parity

point as much as they can reasonably afford, knowing that after that point, more of their payment is going to building equity in their home while less goes to paying interest.

Conclusions

The purpose of writing this paper is to answer two questions that arose during teaching a course on financial management to our undergraduates here at KU. The first question came from a management professor in our department, whereas the second question came from a student during a lecture. We encourage professors to assign this paper as supplemental reading to students who have had a course on calculus, as it shows how this mathematical tool can be used to analyze a business topic. If we are going to require calculus as part of the curriculum, or we find that a significant number of our students have had calculus in high school, then we business professors should provide real-world examples of its application. Specifically, in this paper, we use fairly high-level math to determine when the principal and interest payments on a mortgage are exactly equal. As an extension, we then use calculus to quantify how the parity point between principal and interest changes as a function of the mortgage interest rate.

We're sure that most students nowadays hope to own a house or condo after graduation as soon as they can afford one. Gathergood and Weber (2017) find that mortgage financial literacy predicts home ownership and affects the type of mortgage chosen by younger households. Therefore, teaching them about mortgage mathematics should be a top priority for any introductory course on financial management. Likewise, many schools are offering courses on personal financial planning. Discussion of this paper and the concepts herein would be worthwhile in those courses too. Teaching about mortgages is an important step in advancing financial literacy. To grasp the concept of a mortgage requires not only an understanding of the terms associated with a mortgage, e.g., loan-to-value (LTV) ratio and prepayment penalty, but it also requires an understanding of the quantitative aspects of the loan. Hopefully, this paper will help to further the understanding of mortgages.

References

Bucks, Brian, and Karen Pence. 2008. "Do Borrowers Know Their Mortgage Terms?" *Journal of Urban Economics* 64: 218–233.

Cherry, Harold, and Rick Gorbett. 2008. *ASM Study Manual for Exam FM/Exam 2: Financial Mathematics & Financial Economics (9e).* Westbury, NY: Actuarial Study Materials.

Choi, Youngna, and Steven Spero. 2010. "Undergraduate Research: Mathematical Modeling of Mortgages." *PRIMUS: Problems, Resources and Issues in Mathematics Undergraduate Studies* 20: 698–711.

Fabozzi, Frank J. 1993. *Fixed Income Mathematics Finance.* Chicago, IL: Irwin Professional Publishing.

Foltice, Bryan. 2017. "How to Decrease the Amortization Bias: Experience vs. Rules." *Journal of Financial Education* 43: 273–293.

Gathergood, John, and Jörg Weber. 2017. "Financial Literacy: A Barrier to Home Ownership for the Young?" *Journal of Urban Economics* 99: 62–78.

Goff, Delbert C., and Don R. Cox. 1998. "15-year versus 30-year Mortgage: Which Is the Better Option?" *Journal of Financial Planning* 11: 88–95.

Walker, John S., and Jonathan K. Kramer. 2018. "Clarifying the Meaning of Noninteger N Values in Annuity Calculations." *Journal of Economics and Finance Education* 17: 48–60.

APPENDIX

Assumption Block	
Interest Rate	5.00%
Loan Amount	$ 250,000
Service Fee	0.25%

Month	Beginning mortgage balance	Monthly mortgage payment	Gross interest for month	Servicing fee	Net interest for month	Principal repayment	Cash flow after servicing	Ending mortgage balance
1	$250,000.00	$1,342.05	$1,041.67	$52.08	$989.58	$300.39	$1,289.97	$249,699.61
2	$249,699.61	$1,342.05	$1,040.42	$52.02	$988.39	$301.64	$1,290.03	$249,397.97
3	$249,397.97	$1,342.05	$1,039.16	$51.96	$987.20	$302.90	$1,290.10	$249,095.08
4	$249,095.08	$1,342.05	$1,037.90	$51.89	$986.00	$304.16	$1,290.16	$248,790.92
5	$248,790.92	$1,342.05	$1,036.63	$51.83	$984.80	$305.43	$1,290.22	$248,485.49
6	$248,485.49	$1,342.05	$1,035.36	$51.77	$983.59	$306.70	$1,290.29	$248,178.80
7	$248,178.80	$1,342.05	$1,034.08	$51.70	$982.37	$307.98	$1,290.35	$247,870.82
8	$247,870.82	$1,342.05	$1,032.80	$51.64	$981.16	$309.26	$1,290.41	$247,561.56
9	$247,561.56	$1,342.05	$1,031.51	$51.58	$979.93	$310.55	$1,290.48	$247,251.01
10	$247,251.01	$1,342.05	$1,030.21	$51.51	$978.70	$311.84	$1,290.54	$246,939.17
11	$246,939.17	$1,342.05	$1,028.91	$51.45	$977.47	$313.14	$1,290.61	$246,626.03
12	$246,626.03	$1,342.05	$1,027.61	$51.38	$976.23	$314.45	$1,290.67	$246,311.59
13	$246,311.59	$1,342.05	$1,026.30	$51.31	$974.98	$315.76	$1,290.74	$245,995.83
14	$245,995.83	$1,342.05	$1,024.98	$51.25	$973.73	$317.07	$1,290.80	$245,678.76
15	$245,678.76	$1,342.05	$1,023.66	$51.18	$972.48	$318.39	$1,290.87	$245,360.37
16	$245,360.37	$1,342.05	$1,022.33	$51.12	$971.22	$319.72	$1,290.94	$245,040.65
17	$245,040.65	$1,342.05	$1,021.00	$51.05	$969.95	$321.05	$1,291.00	$244,719.60
18	$244,719.60	$1,342.05	$1,019.66	$50.98	$968.68	$322.39	$1,291.07	$244,397.21
19	$244,397.21	$1,342.05	$1,018.32	$50.92	$967.41	$323.73	$1,291.14	$244,073.47
20	$244,073.47	$1,342.05	$1,016.97	$50.85	$966.12	$325.08	$1,291.21	$243,748.39
21	$243,748.39	$1,342.05	$1,015.62	$50.78	$964.84	$326.44	$1,291.27	$243,421.96
22	$243,421.96	$1,342.05	$1,014.26	$50.71	$963.55	$327.80	$1,291.34	$243,094.16
23	$243,094.16	$1,342.05	$1,012.89	$50.64	$962.25	$329.16	$1,291.41	$242,765.00
24	$242,765.00	$1,342.05	$1,011.52	$50.58	$960.94	$330.53	$1,291.48	$242,434.47
25	$242,434.47	$1,342.05	$1,010.14	$50.51	$959.64	$331.91	$1,291.55	$242,102.56
26	$242,102.56	$1,342.05	$1,008.76	$50.44	$958.32	$333.29	$1,291.62	$241,769.26
27	$241,769.26	$1,342.05	$1,007.37	$50.37	$957.00	$334.68	$1,291.69	$241,434.58
28	$241,434.58	$1,342.05	$1,005.98	$50.30	$955.68	$336.08	$1,291.76	$241,098.50
29	$241,098.50	$1,342.05	$1,004.58	$50.23	$954.35	$337.48	$1,291.83	$240,761.03
30	$240,761.03	$1,342.05	$1,003.17	$50.16	$953.01	$338.88	$1,291.90	$240,422.14
31	$240,422.14	$1,342.05	$1,001.76	$50.09	$951.67	$340.30	$1,291.97	$240,081.85
32	$240,081.85	$1,342.05	$1,000.34	$50.02	$950.32	$341.71	$1,292.04	$239,740.14
33	$239,740.14	$1,342.05	$998.92	$49.95	$948.97	$343.14	$1,292.11	$239,397.00
34	$239,397.00	$1,342.05	$997.49	$49.87	$947.61	$344.57	$1,292.18	$239,052.43
35	$239,052.43	$1,342.05	$996.05	$49.80	$946.25	$346.00	$1,292.25	$238,706.43
36	$238,706.43	$1,342.05	$994.61	$49.73	$944.88	$347.44	$1,292.32	$238,358.99

Teaching the Causes of Great Depression to College Students: Evidence from History, Economics, and Economic History Textbooks

Jeremy Horpedahl, Phillip Magness, and Marcus Witcher[1]

ABSTRACT

We survey the treatment of the Depression in college-level textbooks for courses in US history and economics. History textbooks emphasis on inequality, the stock market crash, and underconsumption as the primary causes does not reflect the consensus of economic historians. Introductory economics textbooks use the Great Depression as an example to illustrate macroeconomic concepts in ways aligned with the research consensus, which emphasizes declining aggregate demand and issues related to monetary policy and the financial system. History textbooks could be improved by focusing more on bank failures, the actions of the Federal Reserve, monetary deflation, and declines in autonomous spending.

Introduction

The Great Depression is the arguably most important macroeconomic event of the 20th century. It is also a defining event in terms of economic policy: following the Great Depression, the federal government took a much more active role in regulating the economy, and not just during economic downturns.

The Depression is usually covered in two college-level courses: introductory economics (usually macroeconomics) and US history survey classes. But the Depression is treated very differently in these two courses. According to the most recently available data, both courses rank among the top ten college courses taken in the United States, with around 40 percent of undergraduate students taking them at some point.[2]

For many students, perception of the Great Depression's causes inform their views on business cycle events in the present. If the Depression is understood to illustrate a failure of free-market capitalism, this belief may shape a student's views about the proper role of government in general economic policy decisions in addition to business cycle events. The market-failure view is common in college-level history textbooks. If instead the Great Depression is understood as a failure of government institutions to properly address a normal business cycle, the policy implications are much different. The government-failure interpretation of the Depression is much more common among economic historians.

The effect of the Depression on economic policy beliefs is readily acknowledged by prominent scholars in each field. Eric Foner, in his best-selling college history textbook, concludes his description of the Great Depression by asserting that it had "discredited" the version of freedom that was summed up at the time by economist and law professor Walton H. Hamilton:

> Liberty of contract has been made the be-all and end-all of personal freedom; … the domain of business has been defended against control from without in the name of freedom. (Foner 2014, p. 804)

[1] Jeremy Horpedahl (corresponding author: jhorpedahl@uca.edu), Associate Professor of Economics, University of Central Arkansas, Conway, AR, USA; Phillip Magness, Senior Research Faculty, American Institute for Economic Research, Great Barrington, MA, USA; and Marcus Witcher, Assistant Professor of History, Huntingdon College, Montgomery, AL, USA.

[2] Adelman (2004, pp. 63-64) has data for 1972, 1982, and 1992 high school graduation cohorts. For US history surveys, the percent of students in each year was 42.6, 36.9, and 43.8 percent. For introductory economics, the same figures are 45.3, 59.0, and 40.9 percent. Siegfried (2000, p. 203) only has data on introductory economics for 1998, but reports a similar figure of 39.5 percent.

Influential educator and education theorist George S. Counts' famous speeches during the Depression argued that the old order, based on competition and capitalism, was no longer viable. Instead, a new order based on cooperation and socialism was inevitable (Violas 1971, p. 26).

> Writing decades after the Depression, economists Milton and Rose Friedman held a similar view:
> In the realm of ideas, the depression persuaded the public that capitalism was an unstable system destined to suffer ever more serious crises. … The change in the public's perception of the proper role of private enterprise … and of the government … provide a major catalyst for the rapid growth of government… to this day. (Friedman and Friedman 1980, p. 70)

Roark et al. (2016, p. 615) describe this view of the Great Depression much more succinctly in their college-level history textbook, calling the Depression a "massive failure of capitalism."

An alternative narrative acknowledging errors in government policy yields an entirely different set of prescriptive outcomes. If, instead, the Great Depression is viewed as a failure of governmental institutions, it becomes less of a morality tale regarding whether free markets are good or bad, and how much regulation they need. Rather, the most pertinent questions involve identifying missteps taken by policymakers both before and during the Depression, as well as directing deeper scrutiny to the institutional arrangements of government that permitted such errors to occur.

The teaching of the Great Depression in the US educational system is thus crucially important for an accurate understanding of the economy and public policy today and in the future. More specifically, the cause or causes of the initial downturn in the Great Depression is especially important for understanding either the instability of private markets, or the inability of government agencies to make and implement appropriate economic policy decisions. Young students may carry in their heads any lessons from this historical episode and apply it to questions of the balance between governments and markets in the world today.

What, then, are students learning about the causes of the Great Depression in classroom textbooks?

Cargill and Mayer (1998, pp. 441-442) surveyed high school history textbooks for causes of the Great Depression, and compared those causes to ones that economists and economic historians identify as important. They found "a large gap between what students are taught through these books and what researchers think about the Great Depression." Cargill and Mayer also indicate that the discussion in high school history textbooks "differs at important points from the prevailing views of present day researchers and presents a misleading view of this period."

The Cargill and Mayer findings were in line with previous research on the topic (Miller and Rose 1983), despite the fact that economists have been trying to present a more balanced view of the causes of the Depression for decades, often targeted at high school teachers. For example, in a 1980 curriculum guide published by the Joint Council on Economic Education, a full page is dedicated to a handout discussing the monetarist explanation of the Great Depression and the role of the Federal Reserve. While noting the existence of some disagreement among economists, it also stated (in a quotation from Savage 1977) that "most economists would agree that the Federal Reserve System should have acted more rapidly and more aggressively to stop or shorten the decline" (O'Neill 1980, p. 100).

High school education may only represent the tip of the proverbial iceberg for its role in perpetuating historical interpretations of the Depression that fall outside of the economic mainstream. In theory, the typical college history textbook is intended for a higher-level readership, and may accordingly be expected to reflect a broader understanding of scholarly consensus among economic historians. Several signs, however, suggest a continuation of the same pattern that Cargill and Meyer observed at the high school level.

We therefore update the Cargill and Mayer paper by applying a similar analysis to college-level U.S. history textbooks. Since their article is more than 20 years old, we incorporate subsequent scholarly contributions from economic historians on the causes of the Depression into our analysis (though not much has changed which would alter that consensus). Doing so gives us a good idea of what US college students are learning about the causes of the Great Depression in their history courses, and whether that aligns with the research of economic historians.

In large part our findings confirm and extend Cargill and Mayer's assessment from over two decades ago. College-level history textbooks contain many of the same errors of both omission and commission about the causes of the Great Depression that were found at the high school level. Such errors are even more troubling at the college level, where textbook selection is further removed from the political influences of the public school system and where instruction is usually provided by academics who claim scholarly expertise in the subjects they teach. Since students are already familiar with the basics of the Depression from their high

school courses, college textbooks should be going into greater detail to convey a deeper understanding of the relevant scholarly literature. And the deeper understanding should be based on the best current research by economists who specialize in studying the events in question.

Defining Consensus

"You're right, we did it. We're very sorry. But thanks to you, we won't do it again." -Ben Bernanke, November 8, 2002 (Bernanke 2002)

The quotation comes from a speech by Ben Bernanke, at the time a member of the Board of Governors of the Federal Reserve. The "you" is Milton Friedman and Anna Schwartz. The "we" is the modern Federal Reserve. The "it" is turning a normal recession into the Great Depression. At the time of the speech, Bernanke was giving a summary of the current state of economic research on the causes of the Great Depression. Bernanke himself was an important contributor to the line of research following Friedman and Schwartz. But it was also a foretelling of the future in some sense. When Bernanke was the Chairman of the Federal Reserve during the 2007-2008 Financial Crisis, he closely followed the advice of Friedman and Schwartz and other monetary economists and economic historians: above all, don't let the money supply contract sharply during a recession.

Bernanke was expressing a professional consensus among economists about the Great Depression. But is this consensus Bernanke and others agree with being represented in college-level US history textbooks? How can we know what the consensus is?

Survey Data of Economic Historians

One method of investigating a consensus is survey data. Whaples (1995) conducted a survey of a random sample of members of the Economic History Association (EHA). Importantly, the EHA has members that are in history departments as well as economics departments, ensuring that we are not only getting the views of economists. It may be that a survey of all academic historians would give different views than historians who are members of the EHA, but we are not aware of any such broader survey. Furthermore, the EHA's membership confines the survey's results to scholars in both department types who specialize in economic history and presumably possess expert competency in that area.

Whaples asked both economists and historians to respond to 40 propositions about US history, and six of those are directly about the Great Depression. Summaries of the propositions, along with the responses, are shown in Figure 1.

The full propositions from Whaples (1995) are as follows:

34: Monetary forces were the primary cause of the Great Depression.

35: The demand for money was falling more rapidly than the supply of money during 1930 and the first three-quarters of 1931.

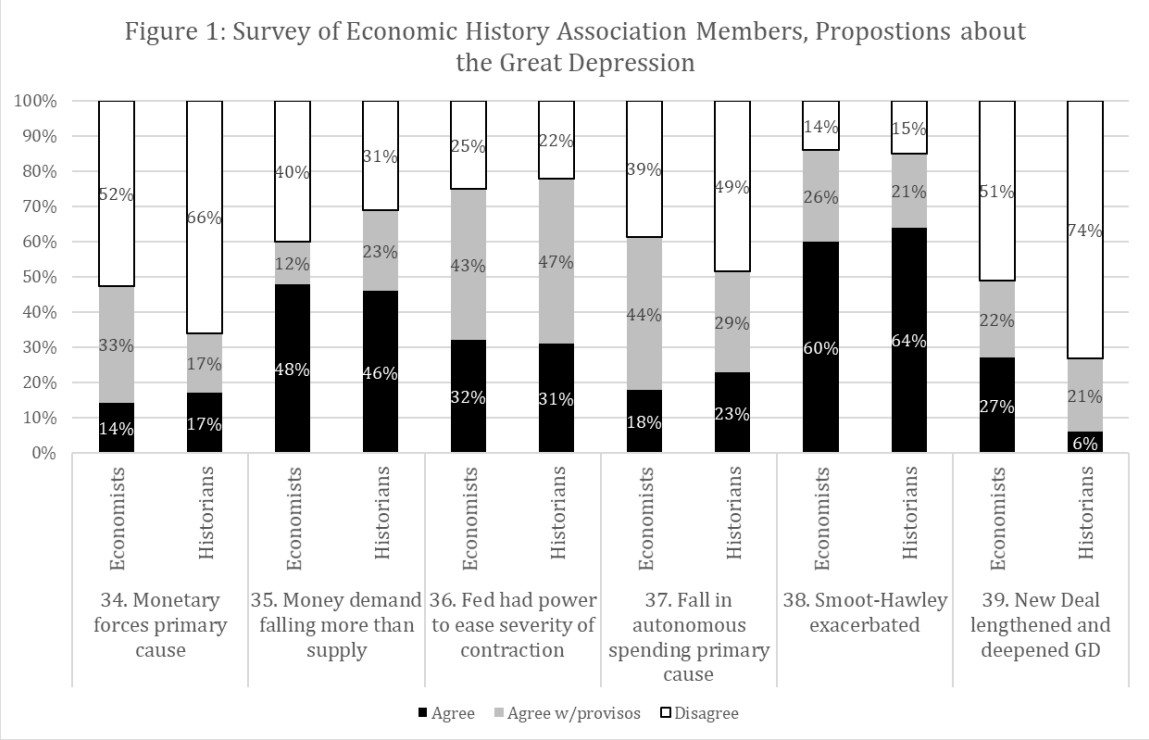
36: Throughout the contractionary period of the Great Depression, the Federal Reserve had ample powers to cut short the process of monetary deflation and banking collapse. Proper action would have eased the severity of the contraction and very likely have brought it to an end at a much earlier date.

37: A fall in autonomous spending, particularly investment, is the primary explanation for the onset of the Great Depression.

38: The passage of the Smoot-Hawley Tariff exacerbated the Great Depression.

39: Taken as a whole, government policies of the New Deal served to lengthen and deepen the Great Depression.

Proposition 39 is primarily about the length and depth of the Great Depression, rather than the initial onset or "causes," but the other five bear directly on what caused the Great Depression. Whaples allowed respondents three choices: agree, agree with provisos, or disagree. For purposes of determining whether there is a consensus, we add the "agree" and "agree with provisos" responses together (we also note the "provisos" group in the Table).



Source: Whaples (1995). The numbering of propositions is from original source.

From these propositions, several observations can be made:

● The clearest consensus position, in terms of responses in one direction and agreement across economists and historians, is that the Smoot-Hawley Tariff of 1930 exacerbated the Great Depression.

● Another consensus position is that the Federal Reserve had the power to prevent some of the worst aspects of the initial downturn, the monetary deflation and bank failures. Both economists and historians agree with this by about a 3:1 margin.

● The biggest disagreement among economists and historians is whether the New Deal lengthened and deepened the Depression. Economists are almost exactly split, but historians reject it by a 3:1 margin.

● Another disagreement is whether monetary forces were the primary cause. Again, economists are split evenly, but historians reject it by about a 2:1 margin. Slightly more economists agree that a fall in aggregate demand (proposition 37) was an important cause, but there is still not clear agreement.

● On most other issues, economists and historians are close in agreeing with each other, but there is no clear consensus on the proposition (a roughly equal split within disciplines).

Several of these propositions come directly from statements made by monetarists Friedman and Schwartz (1965) in propositions 34 and 36, and Keynesian explanations from Temin (1976) in propositions 35 and 37. Clearly both monetarist and Keynesian explanations have some support, but no one side is the overwhelming consensus. Given these responses, we might expect a basic consensus narrative of the Great Depression to go something like this: "Two primary aspects of the initial downturn were monetary deflation and bank failures, and the Federal Reserve could have prevented both of these by expanding the supply of money. Other economists favor a Keynesian explanation, where a fall in aggregate demand and autonomous spending were the more important cause, even though many would still agree that monetary policy was poorly executed. While economic historians continue to debate which of these factors warrants greater emphasis, both were likely important. The Great Depression was further exacerbated by the Smoot-Hawley Tariff."

Survey and Literature Review Articles

As will be clear later in this paper, history textbooks also emphasize several factors that were not mentioned in the Whaples (1995) survey: the stock market crash, income inequality, and overproduction/underconsumption. The "overproduction" or "underconsumption" explanations need some

explanation. Briefly, the overproduction/underconsumption argument holds that economic production outpaced what most consumers could purchase given their low pay, triggering a contractionary event in the form of the Depression. The underconsumption theory is also distinct from Keynesian theories, even though they both focus on spending and consumption.

The combination of overproduction/underconsumption and income inequality is clearly stated in a college-level history textbook by Shi and Tindall (2016, p. 903, italics in the original):

> The once roaring economy fell victim to *overproduction* and *underconsumption*. During the twenties, manufacturing production increased 43 percent, but the purchasing power of consumers did not grow nearly as fast. In essence, the economy was producing more and more products that consumers could not afford to buy … Too many business owners had taken large profits while denying wage increases to employees.

What do economic historians think about these causes? Two survey articles can be used to summarize the views of economists on these issues. Parker (2008) is an entry in the EH.Net Encyclopedia, a project of the EHA edited by Whaples (the same Whaples who conducted the survey discussed above). Temin (2000) is the entry in the *Cambridge Economic History of the United States* on the Great Depression. Temin (2000, p. 301) begins his survey with a clear distinction: older scholarship saw the Depression as evidence of "a great instability in the economy," while more recent work concentrates on shocks to the economy as a cause of the downturn.

Parker (2008) states firmly that:

> Without any doubt, the economics profession would come to a firm consensus around the idea that the economic events of the Great Depression cannot be properly understood without a solid linkage to both the behavior of the supply of money together with Federal Reserve actions on the one hand and the flawed structure of the interwar gold standard on the other.

Monetary policy, both the Federal Reserve and the Gold Standard, take a prominent place in the consensus Parker describes. Parker goes on to state that even to the extent that the stock market crash was important, the crash itself was caused by monetary policy. Hamilton (1987) and Cecchetti (1998) show that the Federal Reserve engaged in contractionary monetary policy in 1928 and 1929, with the explicit goal of ending speculative stock market activity. Parker neatly summarizes it this way: "While popular history treats the crash and the Depression as one and the same event, economists know that they were not." Furthermore, Eichengreen (1996) argues that the Gold Standard actually caused instability and further worsened the crisis, and that the sooner a nation left the Gold Standard, the milder its economic suffering.

Temin agrees with Parker on this importance of monetary policy. The decline in industrial production in 1929 "was caused by contractionary monetary policy" and resulted from the Fed attempting "to arrest what the Fed considered a speculative boom in stock prices" (Temin 2000, p. 304). In an entry on the Great Depression for the *Oxford Encyclopedia of Economic History*, Temin also extensively discusses the importance of the gold standard for transmitting the Depression worldwide and limiting the monetary policy response of affected countries (Temin 2003).

Temin goes on to argue that five other shocks were important: the stock market crash, the Smoot-Hawley tariff, bank failures, worldwide commodity price declines, and the effect of consumer credit on consumption. But Temin is quick to add that "Time has not been kind to the school of thought that blames the Depression on the stock-market crash" (Temin 2000, p. 304).

The stock market crash did have some effect on the downturn, even though economic historians place a much higher priority on Federal Reserve activity. Romer (1990) argues that the October 1929 stock market crash increased consumer uncertainty, leading to decreased spending which contributed to the economic decline. Similarly, Mishkin (1978) shows that a decline in household balance sheets could have caused a decrease in consumption and therefore aggregate demand. Temin (2000) agrees that the stock market crash indeed had some negative effects in propagating the Depression, even if it was much less important than popular belief holds (Temin 2000, p. 305).

Temin also addresses the claim that underconsumption or overproduction caused the downturn. While Temin acknowledges that income inequality peaked at the start of the Depression, he shows that the math just does not work for this as a major cause. While real GDP fell by 30 percent from 1929-1933 and consumption fell by 10 percent in 1930 alone, "the decline in consumption caused by a shift of income was

only 0.5 percent of national income" using reasonable assumptions about the propensity to consume (Temin 2000, p. 303). Not nothing, but not nearly enough to make the Great Depression great.

Parker (2008) does not address inequality or underconsumption/overproduction. Indeed, Cargill and Mayer (1998, p. 450) note that Temin's discussion of this issue is rare because "Such theories were popular in the 1930s, but are no longer taken seriously or even discussed by economists." Though as we will see, and Cargill and Mayer found in high school textbooks, these related issues are discussed as causes in almost every history textbook.

Finally, the recently released *An Economist's Guide to an Economic History*, a 2018 volume edited by Blum and Colvin, contains a chapter on "Financial Crises and Bubbles" by Quinn (2018). Quinn uses the Great Depression as one of the main examples of historical financial crises. Quinn summarizes the Friedman and Schwartz monetarist position, as well as critiques of it by Temin and others. Quinn does also briefly discuss the stock market crash, highlighting Romer's (1990) argument that it increased uncertainty. After giving all sides their due, Quinn concludes "there is no established consensus on whether the Depression was triggered by the Federal Reserve, the Wall Street Crash, or something else entirely" (Quinn 2018, p. 97).

Economic History Textbooks

Another method for examining if there is a consensus on the causes of the Great Depression is to examine college-level textbooks written specifically about economic history, rather than general history textbooks. Three such textbooks are Atack and Passell (1994), Hughes and Cain (2003), and Siegler (2017), as well as an older textbook by Lebergott (1984).

These textbooks are largely in agreement with both Temin (2000) and Parker (2008). On the issue of inequality and underconsumption/overproduction, these textbooks affirm Cargill and Mayer's claim that economists rarely recognize these explanations as causes of the Great Depression. The closest mention is in Hughes and Cain (2003, p. 456). They summarize Galbraith (1979) as listing five major sources of the crash, one of these being unequal distribution of income. Hughes and Cain add immediately afterwards: "But these factors need not have produced a crash in 1929 or at any other time" (2003, p. 456). As they further note, the US had gone through a dozen financial panics before 1929, but none had produced anything like the Great Depression.

These textbooks all discuss the stock market crash, but they leave the reader with the impression that it was neither the initial cause for the downturn, nor the main factor in the decline.

Hughes and Cain do not include a discussion of the stock market in their section "Explanations of the Great Depression" (pp. 486-496). Instead, the stock market is discussed in the prior chapter. Hughes and Cain also preface the "Explanations" with this strong statement: "Whichever view one adopts, the federal government and its agencies played a crucial and usually negative role" (p. 487). They discuss both the monetarist and Keynesian explanations of the Depression, with their key distinction being that "Keynesians agree that the Federal Reserve exacerbated the depression, but they don't believe that it had much to do with the start of the depression" (p. 494).[3]

Like Hughes and Cain, Atack and Passell (1994) discuss the stock market crash *before* their section on "Explaining the Depression." As in Parker (2008), they emphasize that a major reason for the stock market crash was monetary policy itself, with the Fed restricting the money supply beginning in 1928 (pp. 588-589). Atack and Passell begin their "Explanations" section by noting that "There is remarkably little unanimity among economists" (p. 592) about the causes of the Depression (though note that their text was published the year before the Whaples survey of economic historians in 1995). They go on to discuss in detail both the Keynesian (Aggregate Demand-Based Expectations) and Monetarist explanations (including short discussions of both Galbraith and the Austrian view in the Monetarist section). They do not discuss inequality or related issues as causes of the Depression.

Siegler (2017) briefly discusses the stock market crash, noting several common themes from the other textbooks and survey articles. First was the Fed's role in deliberately slowing down stock market speculation (p. 376). Siegler does acknowledge several ways that the stock market crash could have affected the real economy, including Romer's (1990) explanation of consumer uncertainty. But Siegler concludes by bluntly stating that the "stock market crash, however, is not as important as the bank failures and deflation that soon

[3] Earlier editions of Hughes and Cain (such as the 6th edition in 2003) also discuss the Austrian theory of the Depression (from Hayek, Robbins, and Rothbard), but in the latest edition the Austrian view is placed in a footnote where they state that it "has long been out of fashion" (p. 497).

followed" (p. 377). Instead of discussing particular theories of the Depression, Siegler discusses these contributing factors: bank failures, the money supply, the gold standard, and deflation. These are reemphasized in the "Lessons" section in Siegler: 1. good central bank policy is important for short-run stability; 2. fixed-exchange rate systems limit monetary expansion (inhibiting countercyclical monetary policy; and, 3. deflation can have catastrophic consequences. Monetary policy and related real factors are prominent in this explanation, and once again there is no discussion of income inequality.

Regarding the bank failures and bank runs, these can also be attributed to a policy failure, rather than a failure of the free market. For instance, many states had unit banking laws that prohibited competition among banks and experienced large numbers of bank failures (Grossman 1994; Wicker 1996; Witcher and Horton 2013). In contrast, Canada had an extensive branching system in place and experienced zero bank failures during the Great Depression (Friedman and Schwartz 1963, p. 352).

An older economic history textbook by Lebergott (1984) is also worth considering. Lebergott does spend time discussing the stock market crash in detail, but then quotes both Paul Samuelson and Milton Friedman to say that it could have been a "garden-variety" recession (Friedman's phrase) if not for other events (pp. 444-446). Lebergott identifies monetary policy and banking failures (pp. 446-448) and declines in aggregate spending by both consumers and investors (pp. 450-452) as the primary reasons that the recession and stock market crash turned into the Great Depression. While Lebergott's textbook is older and now out-of-print, it presents a clear early version of stating this consensus of the causes, while recognizing that monetary policy and aggregate expenditure problems both were important.

Introductory Macroeconomics Textbooks

Principles of Macroeconomics is one of the most widely taught economics courses for college undergraduates, with around 40 percent taking an introductory economics course (often covering macroeconomics). As with history textbooks, introductory macroeconomics textbooks are a way that the discipline attempts to communicate areas of consensus and disagreement in the profession to students.

Our survey of introductory macroeconomics and principles textbooks is not intended to be comprehensive, but to show examples of how economics textbooks use the Great Depression to illustrate various aspects of macroeconomic theory. Giedeman and Lowen (2008, p. 52) undertake a more comprehensive review of macroeconomics textbooks' use of economic history, and they report that "the Great Depression is the most commonly referenced historical event in macroeconomics textbooks."

Since economics textbooks don't proceed in chronological order, there is not always one chapter that contains a discussion of the Depression, as is usually the case in history textbooks. Rather, textbook authors bring it up when it is relevant to a particular topic they are discussing.

Mankiw (2021) is one of the most widely used introductory textbooks in the United States. He uses the Great Depression to discuss bank runs and aggregate demand. In the chapter on the monetary system, Mankiw (2021, p. 605) uses the bank runs in the Great Depression to show how banks are an important part of the money creation and contraction process, and that it is not solely a function of the Federal Reserve. As bank runs and failures spread across the country, households withdrew savings from still solvent banks out of fear that they too would become failed banks. When households withdraw savings, the money creation process is reversed, contracting the money supply. Mankiw then connects both bank rules and the contraction of the money supply as a cause of economic collapse and high unemployment.

In the chapter on short-run economic fluctuations, Mankiw also uses the Great Depression to illustrate the concept of aggregate demand. Mankiw (2021, p. 708) even references the debate over the causes of the Depression: "Economic historians continue to debate the causes of the Great Depression, but most explanations center on a large decline in aggregate demand." Mankiw then briefly summarizes the debate over the causes of the decline in aggregate demand: monetary explanations versus consumption explanations (with the stock market decline being a factor in reducing household wealth and thus consumption).

Finally, Mankiw brings up the Great Depression several times in a chapter on current debates over macroeconomic policy. In doing so Mankiw is acknowledging a fact that most economists implicitly and sometimes explicitly agree with: in order to have better macroeconomic policy today, we must understand economic history correctly. Mankiw shows how the Great Depression can be used to better understand a current policy debate about macroeconomic stabilization policy (p. 771), and spending increases versus tax cuts to stimulate the economy (pp. 772-773).

Cowen and Tabarrok (2018) take a similar approach to Mankiw in the topics they cover. First they discuss Great Depression bank failures in their chapter on savings, investment, and the financial system (p. 200). They say that the ripple effects from bank failures include declines in household spending and a curtailment of credit for businesses (which further led to small business failures). They also cite Friedman and Schwartz (1963) who blame the bank failures on the Federal Reserve, as well as Bernanke (1983) on the credit squeeze.

Cowen and Tabarrok also devote several pages (pp. 670-673) to the Great Depression and aggregate demand. They cite the pessimism from the stock market crash as contributing to the decline in aggregate demand through a wealth effect felt by households (they also note that the stock market crash was caused in part by tight monetary policy). But they go beyond the general pessimism and lack of confidence in the bank system to more directly blame the Federal Reserve for allowing the money supply to contract even further, calling it "*the largest negative shock to aggregate demand in American history*" (p. 670, italics in original).

Here is how Cowen and Tabarrok sum up what they call "the chain of causal events": monetary contraction reduced aggregate demand, causing bank failures, and causing a decline in the productivity of the financial sector, a real (rather than nominal) shock to the economy (p. 672). The large decline in international trade from the Smoot-Hawley Tariff and retaliations by other countries is also mentioned as a factor lowering both aggregate demand and productivity (p. 672). In their chapter on macroeconomic policy, the Great Depression is the primary example they use of when fiscal policy could have been effectively used to offset a decline in aggregate demand (p. 780).

The undergraduate principles of economics textbook by Gwartney et al. (2018. pp. 635-644) devotes a full chapter to the Great Depression as a case study in macroeconomic policy. They begin by examining the stock market crash as an explanation, but conclude from subsequent stock market patterns that "this view is an exaggeration" (p. 636) that attempts to pinpoint the downturn on a single event.

Instead, they attribute the Depression's cause and severity to a confluence of four major factors: (1) monetary contraction due to Federal Reserve mismanagement, (2) the Smoot-Hawley tariff and subsequent collapse of global trade, (3) contractionary fiscal effects of the Revenue Act of 1932, which raised taxes in an attempt to pay down the federal budget deficit, and (4) price controls adopted under the New Deal, which they credit for unintentionally impeding the recovery.

Other popular textbooks use the Great Depression in many of the same ways as Mankiw (2021), Cowen and Tabarrok (2018), and Gwartney et al. (2018). McConnell et al. (2015) discuss aggregate demand and Keynes' new theory (p. 257), the monetarist view of macroeconomic instability (pp. 426-427), and tariffs (pp. 250, 460) in the context of the Great Depression. On tariffs, they note that "Economic historians generally agree that the Smoot-Hawley Tariff Act was a contributing cause of the Great Depression" (p. 354). McConnell et al. (2015) also use the Great Depression and the stock market crash as an example of the post hoc fallacy: "Many people blame the Great Depression of the 1930s on the stock market crash of 1929. But the crash did not cause the Great Depression. The same severe weaknesses in the economy that cause the crash caused the Great Depression" (p. 19).

Frank et al. (2019), with notable co-author Ben Bernanke, also discuss bank panics and monetary contraction (pp. 259-260) and the decline in aggregate expenditures (pp. 313-14), using the Great Depression as an example. Frank and co-authors also explicitly reject the view that Wall Street speculation caused the Great Depression, instead blaming it on poor economic policymaking (p. 88). Much later in the book (after building up all the tools of macroeconomics) they spell out exactly what those mistakes were: allowing bank failures, monetary contraction, fixed exchange rates (through the gold standard), and increasing tariffs (pp. 482-483).

Table 1 summarizes how several different economics textbooks use the Great Depression to illustrate concepts. Not all of these are necessarily listed as "causes" of the Depression, though many are. Once again, we do not present this list of textbooks as exhaustive, but rather showing the general ways that economics textbooks use the Depression as a teaching tool, and to later show how the approach of economics textbooks is very different from college history textbooks. The economics textbooks we chose partially reflect our attempt to include authors with differing ideological views or methodological approaches.[4]

The macroeconomics textbooks discussed so far in this section have a lot of overlap in the topics they discuss, but one general area is also notable for its absence: income inequality and the related underconsumption theory of the Great Depression, topics that we will see in the next section are heavily emphasized in history textbooks. One relatively new introductory textbook where we might expect to find

[4] We also appreciate suggestions from a referee on several textbooks to include.

inequality discussed is the new CORE economics textbook. CORE is a collaborative textbook with almost two dozen co-authors in the most recent edition (CORE 2017). The textbook is an attempt to redesign the way that undergraduate economics is taught. Its preface makes clear that the book should have a focus on inequality, since students frequently say it is "the most pressing problem that economists should address," showing a "word cloud" where inequality clearly stands out. And they take this approach seriously in the textbook: the title of the very first section (1.1) of the book is "income inequality" and inequality is also one of the six "themes and capstone units" of the book.

Table 1: Use of the Great Depression in College-Level Introductory Economics Textbooks

	Income Inequality	Debt, Credit, Bankruptcies	Stock Market Crash	Federal Reserve Policies	Bank Failures	Monetary Contraction	Gold Standard	Aggregate Demand contraction (Keynesian explanation)	Protectionism/ Smoot-Hawley
Bade and Parkin				X	X	X		X	
Baumol et al.								X	
CORE Team	X		X	X		X		X	
Cowen and Tabarrok		X		X	X	X		X	
Frank et al.				X	X	X	X	X	X
Gwartney et al.				X		X		X	X
Mankiw			X	X	X	X		X	X
McConnell et al.				X		X		X	X
Samuelson and Nordhaus		X	X		X			X	X
Stevenson and Wolfers					X			X	

Note: Some categories listed in Table 2 are not included here because no economics textbook mentioned them. The categories are: underconsumption, overproduction, and lack of federal regulations.

Importantly though, the authors of the various units in the book do not list inequality as a cause of the Great Depression. In Unit 14 of the book (Algan et al. 2017), they use the Great Depression as a way to discuss aggregate demand, much like the more conventional introductory textbooks. Unit 14 lists three factors as contributing to the decline in aggregate demand: uncertainty (from the stock market crash), increased savings, and bank failures and the collapse of credit.

The CORE textbook also devotes a capstone chapter, Unit 17 (Carlin et al. 2017), to "The Great Depression, Golden Age, and Global Financial Crisis." While rising income inequality is discussed as a fact of the 1920s in the introduction to the unit (section 17.1), inequality is not mentioned when they get to the causes of the Great Depression (section 17.2). Instead, they list three causes that very much align with the other textbooks we have discussed: increased savings and declining consumption, bank failures, and monetary deflation. Important descriptions of the causes of the Depression could easily have come from any

other textbook, such as "once the downturn began in 1929, this policy stance [contractionary monetary policy] reinforced, rather than offset, the decline of aggregate demand."

While the CORE textbook is unique in placing discussions of inequality at the front of the textbook and in making it central to the book's focus, most introductory textbooks cover inequality as well. For example, both Mankiw (2021) and Samuelson and Nordhaus (2010) devote full chapters to inequality and related topics, though these come towards the end of the textbook. What separates the discussion of inequality in CORE and other economics textbooks from the history textbooks is whether it is treated as a *cause* of the Depression. Economics textbooks do not. History textbooks largely do treat inequality as a cause, especially when combined with the concepts of underconsumption and overproduction, as we will see in the next section.

The Great Depression in College History Textbooks

As stated earlier, given the responses from the Whaples survey, we might expect that history textbooks describe the Great Depression as follows: Two primary aspects of the initial downturn were monetary deflation and bank failures, and the Federal Reserve could have prevented both of these by expanding the supply of money. Declines in aggregate demand and expenditures are also an important cause, though economic historians disagree whether monetarist or Keynesian theories better explain this decline. But the consensus suggests that monetary policy was poorly performed, even if some economic historians don't think it is the primary cause. The Great Depression was further exacerbated by the Smoot-Hawley Tariff.

For our analysis, we compare nine major college-level textbooks that were written for U.S. history survey courses. We identified these textbooks based on the frequency of their occurrence on college course syllabi as recorded in the Open Syllabus Project database (https://opensyllabus.org/), a collection of over 6 million syllabi, including 179,000 from the field of history. After a careful reading of the nine textbooks, we find that standard narratives of the Great Depression from U.S. history survey classes are not even remotely close to the consensus interpretations found within the economics discipline, and specifically among the economic history subfield. Indeed, these textbooks hardly mention the Federal Reserve, bank failures, or monetary deflation. Instead, they emphasize underconsumption theory, rising income inequality, the accumulation of debt (both foreign and domestic), the stock market crash, and overproduction as the most important causes of the Great Depression. Each textbook offers an array of reasons for the economic downturn, but these five causes dominate their pages. Table 2 summarizes the causes and explanations for the severity of the Great Depression in the history textbooks we surveyed.

By far, the most frequently listed causes for the Great Depression in U.S. history textbooks are the related causes of underconsumption and income inequality. The underconsumption explanation has its roots in contemporaries of the 1920s and 1930s who witnessed large inventories of goods in warehouses sitting unused. The argument was that the economic benefits of the 1920s had been enjoyed by very few Americans. As a result, the market for consumer goods was saturated by 1929 because wages had not increased for most consumers. As the Depression continued, increases in unemployment led to a further decrease in demand for these goods as more and more Americans found themselves without work and as such without a paycheck. In short, the economy was out of balance. It benefited only the wealthy and well connected. The average American was forgotten. Much of this narrative is incorrect based on the findings of the economics literature. For instance, Smiley (2004) has demonstrated that real wages increased during the 1920s.

Norton et al. (2019, p. 627) offers a standard description of the underconsumptionist explanation in *A People and a Nation*:

> When demand leveled off, factory owners had to cut production and pare workforces. Retailers had amassed large inventories that were going unsold and, in turn, they started ordering less from manufacturers. Farm prices continued to sag, leaving farmers with less income to purchase machinery and goods. As wages and employment fell, families could not afford the things they needed and wanted. Thus, by 1929, a sizable population of underconsumers was causing serious repercussions.

Underconsumption theory is often combined with both the overproduction thesis and the income inequality explanation. This makes sense, as underconsumption implies overproduction and also implies that consumers lacked the money to purchase goods. Divine et al. (2013, p. 615) combines all three in *America: Past and Present*, explaining that "the consumer goods revolution" during the 1920s "contained the seeds

of its own demise." Simply put, "the productive capacity of automobile and appliance industries grew faster than the effective demand."

Table 2: Explanations for the Cause or Severity of the Great Depression in College-Level Introductory History Textbooks

	Under-consumption	Income Inequality	Over-production	Debt, Credit, Bankruptcies	Lack of Federal Regulations	Stock Market Crash	Federal Reserve Policies	Bank Failures	Monetary Contraction
Berkin et al.	X	X		X			X		X
Brinkley	X	X				X			
Divine et al.	X	X	X			X			
Faragher et al.	X	X	X			X			
Foner	X	X		X		X			
Henretta et al.	X			X		X			
Norton et al.	X	X		X	X	X			
Roark et al.	X	X		X					
Shi and Tindall	X	X	X				X	X	

Note: Some categories listed in Table 1 are not included here because no history textbook mentioned them. The categories are: the gold standard, aggregate demand, and protectionism.

Divine et al. (2013, p. 615) further explains that "each year after 1924, the rate of increase in the sales of cars and refrigerators and ranges slowed, a natural consequence as more and more people already owned durable goods." They contend that corporate leaders should have responded "by raising wages or lowering prices, both effective ways to stimulate purchasing power and sustain the consumer goods revolution." Or perhaps government, Divine et al. contends, should have "forced a halt in installment buying and slowed bank loans." Likewise, Norton et al. ties the underconsumptionist explanation to income inequality. Norton et al. (2013, p. 627) argues that "underconsumption also resulted from widening divisions in income distribution. As the rich grew richer, middle and lower-income Americans made modest gains at best."

The combination of underconsumption, overproduction, and income inequality have become the predominant explanation for the onset of the Great Depression in history textbooks. It should not be surprising that historians have latched on to these explanations to explain the Great Depression. After all, historians put a heavy emphasis on primary sources, and contemporaries of the 1920s and 1930s claimed underconsumption a major cause of the depression. Indeed, some of Roosevelt's New Deal programs were designed to address the imbalance between production and consumption. Likewise, for historians who may or may not understand the intricacies of monetary policy, the underconsumption, overproduction, and income inequality explanation offers them a familiar and seemingly less imposing explanation. Unfortunately, for all the conveniences it offers historians, this interpretation is rejected by economists and economic historians alike.

Another common explanation for the Great Depression is the large amount of both domestic and international debt that accumulated during the 1920s. It is likely that the large amount of credit issued during the decade, along with the massive reparations being paid by the Weimar Republic, played a role in the economic downturn. Historians rightly label debt as a contributing factor in causing the Great Depression.

The debt explanation is often tied to the stock market crash as an explanation. There is reason for this connection. After all, stocks purchased with credit did contribute to the large increases in the value of the stock market during the decade. According to Henretta et al. (2018, p. 707) in *America's History*, by 1927, the U.S. economy was "sinking in debt," farmers were in a cycle of indebtedness, Americans bought stock on margins, and "consumer lending had become the tenth largest business in the country." Henretta et al. (2018) goes further to claim that the 1920s U.S. economy was largely built on speculation and that the reason

for its prosperity also sowed the seeds of its destruction. They argue that "the risky speculation and easy credit of the 1920s undermined the foundations of the economy. After the 1929 crash, these factors, along with a range of interconnected global conditions discussed in chapter 22, plunged the United States into the Great Depression" (p. 710).

The idea that the stock market was central to the advent of the Great Depression was magnified by the publication of John Kenneth Galbraith's (1955) *The Great Crash*. Galbraith details how rampant speculation led to the stock market crash in October 1929. Although Galbraith mentioned other contributing factors for the Great Depression, and warned against a monocausal explanation, the stock market crash captured the imaginations of historians and became central to their explanation of the Great Depression. Most of today's college history textbooks avoid a monocausal explanation and instead combine the stock market crash with increasing debt, income inequality, underconsumption, and overproduction to offer an explanation for the economic downturn. Interestingly, Galbraith rejects both the underconsumption and overproduction theories. Near the end of *The Great Crash*, he asserts (p. 173):

> Finally, the high production of the twenties did not, as some have suggested, outrun the wants of the people. During these years people were indeed being supplied with an increasing volume of goods. But there is no evidence that their desire for automobiles, clothing, travel, recreation, or even food was sated. On the contrary, all subsequent evidence showed (given the income to spend) a capacity for a large further increase in consumption. A depression was not needed so that people's wants could catch up with their capacity to produce.

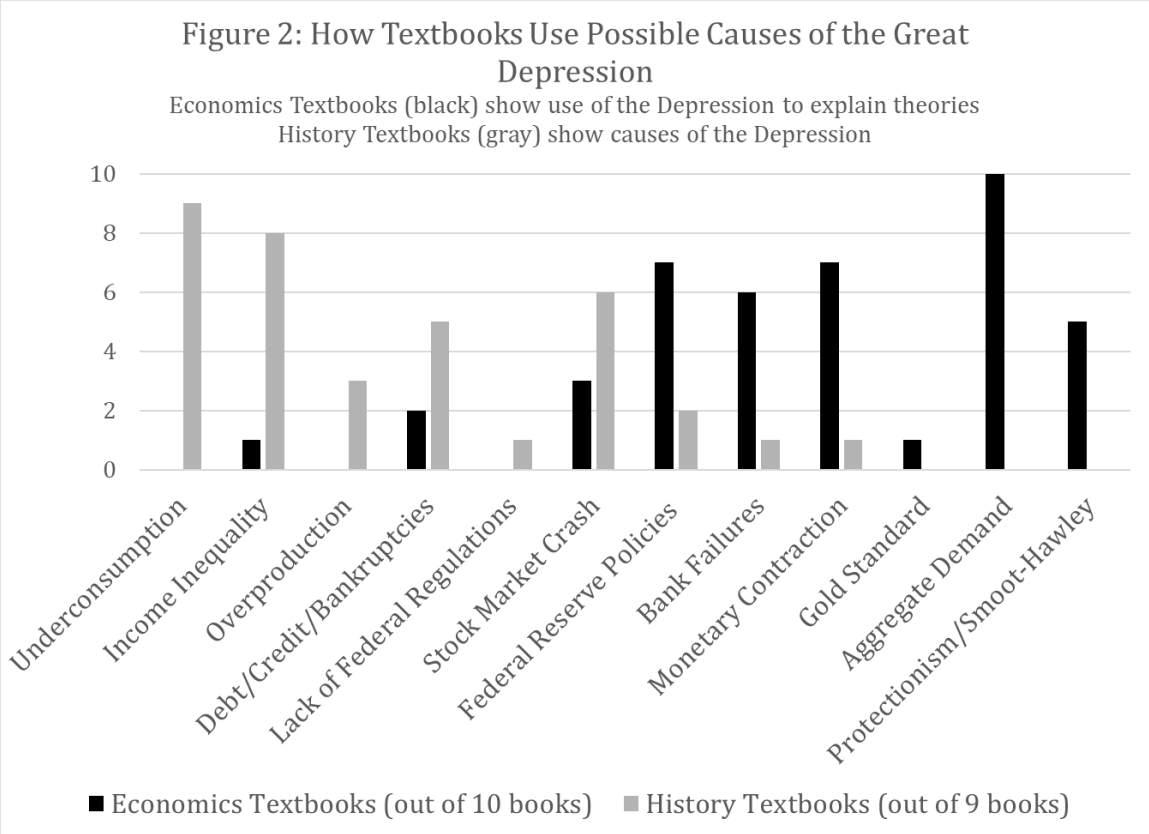
Perhaps even more telling than what the textbooks include is what they do not discuss. Only Shi and Tindall (2016) and Berkin et al. (2015) discuss the Federal Reserve's policies as a cause of the Great Depression. Likewise, only Berkin et al. (2015) provides a detailed discussion of bank failures as a major cause of the Great Depression. This is a striking omission given that 75 percent of economists and 78 percent of economic historians agreed or agreed with provisos in Whaples' 1995 survey that "throughout the contractionary period of the Great Depression, the Federal Reserve had ample powers to cut short the process of monetary deflation and banking collapse. Proper action would have eased the severity of the contraction and very likely would have brought it to an end at a much earlier date." Almost half of economists surveyed by Whaples (47 percent) agreed or agreed with provisos that "monetary forces were the primary cause of the Great Depression," although only 37 percent of historians agreed. That is a considerable percentage, yet the monetary explanation is almost entirely missing from college level history textbooks. Although only 34 percent of economic historians agreed or agreed with provisos that monetary forces were the primary cause, their perspective is not present in one-third of the textbooks we surveyed (Whaples 1995).

Figure 2 summarizes the mentioned causes in both the economics and history textbooks. The differences in coverage of topics causing the Great Depression are stark. Economists never discuss underconsumption (and generally reject this theory, as noted above), but it is universally covered by history textbooks. Likewise, income inequality is discussed as a potential cause in eight out of nine history textbooks, but only one economics textbook. The CORE textbook is the one economics textbook to discuss income inequality, but even here it is not specifically mentioned as a "cause," but rather as a stylized fact of the 1920s. All of the economics textbooks discuss the Keynesian contraction of aggregate demand, but not a single history textbook does so. Monetary contraction, the Federal Reserve, or bank failures are discussed in six or seven of the ten economics textbooks, but one or two of the history textbooks. There is debate among economic historians over whether the monetary factors or the spending factors were more important, but a student reading the history textbooks would get no sense of this debate. Finally, a majority of history textbooks also suggest that the stock market crash was a major cause, while the minority of economics textbooks which use the stock market as an example do so in a narrow way: through the uncertainty it created as well as the wealth effect causing less spending.

Conclusion

After a thorough review of the economic, economic history, and history literature, it is clear that history textbooks do not currently reflect the views of economists and economic historians on what caused the Great Depression. Whereas economists and economic historians primarily emphasize the decline in aggregate demand and the role of the Federal Reserve (in both bringing about the Great Depression and failing to avert

the worst parts of it), college level history textbooks emphasize inequality, the stock market crash, and underconsumption as the primary causes of the downturn.



Many college students are still required to take the American history survey and are likely to be assigned one of the nine history textbooks that we have examined. The Great Depression was the dominant macroeconomic event of the 20th century, and it has captured a unique place in the American psyche. Furthermore, Americans' understanding of the causes of the Great Depression influence what economic policies they are willing to support in the present. Therefore, the way that the Great Depression is taught in both college history and economics courses is of the utmost importance. The wide gulf between history textbooks' explanation and that of economists and economic historians raises some troubling questions about the integration of knowledge between these fields.

While historians should discuss that contemporary observers in the 1920s and 1930s believed underconsumption to be the major cause of the economic downturn, they also have a responsibility to distinguish between the incorrect interpretations of the time and what economists believe to be the cause of the Great Depression today. Both can be done effectively. Historians need to incorporate the economic consensus that the Federal Reserve played a major role in creating the monetary contraction that led to massive bank failures and was the primary cause of the Great Depression. Such a change would be welcome at all levels of education, but especially at the college level. In a college-level course, it is not the students' first time learning about the Great Depression, and many of them have or will take an introductory economics course to give them a better understanding of tools necessary to understand the Federal Reserve and other relevant macroeconomic topics.

References

Adelman, Clifford. 2004. *The Empirical Curriculum: Changes in Postsecondary Course-Taking*. Washington, DC: U.S. Department of Education, Institute of Education.

Algan, Yann, Wendy Carlin, and Paul Segal. 2017. "Unemployment and Fiscal Policy." Unit 14 in The CORE Team, *The Economy*. Available from: http://www.core-econ.org.

Atack, Jeremy, and Peter Passell. 1994. *A New Economic View of American History: From Colonial Times to 1940* (2nd edition). W.W. Norton and Company.

Bade, Robin, and Michael Parkin. 2015. *Foundations of Macroeconomics* (7th edition). Pearson.

Baumol, William J., Alan S. Blinder, and John L. Solow. 2020. *Economics: Principles and Policy* (14th edition). Cengage.

Berkin, Carol, Christopher L. Miller, Robert W. Cherny, and James L. Gormly. 2015. *Making America: A History of the United States* (7th edition). Cengage Learning.

Bernanke, Ben. 1983. "Nonmonetary Effects of the Financial Crisis Propagation of the Great Depression." *American Economic Review* 73: 257-276.

Bernanke, Ben. 2002. "Remarks by Governor Ben S. Bernanke." The Federal Reserve Board. Available from: https://www.federalreserve.gov/BOARDDOCS/SPEECHES/2002/20021108/

Brinkley, Alan. 2016. *The Unfinished Nation: A Concise History of the American People* (volume 2, 8th edition). McGraw-Hill Education.

Cargill, Thomas F., and Thomas Mayer. 1998. "The Great Depression and History Textbooks." *The History Teacher* 31: 441-458.

Carlin, Wendy, Samuel Bowles, and Paul Segal. 2017. "The Great Depression, Golden Age, and Global Financial Crisis." Unit 17 in The CORE Team, *The Economy*. Available from: http://www.core-econ.org

Cecchetti, Stephen G. 1998. "Understanding the Great Depression: Lessons for Current Policy." In *The Economics of the Great Depression*, edited by Mark Wheeler. Kalamazoo, MI: W.E. Upjohn Institute for Employment Research.

Cowen, Tyler, and Alex Tabarrok. 2018. *Modern Principles of Economics* (4th edition). Worth Publishers.

Divine, Robert A., Timothy Hall Breen, R. Hal Williams, Ariela J. Gross, and H.W. Brands. 2013. *America: Past and Present* (combined volume, 10th edition). Pearson Education.

Eichengreen, Barry. 1996. *Golden Fetters: The Gold Standard and the Great Depression, 1919-1939*. Oxford University Press.

Faragher, John Mack, Mari Jo Buhle, Daniel Czitrom, and Susan H. Armitage. 2016. *Out of Many: A History of the American People* (combined volume, 8th edition). Pearson Education.

Foner, Eric. 2014. *Give Me Liberty! An American History* (volume 2, 4th edition). W.W .Norton and Company.

Frank, Robert H., Ben S. Bernanke, Kate Antonovics, and Ori Heffetz. 2019. *Principles of Macroeconomics* (7th edition). McGraw-Hill.

Friedman, Milton, and Rose Friedman. 1980. *Free to Choose: A Personal Statement*. Harcourt Brace.

Friedman, Milton, and Anna J. Schwartz. 1963. *A Monetary History of the United States, 1867-1960.* Princeton, NJ: Princeton University Press.

Galbraith, John Kenneth. 1979. *The Great Crash.* Houghton Mifflin.

Giedeman, Daniel C., and Aaron Lowen. 2008. "The Use of Economic History in Introductory Textbooks." *Journal of Economics and Economic Education Research* 9: 47-66.

Grossman, Richard S. 1994. "The Shoe That Didn't Drop: Explaining Banking Stability During the Great Depression." *The Journal of Economic History*, 54: 654-682.

Gwartney, James D., Richard L. Stroup, Russell S. Sobel, and David A MacPherson. 2018. *Economics: Private and Public Choice.* (16th edition). Cengage Learning.

Hamilton, James D. 1987. "Monetary factors in the Great Depression." *Journal of Monetary Economics* 19: 145-169.

Henretta, James A., Rebecca Edwards, Erick Hinderaker, and Robert O. Self. 2018. *America's History* (volume 2, concise 9th edition). Bedford/St. Martin's.

Hughes, Jonathan, and Louis P. Cain. 2003. *American Economic History* (6th edition). Pearson Education.

Hughes, Jonathan, and Louis P. Cain. 2011. *American Economic History* (8th edition). Pearson Education.

Lebergott, Stanley. 1984. *The Americans: An Economic Record*. New York: W.W. Norton.

Mankiw, N. Gregory. 2021. *Principles of Economics* (9th edition). Cengage Learning.

McConnell, Campbell R., Stanley L. Brue, and Sean M. Flynn. 2015. *Macroeconomics: Principles, Problems, and Policies* (20th edition). McGraw-Hill.

Miller, Steven L., and Stephen A. Rose. 1983. "The Great Depression: A Textbook Case of Problems with American History Textbooks." *Theory & Research in Social Education* 11: 25-39.

Mishkin, Frederic S. 1978. "The Household Balance Sheet and the Great Depression." *The Journal of Economic History* 38: 918-937.

Norton, Mary Beth, Jane Kamensky, Carol Sheriff, David W. Blight, Howard P. Chudacoff, Fredrik Oogevall, and Beth Bailey. 2019. *A People and a Nation: A History of the United States* (volume 2, 11th edition). Cengage Learning.

O'Neill, James B. 1980. *Strategies for Teaching Economics: United States History (Secondary)*. Joint Council on Economic Education.

Parker, Randall. 2008. "An Overview of the Great Depression." In EH.Net Encyclopedia, edited by Robert Whaples. Available from: http://eh.net/encyclopedia/an-overview-of-the-great-depression/

Quinn, William. 2018. "Financial Crises and Bubbles." In *An Economist's Guide to Economic History*, edited by Matthias Blum and Christopher L. Colvin. Palgrave Macmillan.

Roark, James L., Michael P. Johnson, Patricia Cline Cohen, Sarah Stage, and Susan M. Hartmann. 2017. *The American Promise: A History of the United States* (volume 2, 7th edition). Bedford/St. Martin's.

Romer, Christina D. 1990. "The Great Crash and the Onset of the Great Depression." *The Quarterly Journal of Economics* 105: 597-624.

Samuelson, Paul A., and William D. Nordhaus. 2010. *Economics* (19th edition). McGraw-Hill.

Savage, Donald T. 1977. *Money and Banking*. New York: Wiley/Hamilton.

Shi, David E., and George B. Tindall. 2016. *America: A Narrative History* (Brief, 10th edition). W.W. Norton and Company.

Siegfried, John J. 2000. "How Many College Students Are Exposed to Economics?" *The Journal of Economic Education* 31: 202-204.

Siegler, Mark V. 2017. *An Economic History of the United States: Connecting the Present with the Past*. Palgrave.

Smiley, Gene. 2004. "The U.S. Economy in the 1920s." In EH-Net Encyclopedia, edited by Robert Whaples. Available from: https://eh.net/encyclopedia/the-u-s-economy-in-the-1920s/

Stevenson, Betsey, and Justin Wolfers. 2020. *Principles of Macroeconomics*. Worth Publishers.
Temin, Peter. 1976. *Did Monetary Forces Cause the Great Depression?* New York: W.W. Norton.

Temin, Peter. 2000. "The Great Depression." In *The Cambridge Economic History of the United States, Volume III*, edited by Stanley L. Engerman and Robert E. Gallman. Cambridge University Press.

Temin, Peter. 2003. "Great Depression." In *The Oxford Encyclopedia of Economic History*, edited by Joel Mokyr. Oxford University Press.

Violas, Paul C. 1971. "The Indoctrination Debate and The Great Depression." *The History Teacher* 4: 25-35.
Whaples, Robert. 1995. "Where Is There Consensus Among American Economic Historians? The Results of a Survey on Forty Propositions." *The Journal of Economic History* 55: 139-154.

Wicker, Elmus. 1996. *The Banking Panics of the Great Depression*. Cambridge University Press.

Witcher, Marcus, and Joseph Horton. 2013. "From Prosperity to Poverty: The Story of American Economic Decline During the 1920s." *Journal of Applied Business and Economics* 14: 79-87.

Advancing the Introduction to Both Narratives of Aggregate Output: A New Model

Stephen Kirk Elwood[1]

ABSTRACT

There are two prominent theoretical perspectives on the determination of aggregate output: The general equilibrium perspective and the reactive, circular flow perspective that relates economic activity instigating further activity. The former replaced the latter as the dominant view within macroeconomics over 30 years ago, but the lack of general equilibrium models (i.e., dynamic stochastic general equilibrium models) suitable for undergraduates has kept such models out of the undergraduate curriculum. This paper presents a model that captures both narratives within the same model environment that makes their fundamental theoretical differences manifestly clear in a way that is accessible to undergraduates.

Introduction

This paper proposes a model that straightforwardly relates and displays the reactive, circular flow narrative of the determination of output as well as the general equilibrium narrative within the same model setting. The reactive narrative describes what has become known as "short run" equilibrium output, in which prices have not adjusted (i.e., prices are deemed to be "sticky"), while the general equilibrium narrative captures so-called "long-run" equilibrium output that assumes prices have fully adjusted. The dual-purpose model is particularly valuable because its determination of general equilibrium output is mathematically simple enough to introduce in undergraduate macroeconomic textbooks. This fills a need in the undergraduate curriculum because macroeconomic theory's models that capture general equilibrium output – i.e., dynamic stochastic general equilibrium models – are considered too complicated for the great majority of undergraduates. The new model also includes a role for money in the reactive determination of output, and it specifies two goods rather than a single homogeneous good so that a more realistic representation of aggregate output is provided.

The paper proceeds by reviewing how mainstream macroeconomics' conception of output has changed over the years and recognizing the models of output that have been used. Next, the model is introduced and shown to capture the reactive nature of output before illustrating – within the exact same environment – a simple, yet authentic, general equilibrium determination of output. The price adjustment forces that cause the short run reactive equilibrium level of output to approach the long run general equilibrium level are presented. The model is then employed to analyze the effects of monetary and fiscal policies before concluding.

Two Visions of the Determination of Aggregate Output

There are many economists who remember when the primary macroeconomic theory of output characterized macroeconomic activity as a reactive process of activity instigating further activity, e.g., a butcher buying something from a baker that provides the baker with the funds to buy something from the brewer, and so on. This reactive nature of output was foundational to quantity theory, which even allowed for variations in the speed of reactive activity by specifying its velocity. The acknowledged strength of the simple Keynesian cross model (which can be found under other names) has always been its capturing of reactive behavior. The reactive narrative has had a huge impact on economic policy by providing cogent

[1] Professor of Economics, James Madison University, Harrisonburg, VA 22807, elwoodsk@jmu.edu. The author would like to thank two anonymous referees for many valuable comments.

explanations of how spending multipliers work and how stimulus policies stimulate further activity. This conceptualization of macroeconomic activity was generally held by economists regardless of their views on whether government should be more activist or more laissez-faire. For example, Milton Friedman based his objection to activist monetary policy on its "long and variable lags" rather than doubts about its ability to influence output (see Friedman 1958).

During the reign of the reactive narrative, the concept of general equilibrium output could be implicitly present as "full-employment" output (aka, "long-run" output) and was specified as exogenous in reactive models of output. It was effectively assumed that a clearing labor market is sufficient to assure general equilibrium (Abel et al. 2020 exemplifies this interpretation). Full-employment output was treated as a potential condition that an economy gravitated towards over time as prices in all of an economy's markets adjusted. Whereas the idea that fully adjusted prices would bring about full-employment existed, the idea that all prices would adjust rapidly enough that general equilibrium would prevail at each perceived moment was rarely considered realistic.

A new narrative advanced in the 1980s, in which expectations strongly influenced behavior and aggregate output was characterized as sequential general equilibria: All markets clear in period *t*, then again in *t+1*, and then again in *t+2*. The dynamic process accounts for a period's general equilibrium influencing the next period's, as well as for expectations of the future affecting the current general equilibrium (see Kydland and Prescott 1982 and Long and Plosser 1983 for the seminal papers that introduced models that capture this process). One imagines a Walrasian auctioneer simultaneously determining the general equilibrium prices of all goods so that the market clearing transactions can occur, immediately followed by the auctioneer repeating the *tatonnement* process of uncovering the market clearing prices of all goods for the next recognized discrete moment. But the path of output predicted by this general equilibrium thinking was much less volatile than what was observed, so the models assumed that each period suffered one or more exogenous stochastic shocks to help make the resulting simulations by these models mimic the observed data.[2] Whereas the early models were referred to as *real business cycle* models, they grew to become known as dynamic stochastic general equilibrium (DSGE) models.

The new story and its DSGE models soon dominated theoretical conceptions of the determination of output in mainstream macroeconomic theory and have continued to do so. Models have been modified to specify a nominal rigidity (often Calvo pricing) that prevents money from being neutral and are regarded as New Keynesian DSGE models (e.g., see Clarida et al. 1999 and Woodford 2003). Then the models' failure to credibly depict the depth of the Great Recession led to a wave of DSGE models that also specify financial markets with financial frictions (e.g., maturity mismatches by a large shadow banking sector, the tightening of lending criteria by creditors, or fluctuations in risk) to better capture more severe economic downturns (e.g., see Gertler and Kiyotaki 2015 and Christiano et al. 2014). Whereas models which have nominal rigidities and financial frictions might be considered inconsistent with general equilibrium given their assumed disruption of an important market, the models continue to be referred to as DSGE models because they are primarily driven by general equilibrium forces.

While the hegemony of the DSGE story has been formidable over the last three decades, circular, reactive behavior continues to be modestly represented within mainstream macroeconomics by agent-based computational economic (ACE) models (e.g., Gaffeo et al. 2008 or Lengnick 2013). Their ability to survive despite the dominance of the DSGE models is impressive given they have no solutions that identify equilibria. Instead, they merely specify reactive processes without clear equilibria. Not being constrained to produce equilibria allows ACE models to capture ongoing circular flow activity propelled by reactive behavior.

Accompanying the dominance of DSGE methodology is the problem it creates for the undergraduate economics curriculum. Mainstream macroeconomic theory did not just welcome DSGE models, it adopted the specification of microfoundations within solvable DSGE systems as a requisite standard for models of equilibrium output. This demanding standard excludes the great majority of mainstream macroeconomic theory from anyone not prepared for graduate studies in economics because DSGE models are considered too mathematically complicated for most undergraduates. This means that the undergraduate curriculum has no model of equilibrium output to put before students that captures how mainstream macroeconomic theory has viewed output since the 1980s. Efforts have been made to find a rendition of DSGE methods that is sufficiently simple for interested undergraduates (see Solis-Garcia 2018 for a design of a course on DSGE modeling for undergraduates and Costa Junior and Garcia-Cintado 2018 for what they call the "simplest

[2] The acclaimed model by Smets and Wouters (2007) specifies not just one, or two, but *seven* different types of shocks that obey assumed highly autocorrelated processes.

frictionless RBC-type DSGE model"). But such models remain significantly more involved than any other models found in undergraduate economics textbooks.

The approach taken by many undergraduate macroeconomic textbooks is to continue introducing one or more of the same models that were found in textbooks when DSGE models came on the scene, i.e., the Keynesian cross, the IS/LM, and aggregate demand/aggregate supply (AD/AS) models. These traditional models that linger in the undergraduate curriculum were unambiguously developed under the influence of the reactive narrative during the pre-DSGE era. Indeed, the Keynesian cross' singular strength is that it concisely captures the reactive flow story. The majority of introductory textbooks that discuss expenditure multipliers do so shortly after presenting the Keynesian cross model. For example, see Frank et al. (2019), Krugman and Wells (2018), and Schiller (2019). Mankiw (2018) provides the Keynesian cross' multiplier effect without fully presenting the model. Colander (2019) represents an exception by describing the multiplier effect before introducing the Keynesian cross model later in an online supplemental chapter to the textbook. The IS curve from the IS/LM model also exhibits the reactive property whether it is explicitly developed by building upon the Keynesian cross or not since its determination of goods market equilibrium specifies output and consumption as interdependent. The usual intermediate theory textbook derivation of the aggregated demand curve builds upon the IS curve and, therefore, also incorporates it.

There are also a few modified versions of these older models that have gained a place in more than one undergraduate textbook. One attributed to Taylor (see Taylor and Weerapana 2018 or Frank et al. 2019) has familiar-looking aggregate demand and aggregate supply curves, but with the price level replaced by inflation as negatively correlated with the level of aggregate demand. This modified aggregate demand curve simply represents a monetary policy reaction function in which the money supply and, therefore, aggregate demand are contracted in response to an increase in inflation. A modification of the IS/LM model has appeared (see Blanchard 2021 or Jones 2021) in which the LM curve (referred to as a MP curve in Jones 2021) is assumed to incorporate a monetary policy reaction function that renders it horizontal. But neither tweak produces a model that exhibits the reactive narrative as effectively as the Keynesian cross.

In the last decade, there has been a model found in a few intermediate textbooks referred to as the "three-equation New Keynesian model" advanced by Carlin and Soskice (2006) (e.g., see Mankiw 2019 or Williamson 2018) which goes by other names (e.g., the dynamic IS-PC-MR or the IS-AS model). It is thought to qualify as New Keynesian because it accounts for expectations of future conditions in its determination of current output, but it does not specify microfoundations or general equilibrium and is effectively a reworking of the AD/AS model (where the AS curve is referred to as the New Keynesian Phillips curve) appended with a monetary policy reaction function and explicit expectations.

A conspicuous weakness of all of these models is that the level of full-employment output – i.e., general equilibrium output – is exogenous. Whether a value for it is simply assumed or it is derived from a previous step as the outcome of a clearing labor market and a production function, it enters into these undergraduate models as predetermined. In other words, these models of output rely on an exogenous assertion of what general equilibrium output is, and, therefore, they provide no insight to the nature of general equilibrium. All the models do is show how output can deviate from the given full-employment output in response to shocks if prices are sticky and do not promptly adjust. While these models certainly fail to meet the DSGE standards, they are being grudgingly accepted within the undergraduate economics curriculum until there is an alternative model that meets the high standards and is sufficiently simple.[3]

The following section presents a new model with two orientations: The first captures and helps build intuition regarding the underlying nature of the reactive narrative while the second does the same for the general equilibrium conception of aggregate output. It is only a teaching model that is too simple to simulate data that can be compared with actual data as done with DSGE models. But a teaching model that captures both narratives is what the undergraduate economics curriculum is lacking.

The Model

The model specifies an economy composed of two agents (*A* and *B*) and two goods (*a* and *b*), where *A* has a comparative advantage in the production of *a*, *B* has the comparative advantage in the production of *b*,

[3] ACE models are mathematically accessible, but their lack of equilibria prevents them from matching the ability of the currently used teaching models to identify an equilibrium level of output. Given that ACE models have not replaced – or even joined – the more traditional models in the undergraduate macroeconomics curriculum allows one to infer that economists prefer introducing the concept of aggregate output using obsolete models that have equilibria over cutting-edge models that do not.

and they specialize accordingly. Each agent fulfills its consumption desires by purchasing the other agent's specialized production (financed by income from sales of its output to that other agent) and consuming homemade output. This backdrop is consistent with economies where a significant portion of output is homemade and never sold or included in measures of gross domestic product (GDP). *A* purchases *b* and *B* purchases *a* using money where the prices of *a* and *b* are P_a and P_b, respectively. *A*'s nominal income is necessarily equal to *B*'s nominal expenditure, with both being equal to $P_a(E_{Ba})$ where E_{Ba} is *B*'s purchases of *a* (from *A*). Accordingly, E_{Ba} is simultaneously *B*'s real expenditure and *A*'s real income in terms of *a*. Similarly, *B*'s nominal income is *A*'s nominal expenditure and equals $P_b(E_{Ab})$ where E_{Ab} is *A*'s purchases of b (from *B*). Thus, E_{Ab} is both *A*'s real expenditure and *B*'s real income in terms of *b*.

Equilibrium within the model consists of equilibrium levels of both E_{Ab} and E_{Ba} which together represent equilibrium aggregate (marketed) output for the two-good economy. The model environment is depicted by a diagram with the quantity of *a* indicated along the horizontal axis and the amount of *b* on the vertical axis (e.g., see Figure 1). The recognition that measured output does not consist of a single homogeneous good is a strength of the new model relative to models of output that employ the simplifying assumption. It invites a discussion about the compromises that accompany any attempt to aggregate the sum of heterogeneous goods into a single number representing *real* aggregate output, as is done by measures of real GDP.

The Determination of Output According to the Reactive Narrative

The model's depiction of the reactive narrative's determination of output specifies *A*'s purchases of *b* as:

$$E_{Ab} = E^0_A/P_b + R_A P_a(E_{Ba})/P_b = E^0_A/P_b + R_A(E_{Ba})p \quad (1)$$

where E^0_A is *A*'s nominal autonomous spending, R_A is the share of *A*'s income that is spent buying *b*, and *p* is the relative price P_a/P_b. The first component of *A*'s expenditures is independent of her income and is only possible because of money balances she holds when the period begins, not that she necessarily spends all her money holdings. One would expect that the amount she spends of her money holdings would vary positively with the size of those holdings (since they constitute wealth) as well as with her consumer confidence. The second term is simply a fraction of her real income spent on *b* during the period (where, as discussed above, E_{Ba} constitutes *A*'s real income in terms of *a*). For example, if R_A=0.70, then *A* spends 70% of her income on *b*. R_A is effectively *A*'s marginal propensity to consume (*mpc*) from her personal consumption function.

Figure 1 illustrates *A*'s expenditure function (i.e., E_{Ab}). Note that Figure 1 provides a line labeled *P* with a slope of *p* (i.e., P_a/P_b) as a reference that indicates the market value of each amount of *a* in terms of *b*, and vice versa. Equation (1) shows that *A* would buy E^0_A/P_b units of *b* if she were to have no income from selling *a* to *B* (i.e., $E_{Ba} = 0$), as represented by her spending line's intercept. Then income from her sales of *a* (i.e., E_{Ba}) causes her to spend more on *b* (i.e., increases E_{Ab}). Her E_{Ab} line would have the same slope of the *P* line *if* $R_A = 1.0$, and it would be horizontal if $R_A = 0$. More generally, the slope of her expenditure line is $(R_A)p$. The line in Figure 1 shows a case where $1.0 > R_A > 0$. As long as $R_A>0$ then *A*'s spending is a positive function of *A*'s income. Note that her nominal income that does not finance purchases of *b* adds to her money holdings.

B's spending on *a* is comparable:

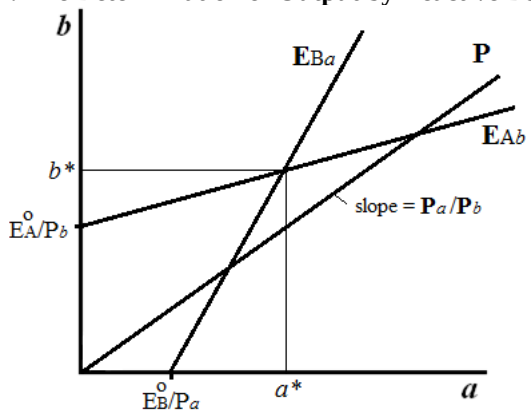
$$E_{Ba} = E^0_B/P_a + R_B P_b(E_{Ab})/P_a = E^0_B/P_a + R_B(E_{Ab})/p \quad (2)$$

where E^0_B is *B*'s nominal autonomous spending financed by his money holdings, R_B is *B*'s share of income that he spends (i.e., his personal *mpc*), and E_{Ab} represents *B*'s real income in terms of *b*. This spending behavior is represented in Figure 1 by the E_{Ba} line which intersects the horizontal (or *a*) axis at E^0_B/P_a and has the slope p/R_B. As with *A*, *B*'s spending is a positive function of his income as long as $R_B > 0$.

Combining these two agents' spending behaviors creates reactive activity: Increases in *A*'s income causes her to spend more, which increases *B*'s income and, therefore, his spending, which increases *A*'s income, and so on. The joint spending behavior will bring about equilibrium levels of *a* and *b* produced and sold, denoted in Figure 1 as *a** and *b** and found where the two expenditure lines E_{Ab} and E_{Ba} intersect.[4] (It is theoretically possible that *a** and *b** would both be infinite, but it would require both R_A and R_B to equal 1.00.)

[4] Finding the intersection of E_{Ab} and E_{Ba} requires solving the system composed of Equations (1) and (2). For example, given $P_a = 4$ and $P_b = 5$ (so that $p = 0.8$), $E^0_A = 400$, $E^0_B = 40$, $R_A = 0.5$ and $R_B = 0.6$, then the equilibrium level of E_{Ba} (= *a**) = 100 and the equilibrium level of E_{Ab} (= *b**) = 120.

Figure 1: The Determination of Output by Reactive Behavior



Note: Output (i.e., $a^* + b^*$) is determined by the intersection of A's and B's reaction functions (E_{Ab} and E_{Ba}, respectively) where both agents take P_a and P_b as given and choose their respective levels of autonomous expenditures (i.e., E^0_A and E^0_B) as well as their respective *mpc*'s (R_A and R_B).

If the intersection of the two expenditure lines fell on the P line then the money balances each agent holds would be unaffected by the period's activity because each agent's nominal autonomous spending would equal the money balances acquired by not spending their entire incomes. But it is possible that the intersection of their spending lines is not on the P line (as is the case in Figure 1) and money balances change hands by the end of the period. For example, Figure 1 illustrates a case where A ends up with fewer money balances than she began the period: Although she accumulates money balances with her income since R_A<1.00, her E^0_A exceeds the balances she recoups so the net change in her balances is negative. To be precise, her change in money balances (ΔM_A) is the amount she earns minus what she spends, i.e.,

$$\Delta M_A = P_a a^* - P_b b^* \tag{3}$$

which is negative in this case. Correspondingly, B's change in money balances (ΔM_B) is

$$\Delta M_B = P_b b^* - P_a a^* \tag{4}$$

so that $\Delta M_B = -\Delta M_A$ (which is assured because the entire exogenous money supply is held between the two agents).

While the intersection of the two spending lines may not fall on P, there will be forces at work promoting that outcome over successive periods. To explain, reconsider the case illustrated by Figure 1. The resulting change in money balances will have a wealth effect on both parties. For A, the reduction in money balances will likely reduce her next E^0_A as well as her R_A. The former would lower the intercept of her expenditure line (E_{Ab}) and the latter would make it flatter, both of which would lead to its intersection with B's expenditure line (E_{Ba}) closer to the P line. At the same time, the wealth effect from the increase in B's money balances would likely cause his E^0_B and R_B to increase, which would both move B's expenditure line to an intersection with A's that is closer to the P line. Therefore, while it is too much to assume that the intersection of the two expenditure lines falls on the P line, there are forces promoting that outcome over time.[5]

The model's determination of equilibrium values of two goods instead of just one distinguishes it from other simple models of output. This requires some form of aggregation of the two to come up with a measure of real aggregate output. While nominal output is simply $P_a a^* + P_b b^*$, real output can be measured in different ways. For example, it could be completely in terms of a (i.e., $a^* + P_b b^*/P_a$), in terms of b (i.e., $b^* + a^* P_a/P_b$), or in terms of some basket of a^* and b^* (i.e., a linear combination of the two extremes). The multiple possible measures of aggregate output reflects a strength of the model since it underlines the aggregation issues that beset actual measures of real aggregate output (i.e., real GDP).

[5] The possible changing hands of money balances by the end of a period is the only way that the model allows for actions in the current period to influence future conditions. This means that the model does account for the effects of current investment on expected future conditions. The model could be modified to specify expectations and investment (and, therefore, capital), but it would be more complicated without contrasting the differences between the reactive and general equilibrium narratives more effectively.

Before presenting the general equilibrium side of the new model, it is worth commenting on how money's role in determining output differs between the two narratives. General equilibrium ignores any consequences from the way money circulates within an economy. It implicitly assumes that a period's trades are independent of any order in which they might occur as if they all occur simultaneously, even if that period is a year in duration. Such independence would only apply if economic activity occurred *as if* a Walrasian auctioneer were in charge and no trades transpire until it is predetermined how each would fit within an assured general equilibrium. Alternatively, the sequential nature of reactive activity presupposes some means of payment that increases the sellers' wealth and precipitates the next transaction in the circular flow. For example, it is because the butcher buys the baker's wares that the baker has the income to purchase something further, and so on. Although this is evidently true in the presence of a cash-in-advance constraint in which money payments finance all purchases, it is also true for those whose expenditures are constrained by their realized income regardless of whether the transactions are settled with cash or credit.

An aspect of money that is inseparable from the notions of reactive behavior and spending multipliers is money's capacity to serve as a reserve of potential aggregate demand. For example, a butcher that holds money balances possesses the means to increase aggregate demand simply by spending them. No correlated aggregate supply activity need occur due to the spending, although it will usually promote greater production given the reduction in a firm's inventory. It is true that the money spent by one person is acquired by another so that no net change in aggregate money balances transpires from the transaction. But the fact that aggregate money balances do not change does not negate the transaction's effect on aggregate demand.[6]

The Determination of Output According to the General Equilibrium Narrative

If supply always responded passively to demand – e.g., if the baker only made more bread upon sales (that reduced inventory) – then the above model's reactive activity would capture the phenomenon that determines output. But scarcity that pressures prices to change in response to changes in spending also need to be taken into account. This section presents the general equilibrium side of the new model which captures scarcity's effects on prices within the same setting that the reactive narrative was just presented. I.e., the two-agent, two-good environment accommodates the agents' offer curves that together determine the equilibrium relative price between the goods and the volume of trade that a Walrasian auctioneer would coordinate. In doing this, the model provides an endogenous determination of output that transparently depicts the nature of general equilibrium. Figure 2 shows the offer curves for agents *A* and *B* (Y_A and Y_B, respectively) whose intersection identifies the market clearing, general equilibrium relative price (or p^{**}, which is seen as the slope of the $\bar{P}$ line) along with the general equilibrium levels of *a* and *b* that are produced and exchanged, i.e., a^{**} and b^{**}, respectively.

Although offer curves are most commonly associated with international trade theory,[7] they capture the same venerable economic concept of reciprocal demand in which one entity demands good X by supplying another good while a second entity supplies good X in demanding the other. Specifying each agent's productive capacity (and the resulting production possibilities frontier) and a relative price between the goods makes the microfoundations of each agent's demand for the other's good and supply of their own good a simple matter of maximizing their utility. The model's determination of an equilibrium relative price between the goods and their equilibrium quantities presents a simple yet sound example of general equilibrium.

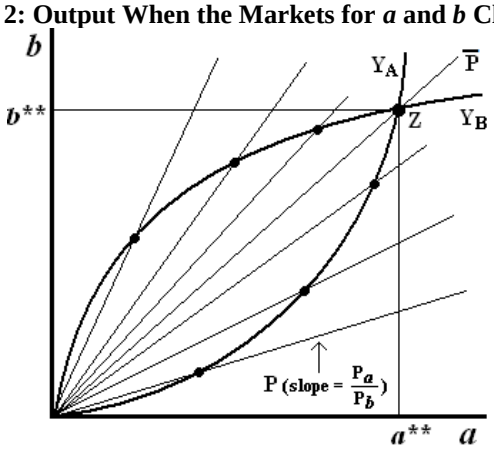
The model adroitly captures shocks to productivity. For example, an increase in *A*'s productivity would increase the opportunity cost of her leisure and she would produce more *a* at every possible relative price ($p = P_a/P_b$), i.e., her Y_A curve would shift out, causing both a^{**} and b^{**} to increase. An increase in *B*'s productivity would cause a corresponding outward shift of Y_B. Given that productivity shocks are the cause of fluctuations in output as well as its growth under the general equilibrium narrative, the model helps explain both phenomena from a general equilibrium perspective. One only need imagine sequential general equilibria

[6] Although this way of thinking about money balances is reminiscent of Monetarism, it is not the Monetarism advanced by Milton Friedman, but is more accurately an offspring of Quantity Theory before it was "restated" by Friedman (1956). The primary difference relevant here is that Friedman regarded velocity as stable, whereas it was not according to the earlier theorists (see Tavlas 1998). The distinction may not seem significant, but it dramatically alters the approach one takes to policy and, of course, Friedman's restated theory was effectively falsified by the significant drop in velocity during the recession of the early 1980's. But the Quantity Theory idea that changes in money supply (or demand) affects the circulation of money and, therefore, economic activity is present in the reactive-behavior narrative.

[7] For example, see Appleyard and Field (2017) or Salvatore (2019).

to see how productivity shocks impact output over time. And, again, a measure of changes in real output over time would have to be in terms of an accepted aggregation of a^{**} and b^{**}.

Figure 2: Output When the Markets for *a* and *b* Clear



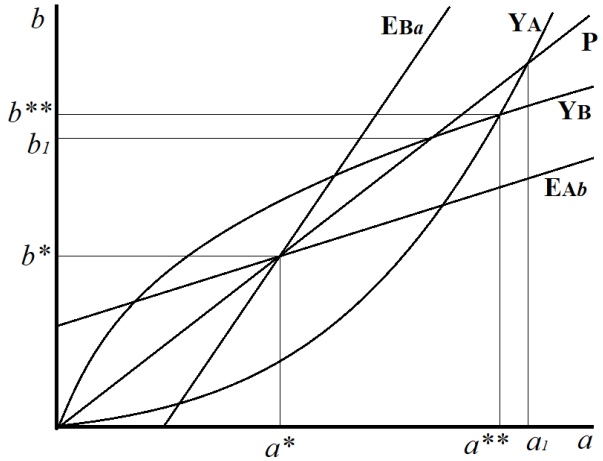
Note: General equilibrium output (i.e., $a^{**} + b^{**}$) is determined by the intersection of *A*'s and *B*'s offer curves (Y_A and Y_B, respectively). Flexible prices adjust until both *A*'s and *B*'s demands for the other's good equals the amount the other wants to supply.

"Long Run" General Equilibrium Forces On "Short Run" Reactive Output

The next step is to examine how the potential of general equilibrium influences an economy that has achieved a reactive equilibrium with levels of a^* and b^* that differ from general equilibrium's a^{**} and b^{**}. This assumes the reactive narrative occurs without prices fully adjusting to their general equilibrium levels. Thus, the reactive narrative provides what is commonly referred to in macroeconomic theory as a "short run equilibrium output." The term "long run equilibrium output" is then used as another way to describe general equilibrium that will exist after prices have fully adjusted.

For example, consider the situation in which both a^* and b^* from the reactive side of the model are less than the a^{**} and b^{**} that would transpire under general equilibrium. Figure 3 illustrates precisely this recessionary case in which both agents are producing and consuming less than what their offer curves reveal they would ideally choose at the going prices. Their dissatisfaction would give them both a reason to reduce the price of their good before the subsequent period – *A* reducing P_a and *B* reducing P_b – to generate more business, and this would affect the next period's output in two ways.

Figure 3: Reactive Behavior Determination of Output that Falls Short of Market Clearing, General Equilibrium Output



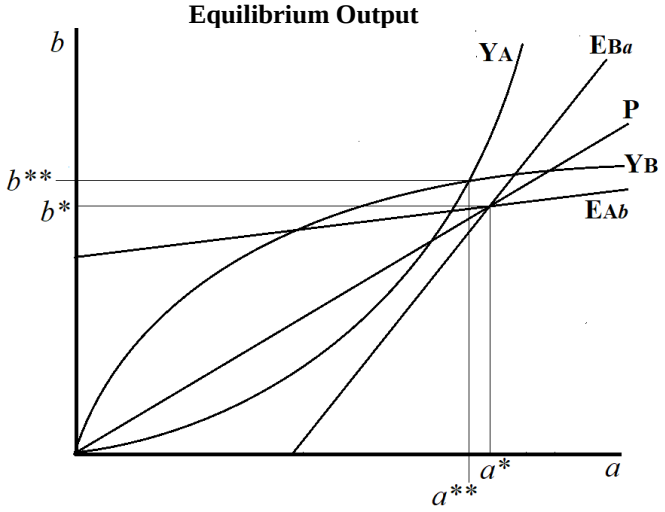
Note: Reactive behavior determination of output ($a^* + b^*$) that is less than output if prices fully adjusted ($a^{**} + b^{**}$) leaves both agents experiencing less sales that desired given their reactions to the relative price (i.e., $a^* < a_1$ and $b^* < b_1$) and provides each with the incentive to lower their prices.

First, the reduction of the two prices would increase the purchasing power of both agents' autonomous expenditures (i.e., E^0_A and E^0_B) that would increase the intercepts of their expenditure lines, with each increasing both a^* and b^* (towards a^{**} and b^{**}). Second, the adjusted prices could change the slope of the P line. It is arguable – though not certain – that in situations such as that found in Figure 3 that the drop in P_a would exceed the drop in P_b. This is because A's desired production given the relative prices captured by the slope of P (i.e., a_1 in Figure 3) exceeds A's actual production (i.e., a^* in Figure 3) to a greater degree than B's preferred production (b_1 in Figure 3) exceeds B's production (i.e., b^*). The greater (percentage) drop in P_a would cause the slope of P ($=P_a/P_b = p$) to fall towards the P that would prevail in general equilibrium. Thus, the economic forces pushing prices down would push the relative price towards its long run, general equilibrium value as well as promote increased output.

The analysis of an economic boom in which a^* and b^* both exceed a^{**} and b^{**} respectively would relay the opposite, yet symmetric story: The undesirably high production would cause both agents to raise their prices, which would reduce the purchasing power of their autonomous spending that would promote less output and would likely push the relative price towards what a Walrasian auctioneer would uncover.

Consider a third case in which one agent is producing more than would be desired under general equilibrium and the other is producing less as depicted in Figure 4. A would have reason under this circumstance to raise the price of P_a when feasible (i.e., before the next period) given that she is producing more than desired at that p. It would also give B a reason to lower P_b to gain more sales since he is producing less than desired. The combination of price changes would unambiguously increase the slope of P (i.e., p) towards its long run, general equilibrium value.

Figure 4: Reactive Behavior Determination of Output that Differs From Market Clearing, General Equilibrium Output



Note: Reactive behavior determination of output ($a^* + b^*$) that differs from output if prices fully adjust ($a^{**} + b^{**}$) which does not necessarily depict a recession or economic boom. Agent A experiencing more sales than desired given the relative price (i.e., $a^* > a^{**}$) gives her the incentive to raise P_a, and, agent B experiencing b less (i.e., $b^* < b^{**}$) pushes him to lower P_b. Both will make the P line steeper, i.e., make the relative price closer to what would prevail in general equilibrium.

Monetary and Fiscal Policy

Monetary and fiscal policies designed to counter the business cycle are based primarily on the reactive narrative and attempt to alter short run output (more rapidly than will occur as prices adjust) by changing expenditures. For example, consider expansionary monetary policy. The increased money balances held by A and/or B could lead one or both of them to increase their autonomous spending and/or their share of respending. Increases in E^0_A will shift A's expenditure line (E_{Ab}) up and increases in R_A will pivot it out, just as increases in E^0_B and R_B will shift and pivot B's expenditure line out, respectively, any of which will cause both a^* and b^* to increase. Of course, increasing the money supply will not have any impact unless it affects at least one of these agents' behavior; an increase in the money supply that does not alter someone's autonomous spending or share of respending (i.e., in the case that money demand rises commensurately) would have no effect.

The analysis of changes in money demand employs similar reasoning. For example, increases in money demand by an individual (perhaps due to a decrease in consumer confidence) will decrease the individual's autonomous spending and/or personal *mpc*. The former reduces the intercept of the individual's expenditure line and the latter pivots it towards the *P* line. Both will result in an intersection of the two agents' expenditure lines at lower levels of both *a** and *b**. The ability of changes in the money supply to counter changes in money demand strongly endorses the use of monetary policy to accommodate changes in money demand.

Examining changes in credit using the model is very similar to the above analysis. A dependable feature of credit is that the borrower spends the proceeds of the loan. It could be that *A*, instead of holding money balances, lends them to *B*, who then spends them. The increase in *B*'s autonomous spending will stimulate economic activity (as captured by the outward shift of *B*'s expenditure line). Bank lending effectively does this without requiring that *A* take on the risk of lending, i.e., *A*'s deposit in the bank can simultaneously allow *A* to hold money balances (as deposits) while the bank lends money to *B* to spend.

The effects of fiscal policy are a little tricky given that introducing government activity requires accommodating a third entity. One way to fit government purchases into the two-dimensional diagram is to include it as a form of autonomous spending in the economy. Assuming that the government would purchase a variety of products, it is appropriate to specify spending on both *a* and *b*. The expenditure lines would then not only represent *A*'s and *B*'s spending, but their spending augmented by government purchases. The autonomous spending on *b* and *a* would increase from E^0_A/P_b and E^0_B/P_a to $(E^0_A + G_b)/P_b$ and $(E^0_B + G_a)/P_a$, respectively, where G_b and G_a are the nominal amount spent by the government on *b* and *a*, respectively. Figure 5 shows the expenditure by society on good *b*:

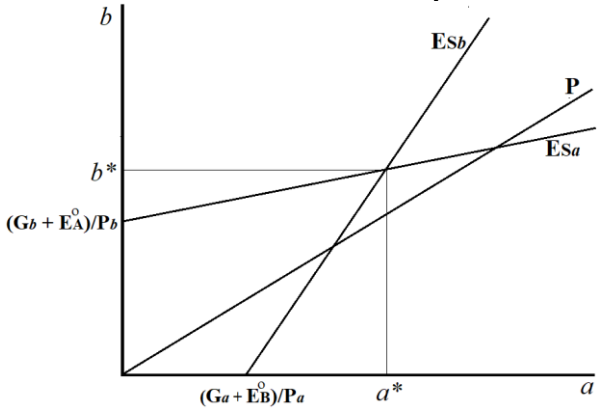
$$E_{Sb} = (G_b + E^0_A)/P_b + R_A P_a(E_{Ba})/P_b = (G_b + E^0_A)/P_b + R_A p(E_{Ba}) \quad (5)$$

and the expenditure by society on good *a*:

$$E_{Sa} = (G_a + E^0_B)/P_a + R_B P_b(E_{Ab})/P_a = (G_a + E^0_B)/P_a + R_B b_A(E_{Ab})/p \quad (6)$$

which is very similar to the model depicted by Figure 1, except the government spending increases the intercepts by G_b/P_b and G_a/P_a, respectively, which would unambiguously increase both *a** and *b**.

Figure 5: Reactive Behavior's Determination of Output with Government Purchases



Note: Government purchases of *a* and *b* contribute to the nominal autonomous expenditures on both goods and, therefore, influence the determination of *a** and *b**.

Income taxes are captured by specifying a second reason why income is not respent; each agent's respending is reduced due to the decrease in disposable income from the taxes. The taxes' attenuation of the reactive effects in the model is represented by pivoting the expenditure lines towards the *P* line. Adjusting for taxes and again including G_a and G_b now cause the two expenditure functions by society to be

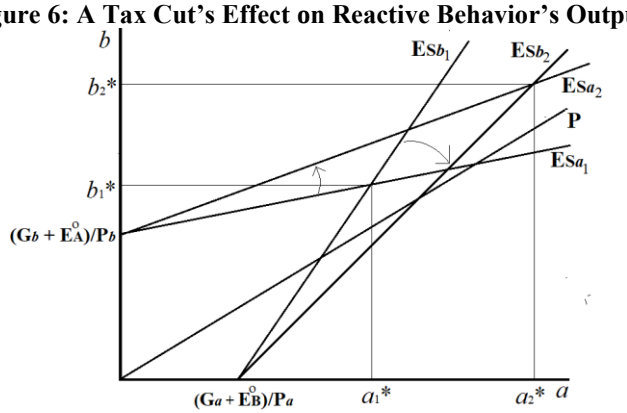
$$E_{Sb} = (G_b + E^0_A)/P_b + R_A(1 - t)p(E_{Ba}) \quad (7)$$

and

$$E_{Sa} = (G_a + E^0_B)/P_a + R_B(1- t)(E_{Ab})/p \quad (8)$$

where t is the income tax rate. Figure 6 illustrates the effect of a *decrease* in the tax rate that causes the expenditure lines to pivot away from the P line so that both a^* and b^* increase (from a_1^* to a_2^* and from b_1^* to b_2^*). Of course, the net effect on output of an increase in government spending accompanied by a tax increase depends on their relative strengths. Note that the specification permits the representation of Ricardian equivalence if a tax cut (that might otherwise increase spending) is directed completely to augmenting money balances (or purchasing the government bonds issued to replace the financing lost by the tax cut).

Figure 6: A Tax Cut's Effect on Reactive Behavior's Output



Note: Tax rate decreases that cause agents to increase the share of their sales (i.e., their pre-tax incomes) that they spend on the other's good will cause the E_{Sa} and E_{Sb} lines to pivot out. Equilibrium output of both goods will rise (i.e., a^* increases from a^*_1 to a^*_2 and b^* increases from b^*_1 to b^*_2).

Conclusion

This paper has introduced a simple model of output that is able to relate and effectively contrast the two major narratives used to characterize macroeconomic activity: The reactive, circular flow story and the general equilibrium story. The latter's hegemony in current mainstream macroeconomic theory for over three decades has not caused the former to disappear for two reasons. One, because it survives in many economists' intuitions and can be found in ACE models. But it also persists because the models used to capture general equilibrium remain too complicated for undergraduate students, so that the undergraduate curriculum continues to rely on the more accessible older models that are consistent with the reactive narrative. The problem is that these traditional models offer no insights to the determination of general equilibrium output and specify it as an exogenous variable (that somehow occurs when the labor market clears). In contrast, general equilibrium output is endogenous in the simple model presented in this paper and is determined in the same environment as the reactive level of output.

Additional macroeconomic perspectives are represented by the model: Short run influences on output are captured by the reactive, circular flow side of the model, and, long run intuitions under the assumption of fully adjusted prices are maintained by the specified general equilibrium level of output, where the long run general equilibrium forces exert influence on subsequent short run, reactive levels of output. The model is also effective at showing how both money supply and money demand affect spending activity and, therefore, short run output. A further valuable feature is its highlighting of the aggregation problem that exists when constructing a measure of real output for an economy with more than one good.

The most appropriate place to introduce the model in an undergraduate economics curriculum is not obvious. The reactive side of the model is modestly more complicated than the Keynesian cross and could be taught in introductory macroeconomics courses. The reactive diagrams could stand alone, however: solving for values of a^* and b^*, as well as multipliers, only requires knowledge of basic algebra (i.e., the ability to solve systems of two equations with two unknowns). The general equilibrium side of the model requires understanding offer curves, which are most commonly taught in upper-level international trade courses, and to introduce them in an introductory macroeconomics class might be considered too ambitious. Intermediate macroeconomics classes would not have difficulty mastering the reactive side of the model, but

introducing offer curves could be a little time consuming, particularly in classes where the students have not already had intermediate microeconomics or learned about indifference curves.

References

Abel, Andrew B., Ben S. Bernanke, and Dean Croushore. 2020. *Macroeconomics*, 10th ed. New York: Pearson.

Appleyard, Dennis R., and Alfred J. Field, Jr. 2017. *International Economics*, 9th ed. New York: McGraw Hill.

Blanchard, Olivier J. 2021. *Macroeconomics*, 8th ed. Boston: Pearson.

Carlin, Wendy J., and David W. Soskice. 2006. *Macroeconomics: Imperfections, Institutions, and Policies*. New York: Oxford University Press.

Clarida, Richard H., Jordi Galí, and Mark L. Gertler. 1999. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature* 37: 1661–1707.

Colander, David C. 2019. *Macroeconomics*, 11th ed. New York: McGraw-Hill.

Costa Junior, Celso J., and Alejandro C. Garcia-Cintado. 2018. "Teaching DSGE Models to Undergraduates." *EconomiA* 19: 424-444.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno. 2014. "Risk Shocks." *American Economic Review* 104(1): 27–65.

Frank, Robert H., Ben S. Bernanke, Kate Antonovics, and Ori Heffetz. 2019. *Principles of Macroeconomics*, 7th ed. New York: McGraw Hill.

Friedman, Milton. 1956. "The Quantity Theory of Money: A Restatement." In *Studies in the Quantity Theory of Money*, edited by Milton Friedman. Chicago: Chicago University Press.

Friedman, Milton. 1958. "The Supply of Money and Changes in Prices and Output." In: The Relationship of Prices to Economics Stability and Growth, 85th Congress, 2nd Session, Joint Economic Committee. Washington, DC: U.S. Printing Office. Reprinted in: *The Optimum Quantity of Money and Other Essays*, edited by Milton Friedman, pp.171-87. Chicago: Adeline.

Gaffeo, Edoardo, Domenico Delli Gatti, Saul Desiderio, and Mauro Gallegati. 2008. "Adaptive Microfoundations for Emergent Macroeconomics." *Eastern Economic Journal* 34: 441-463.

Gertler, Mark L., and Nobuhiro Kiyotaki. 2015. "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy." *American Economic Review* 105: 2011–2043.

Jones, Charles I. 2021. *Macroeconomics*, 5th ed. New York: Norton.

Krugman, Paul R., and Robin E. Wells. 2018. *Macroeconomics*, 5th ed. New York: Worth.

Kydland, Finn E., and Edward C. Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica* 50: 1345–70.

Lengnick, Matthias. 2013. "Agent-Based Macroeconomics: A Baseline Model." *Journal of Economic Behavior and Organization* 86: 102-120.

Long, John B., and Charles I. Plosser. 1983. "Real Business Cycles." *Journal of Political Economy* 91: 39–69.

Mankiw, N. Gregory. 2018. *Principles of Macroeconomics*, 8th ed. Stamford: Cengage.

Mankiw, N. Gregory. 2019. *Macroeconomics*, 10th ed. New York: Macmillan.

Salvatore, Dominick. 2019. *International Economics*, 13th ed. Hoboken: Wiley.

Schiller, Bradley R. 2019. *The Macroeconomy Today*, 15th ed. New York: McGraw-Hill.

Smets, Frank R., and Rafael Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review* 97: 586-606.

Solis-Garcia, Mario. 2018. "The Macro Pedagogy Debate: Teaching DSGE to Undergraduates Symposium." *Journal of Economic Education* 49: 226-236.

Tavlas, George S. 1998. "Retrospectives: Was the Monetarist Tradition Invented?" *Journal of Economic Perspectives* 12: 211-222.

Taylor, John B., and Akila Weerapana. 2018. *Principles of Macroeconomics*, 8th ed. Boston: Flatworld.

Williamson, Stephen D. 2018. *Macroeconomics*, 6th ed. New York: Pearson.

Woodford, Michael D. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton: University Press.

The Cost of Underperforming Investments

William A. Ogden, Pedro Sottile, and Yom Bui[1]

ABSTRACT

Underperforming investments change a firm's equity value, cost of equity, capital structure and cost of capital. We show how to correctly adjust the cost of capital and correctly value a firm with underperforming investments. The correct equity estimate equates the equity value derived from the Economic Profit Model with the equity used to estimate the WACC. We demonstrate how underperforming investments can reduce equity value even though earnings may increase. We confirm our results with the Residual Earnings Model and show that our approach for valuing a company with underperforming investments also works well with investments that increase equity value.

Introduction

The pressure for a company to grow can be substantial. Lauenstein (1983) summarizes a litany of sources of pressure for growth, including personal motivation of management, interests of supporting units (lawyers, auditors, commercial bankers, investment bankers, and consultants), boards of directors, and shareholders. Organic growth opportunities can be difficult to find, particularly for companies in low or no growth industries (such as steel). This can tempt companies to venture into unwise acquisitions. Jackson (2007) notes that literally thousands of private equity firms are competing in the market for good companies. The premiums that bidding companies offer for targets will limit the number of buyers whose acquisitions create value for their shareholders.

The emphasis on growth and reliance on earnings as a performance measure can lead to underperforming investments. The change in company value that results from underproductive investments is reflected in lost equity value. To fully understand the wealth effect of an underperforming investment, we need to accurately estimate its impact on the company's cost of capital. With an accurate estimate of the cost of capital, we can value the company correctly.

In corporate finance courses, we teach the basics for estimating the cost of capital. We also teach how changes to the capital structure impact the cost of capital. However, the changes in the capital structure that we typically analyze are intentional changes such as stock buybacks or issuing stock to retire debt (for examples, see Brealey et al. 2023). How do you adjust the cost of capital when the change in the capital structure is the result of underperforming investments? We can use our fundamental cost of capital tools to identify underperforming investments and correctly adjust the company's cost of capital. Estimating the correct cost of capital requires iteration between cost of capital calculations and the valuation models.

As noted by Fernandez (2020), only the Adjusted Present Value (APV) does not require iterating in order to properly estimate enterprise value. He argues that "many valuations are incorrect because the authors do not iterate and, therefore, the four methods do not provide the same value" (Fernandez 2020, p. 18). One of the four methods cited by Fernandez that can lead to incorrect valuations without iterating is the Capital Cash Flow (CCF) model by Ruback (2002). Ruback's paper includes a table that highlights the percentage valuation errors introduced by CCF because it deviates from the APV method in that CCF discounts the interest tax shields at the unlevered cost of capital instead of the cost of debt (as per the APV). A study by Markou and Taylor (2014) of valuation methodologies at three large global investment banks indicated that analysts employed Discounted Cash Flow (DCF) and multiples-based models to arrive at target stock prices, but made no mention of APV or iterating.

[1]Ogden: Professor of Finance, College of Business, University of Wisconsin-Eau Claire, WI 54701. Sottile: Associate Professor of Finance, College of Business, University of Wisconsin-Eau Claire, WI 54701. Bui: Assistant Professor of Finance, College of Business, University of Wisconsin-Eau Claire, WI 54701.

In our corporate finance courses, we introduce and utilize the economic profit, free cash flow, and cash flow to equity valuation models. We present corporate finance as two distinct courses, working capital management and long–term financial management. After completing these courses, students are highly proficient with Excel, including optimizations via the Solver and Goal Seek functions. At this point in their course work, many students have also completed the portfolio management and financial modeling courses, where they also apply Excel optimization techniques. Concepts such as the WACC, levered, and unlevered cost of equity are components in the cost of capital unit of the long–term financial management course. Rather than introducing the concept of iterative estimating in the long–term financial management course when we first introduce valuation models, we suggest students apply the Net Present Value (NPV) if the constant debt/value assumption is met or approximately met and estimate value with the APV if the debt/value ratio is not constant or approximately constant, as suggested by Brigham and Daves (2017). In the corporate valuation course, we introduce the iterative process. We set up iterative processes for the Free Cash Flow (FCF), Economic Profit (EP), and the Residual Earnings (RE) models while valuing an example company. With an understanding of the economic profit model, the similarly structured residual earnings model (that we employ to verify equity value) is not difficult for students to learn. We cannot find strong support that practitioners frequently employ iterative valuation methods. However, the exercise that we introduce in the following sections serves other purposes by:

- Reinforcing that investments must earn the WACC in order to increase shareholder value.
- Demonstrating that all models should lead to the same valuation if given consistent input values.
- Familiarizing students with iterative estimating, which is useful for other purposes, such as estimating the equity risk premium following the Excel setup provided by Damodaran (2021).

While we stress that a company's investments must earn at least the WACC, the example in the following section makes it clear that the loss in value resulting from underperforming investments accrues to the shareholders. Another point that the example reinforces is that increasing earnings does not necessarily increase value. In the next section, we analyze an example of a company's expansion via underperforming investments. In addition, we show that the same approach to finding the correct cost of capital for underperforming investments works well with strongly performing investments.

Underperforming Investments, an Example

In this section, we present an example of a company in a no–growth industry. The Spellman Company has an EBIT of $40, a federal plus state tax rate of 25% and $160 of 4% debt, giving a net income of $25.20 as shown in Figure 1.

Figure 1: Initial Spellman Company Financial Statements

Spellman, Inc.

Income Statement	
EBIT	40
Interest	6.4
EBT	33.6
Taxes (25%)	8.4
Earnings	25.2

Balance Sheet			
Assets	400	Debt	160
		Equity	240
			400

Valuation of the Company based on the Economic Profit Model

The company's equity market value equals its book value of $240 for a total capital value of $160 (debt) + $240 (equity) = $400. With an initial equity cost of capital (r_E) of 10.50%, the company's WACC is:

WACC = (Debt/Value)(r_D)(1 – T) + (Equity/Value)(r_E)
= 160/400*4.0%*(1 – 25.0%) + 240/400*10.50%
= 7.50%

Assuming that the company does not grow (or make capital investments), the value of the company following the Economic Profit Model is:

Value = [EBIT(1 – T) – (Beginning Capital)*(WACC)] / (WACC – g) + Beginning Capital,

where:
EBIT: Earnings before Interest and Taxes = 40.
T: Tax rate (federal + state = 25%).
Beginning Capital: Investor Capital = Debt + Equity = 160 + 240 = 400.
WACC: Weighted Average Cost of Capital = 7.50%, calculated above.
g: Growth rate of Economic Profits = 0%.

Value = [40(1–25.0%) – 400*7.50%] / (7.50%–0%) + 400
= (30 – 30)/7.50% + 400
= $400

Thus, Spellman's market and book values are equal at $400.

New Capital Investment

In an attempt to increase its value, Spellman invests an additional $300. The new capital will be scale enhancing; thus, the company will borrow 160/400 × 300 = $120 (at 4%) and issue new stock of 240/400 × 300 = $180. This maintains the existing capital structure of 40% debt and 60% equity. The new investment raises total EBIT to $60. As before, the investment is in the same no–growth industry. The resulting net income is $36.60 as shown in Figure 2:

Figure 2: Spellman Company Financial Statements after $300 of New Capital

Spellman, Inc.

Income Statement

EBIT	60
Interest	11.2
EBT	48.8
Taxes (25%)	12.2
Earnings	36.6

Balance Sheet

Assets	700	Debt	280
		Equity	420
			700

When valuing the company with $700 of capital and no growth opportunities, Spellman's value is:

Value = [EBIT(1–T) – (Beginning Capital)*(WACC)] / (WACC – g) + Beginning Capital
= [60*(1 – 25%) – 700*7.50%]/(7.50% – 0%) + 700
= (45 – 52.5)/7.50% + 700
= $600.

With $280 in debt, the new equity value is:

Equity = Value – Debt
= 600 – 280
= $320.

Shareholders invested an additional $180, but their value increased by only 320–240 = $80.

We argue that the company is not valued correctly. The WACC is calculated based on the market values of the capital components. Koller et al. (2020) point out the circularity of the equity valuation process. In order to obtain an accurate valuation, you must value simultaneously the company and its equity. Most valuations of publicly traded companies estimate the equity value for the WACC equation by multiplying the

number of shares of common stock by the price per share. Thus, the analyst has already assumed a value for what he or she is trying to estimate.

Based on the above valuation of Spellman, the company has a total value of $600 with $320 or 320/600 = 53% equity and (1 – 53%) = 47% debt, not 40% debt as we assumed for the WACC estimate. We estimated the WACC with an equity value of $420 and arrived at an Economic Profit Model Equity value of $320. Our goal is to produce an equity valuation that matches the value we use in the WACC estimate. In order to accomplish this, we must resolve the circularity issue identified by Koller et al. (2020). In the following section, we present a process for resolving it.

The Unlevered Cost of Capital and Revaluation of the Company

How do we obtain an accurate cost of capital estimate? Our approach is to begin with a value that does not change as debt and equity values change: the unlevered cost of capital (r_U). We define (r_U) as the cost of capital with all equity financing. We calculate r_U with the following equation:

r_U = Debt/Value * r_D + Equity/Value * r_E,

where:
r_D is the cost of debt.
r_E is the cost of equity.
Value is the sum of Debt and Equity capital at market values.

We calculate r_U based on its initial capital values (prior to the addition of $300 in new capital):

r_U = 160/400*4.00% + 240/400*10.50% = 7.90%

To find the cost of equity (r_E) with a total capital of $700, we re-lever r_U using the equity value that we found in our initial valuation with the $300 in new capital (equity = $320):

$r_E = r_U + (r_U - r_D)$*Debt/Equity = 7.90% + (7.90% – 4.00%) * 280/320 = 11.31%.

The WACC based on our initial valuation with $700 of investor–provided capital (value = $600) is:

WACC = Debt/Value (r_D)(1 – T) + Equity/Value (r_E)
= 280/600(4%)(1 – 25%) + 320/600(11.31%)
= 7.43%.

Revaluing Spellman using a *WACC* of 7.43%, we obtain a value of:

Value = [60*(1 – 25%)–600*7.43%]/(7.43% – 0%) + 600
= 5.65 + 600
= $605.65.

The imputed *equity* value is:

Equity = Value – Debt = 605.65 – 280 = $325.65.

Now we find an *equity* value of $325.65. Our initial difference between $420 (based on capital provided by shareholders) and $320 (found from the Economic Profit Model) was $100. After re–estimating the WACC, the Economic Profit Model Equity value and the equity value used in the WACC equation now differ by only $5.65. In the following section, we introduce an iterative approach to finding the correct WACC and valuation for the Spellman Company. The *WACC* of 7.50% and the associated *Equity* value of $320 will be the beginning values in the iterative process.

Obtaining Equal Equity Values from the Economic Profit Model and WACC Equation

We are now relatively close to having the same equity value from the Economic Profit model as we are employing in the WACC calculation. Rather than continuing this trial-and-error process, we can set up the worksheet in Figure 3.

Figure 3: Valuations of Spellman Initial Setup

	A	B	C	D	E	F
1	Capital Charge based on Initial Investor Capital					
2						
3	Calculation of Earnings	Scenario	Initial	1	2	3
4	EBIT		40	60	70	80
5	Interest		6.40	11.20	11.20	11.20
6	Before-Tax Income		33.60	48.80	58.80	68.80
7	Taxes		8.40	12.20	14.70	17.20
8	Earnings		25.20	36.60	44.10	51.60
9	% change in Earnings			45%	75%	105%
10						
11	WACC Calculations					
12	WACC Equity	Solve for	240.00	420.00	420.00	420.00
13	WACC Debt	Given	160.00	280.00	280.00	280.00
14	WACC Value	WACC Debt + Equity	400.00	700.00	700.00	700.00
15	rU	Previously calculated	7.90%	7.90%	7.90%	7.90%
16	rE (WACC)	rU + (rU – rD)D/E	10.50%	10.50%	10.50%	10.50%
17	rD (WACC)	Given	4.00%	4.00%	4.00%	4.00%
18	WACC Debt/Value	WACC Debt/Value	40.00%	40.00%	40.00%	40.00%
19	WACC Equity/Value	WACC Equity/Value	60.00%	60.00%	60.00%	60.00%
20	WACC	D/V*rD*(1 – T) + E/V*rE	7.50%	7.50%	7.50%	7.50%
21						
22	WACC Equation Value - Economic Profit Model Value		0.00	100.00	0.00	-100.00
23	(Row 14 – Row 28)					
24	Economic Profit Model Calculations:					
25	Beginning Capital	Given	400	700	700	700
26	NOPAT	EBIT(1 – T)	30.00	45.00	52.50	60.00
27	Economic Profit (EP)	NOPAT – WACC*Beg Cap	0.00	-7.50	0.00	7.50
28	Value	EP/(WACC – g) + Beg Cap	400.00	600.00	700.00	800.00
29	Less: Debt	Given	160.00	280.00	280.00	280.00
30	Equity	Value – Debt	240.00	320.00	420.00	520.00
31	Equity from Investors		240	420	420	420
32	% change in Equity value			-23.81%	0.00%	23.81%
33						
34	Residual Earnings Equity Valuation					
35	NOPAT	From above	30.00	45.00	52.50	60.00
36	Interest A/T	Debt*rD*(1 – T)	4.80	8.40	8.40	8.40
37	Flow to Equity (FTE)	NOPAT – Interest A/T	25.20	36.60	44.10	51.60
38	Equity Capital Charge	Beginning Equity*rE	25.20	44.10	44.10	44.10
39	Residual Earnings (RE)	FTE – Equity Cap Chg	0.00	-7.50	0.00	7.50
40	PV RE	RE/(rE – g)	0.00	-71.43	0.00	71.43
41	Beginning Equity Value	Given	240.00	420.00	420.00	420.00
42	Ending Equity Value	Initial Equity + PV RE	240.00	348.57	420.00	491.43

Note: WACC equity values in row 14 and the equity values derived from the Economic Profit model in row 28 differ for EBIT values of 60 and 80.

In column C, we show the initial valuation of Spellman with $400 in capital. In columns D, E, and F we have valuations with $700 of capital. We value the company following three scenarios: *EBIT* increasing to $60, $70, and $80 for Scenarios 1, 2, and 3, respectively, as a result of increasing capital by $300. We set the equity value to $420 in cells D12, E12, and F12 (the amount shareholders have contributed). Valuing

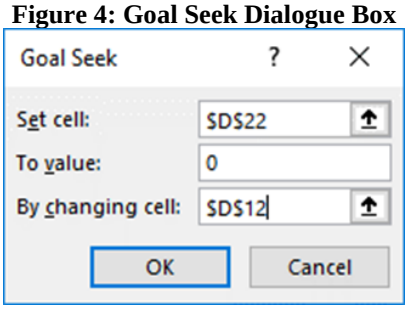
Spellman in Scenario 1 with cell D12 set at $420, the WACC company value is $700 (cell D14), while the Economic Profit Model value based on a *NOPAT* of $45, *investor capital* of $700, and a *WACC* of 7.50% yields a value of $600 (cell D28):

Value = [EBIT(1–T) – (Beginning Capital)(WACC)] / (WACC – g) + Beginning Capital
= [60*(1 – 25%) – 700*7.50%]/(7.50% – 0%) + 700
= (45 – 52.5)/7.50% + 700
= $600.

The $100 difference between the Economic Profit Model and WACC equation corporate values is in cell D22.

A correct valuation in Scenario 1 will result in cells D14 and D28 having equal values and D22 equaling 0. We accomplish this by employing either Excel's Goal Seek or Solver function. Since no constraints are required, Goal Seek is sufficient. In Goal Seek, we set the value of Cell D22 to a value of 0 by changing Cell D12 as shown in Figure 4.

Figure 4: Goal Seek Dialogue Box



After clicking "OK", Goal Seek iterates the value in cell D22 to 0 by changing cell D12 to a final value of 325.06 as shown in Figure 5 under Scenario 1. While shareholders contributed an additional $420 – $240 = $180, equity value has increased only $325.06 – $240 = $85.06, for a loss in value of $94.94. In contrast, Spellman's creditors that contributed $160 initially and an additional $120 to fund the expansion find the value of their loans remains at $280. Thus, the loss in Spellman's value of $700 – $605.06 = $94.94 comes entirely from the shareholders. Also in this scenario, the expansion increased EBIT to $60 from $40 and earnings to $36.60 from $25.20, or by 45%. From this outcome, students have an example of how increased earnings do not necessarily translate to greater shareholder value. We emphasize using the economic profit model to determine if a company is creating value rather than relying on earnings. Economic profit in Scenario 1 is – $7.06 because the after-tax operating profit *(EBIT)(1 – T)* = $60(1 – 25%) = $45 is less than the capital charge *(Beginning Capital)*WACC* = $700*7.44% = $52.06. Economic profit in Scenario 1 is – $7.06, indicating a loss in value to the company's shareholders (if interest exceeds EBIT, creditors would also lose value).

To verify the Scenario 1 results, we also value equity with the Residual Earnings Model. The model bases equity value on estimates of the cash flows available to shareholders. Against the cash flow available to investors, the model assesses a capital charge (product of the cost of equity times the equity capital contribution). From Figure 3, the Residual Earnings Model values Spellman's Equity as follows:

Earnings Available to Equity = NOPAT – Interest After Taxes = 60*(1 – 25%) – 280*4%*(1–25%) = $36.60.

Equity Capital Charge = Beginning Equity*r_E = 420*10.5% = $44.10.

Residual Earnings = Earnings Available to Equity – Equity capital Charge = 36.60 – 44.10 = – $7.50.

With –$7.50 in no growth residual earnings, the value of equity changes by:

Present Value of Residual Earnings = RE/(r_E – g) = –7.50 / (10.50% – 0) = – $71.43.

Equity = Beginning Equity + Present Value of Residual Earnings = 420 – 71.43 = $348.57.

In Figure 3, Scenario 1 has three different equity values: $420 (cell D12, WACC equation), $320 (cell D30, Economic Profit model), and $348.57 (cell D42, Residual Earnings model). Goal Seek in Figure 4 iterates all three to a value of $325.06 as shown in Figure 5.

Figure 5: Completed Spellman Valuations with Nominal Values of Investor Capital

	A	B	C	D	E	F
1	Capital Charge based on Initial Investor Capital					
2						
3	Calculation of Earnings	Scenario	Initial	1	2	3
4	EBIT		40	60	70	80
5	Interest		6.40	11.20	11.20	11.20
6	Before-Tax Income		33.60	48.80	58.80	68.80
7	Taxes		8.40	12.20	14.70	17.20
8	Earnings		25.20	36.60	44.10	51.60
9	% change in Earnings			45%	75%	105%
10						
11	WACC Calculations					
12	WACC Equity	Solve for	240.00	325.06	420.00	514.94
13	WACC Debt	Given	160.00	280.00	280.00	280.00
14	WACC Value	WACC Debt + Equity	400.00	605.06	700.00	794.94
15	rU	Previously calculated	7.90%	7.90%	7.90%	7.90%
16	rE (WACC)	rU + (rU – rD)D/E	10.50%	11.26%	10.50%	10.02%
17	rD (WACC)	Given	4.00%	4.00%	4.00%	4.00%
18	WACC Debt/Value	WACC Debt/Value	40.00%	46.28%	40.00%	35.22%
19	WACC Equity/Value	WACC Equity/Value	60.00%	53.72%	60.00%	64.78%
20	WACC	D/V*rD*(1 – T) + E/V*rE	7.50%	7.44%	7.50%	7.55%
21						
22	WACC Equation Value - Economic Profit Model Value		0.00	0.00	0.00	0.00
23	(Row 14 – Row 28)					
24	Economic Profit Model Calculations:					
25	Beginning Capital	Given	400	700	700	700
26	NOPAT	EBIT(1 – T)	30.00	45.00	52.50	60.00
27	Economic Profit (EP)	NOPAT – WACC*Beg Cap	0.00	-7.06	0.00	7.17
28	Value	EP/(WACC – g) + Beg Cap	400.00	605.06	700.00	794.94
29	Less: Debt	Given	160.00	280.00	280.00	280.00
30	Equity	Value – Debt	240.00	325.06	420.00	514.94
31	Equity from Investors		240	420	420	420
32	% change in Equity value			-22.60%	0.00%	22.60%
33						
34	Residual Earnings Equity Valuation					
35	NOPAT	From above	30.00	45.00	52.50	60.00
36	Interest A/T	Debt*rD*(1 – T)	4.80	8.40	8.40	8.40
37	Flow to Equity (FTE)	NOPAT – Interest A/T	25.20	36.60	44.10	51.60
38	Equity Capital Charge	Beginning Equity*rE	25.20	47.29	44.10	42.09
39	Residual Earnings (RE)	FTE – Equity Cap Chg	0.00	-10.69	0.00	9.51
40	PV RE	RE/(rE – g)	0.00	-94.94	0.00	94.94
41	Beginning Equity Value	Given	240.00	420.00	420.00	420.00
42	Ending Equity Value	Initial Equity + PV RE	240.00	325.06	420.00	514.94

Note: WACC equity values in row 14 and the equity values derived from the Economic Profit model in row 28 are equal for all EBIT values.

In Scenario 2, the new $300 of capital earns exactly the WACC. Equity values in the WACC equation, the Economic Profit, and Residual Earnings models all agree with a value of $420, or exactly the amount that shareholders have invested in Spellman. Economic profit and residual earnings are both $0.00.

With an *EBIT* of $80 in Scenario 3 on Figure 5, economic profits and residual earnings are positive, adding to shareholder value. Compared to what shareholders invested in Spellman, the value of their capital increases by 22.60% (cell F32).

Capital Charges Based on Market Values of Capital

A different approach to valuing Spellman recognizes that the capital charges (both Economic Profit and Residual Earnings models) are based on the nominal value of the capital provided, not its market value. In Figure 6, we assess the capital charges based on market values. All scenarios have $0.00 economic profits and residual earnings. All numbers in the economic profits and residual earnings calculations are displayed with 2 digits in Excel, but the calculated value might exceed two digits.

Figure 6: Spellman Valuations with Capital Charges Based on Market Value of Investor Capital

	A	B	C	D	E	F
1	Capital Charge based on Market Value of Capital					
2						
3	Calculation of Earnings	Scenario	Initial	1	2	3
4	EBIT		40	60	70	80
5	Interest		6.40	11.20	11.20	11.20
6	Before-Tax Income		33.60	48.80	58.80	68.80
7	Taxes		8.40	12.20	14.70	17.20
8	Earnings		25.20	36.60	44.10	51.60
9	% change in Earnings			45%	75%	105%
10						
11	WACC Calculations					
12	WACC Equity	Solve for	240.00	325.06	420.00	514.94
13	WACC Debt	Given	160.00	280.00	280.00	280.00
14	WACC Value	WACC Debt + Equity	400.00	605.06	700.00	794.94
15	rU	Previously calculated	7.90%	7.90%	7.90%	7.90%
16	rE (WACC)	rU+(rU-rD)D/E	10.50%	11.26%	10.50%	10.02%
17	rD (WACC)	Given	4.00%	4.00%	4.00%	4.00%
18	WACC Debt/Value	WACC Debt/Value	40.00%	46.28%	40.00%	35.22%
19	WACC Equity/Value	WACC Equity/Value	60.00%	53.72%	60.00%	64.78%
20	WACC	D/V*rD*(1-T)+E/V*rE	7.50%	7.44%	7.50%	7.55%
21						
22	WACC Equation Value - Economic Profit Model Value		0.00	0.00	0.00	0.00
23	(Row 14 - Row 28)					
24	Economic Profit Model Calculations:					
25	Beginning Capital	Given	400	700	700	700
26	NOPAT	EBIT(1-T)	30.00	45.00	52.50	60.00
27	Economic Profit (EP)	NOPAT - WACC*MV of Cap	0.00	0.00	0.00	0.00
28	Value	EP/(WACC - g) + MV of Cap	400.00	605.06	700.00	794.94
29	Less: Debt	Given	160.00	280.00	280.00	280.00
30	Equity	Value - Debt	240.00	325.06	420.00	514.94
31	Equity from Investors		240	420	420	420
32	% change in Equity value			-22.60%	0.00%	22.60%
33						
34	Residual Earnings Equity Valuation					
35	NOPAT	From above	30.00	45.00	52.50	60.00
36	Interest A/T	Debt*rD*(1-T)	4.80	8.40	8.40	8.40
37	Flow to Equity (FTE)	NOPAT - Interest A/T	25.20	36.60	44.10	51.60
38	Equity Capital Charge	Market value of Equity*rE	25.20	36.60	44.10	51.60
39	Residual Earnings (RE)	FTE - Equity Cap Chg	0.00	0.00	0.00	0.00
40	PV RE	RE/(rE-g)	0.00	0.00	0.00	0.00
41	Market Equity Value	Calculated	240.00	325.06	420.00	514.94
42	Ending Equity Value	Initial Equity + PV RE	240.00	325.06	420.00	514.94

Note: WACC equity values in row 14 and the equity values derived from the Economic Profit model in row 28 are equal for all EBIT values.

Comparing the economic profits (EP) for Scenario 2 in Figures 5 and 6 we have:

From Figure 5: EP = EBIT(1 – T) – (Beginning Capital)(WACC) = 60*(1 – 25%) – 700*7.44% = – $7.06.

From Figure 6: EP = 60*(1 – 25%) – 605.06*7.44% = $0.00.

Comparing residual earnings (RE):

From Figure 5: RE = [NOPAT – Interest*(1 – T)] – (Beginning Equity)(r_E)
= [45 – 11.20*(1 – 25%)] – 420*11.26%
= – $10.69.

From Figure 6: RE = [NOPAT – Interest*(1 – T)] – (Market Value of Equity)(r_E)
= [45 – 11.20*(1 – 25%)] – 325.06*11.26%
= $0.00.

All other calculations are the same in both figures. We can argue that since the equity is not worth the $420 that shareholders have contributed, the capital charges should not be based on the nominal value. The *WACC* of 7.44% is based on an *equity* value of $325.06, not $420. The same argument holds for Scenario 3. The nominal value of the equity again is $420, but the market value is $514.94. Essentially, we have adjusted the value of the equity to its market value, which eliminates economic profits and residual earnings. Regardless of the capital charge assumption, the resulting valuations are identical.

We compare the three scenarios based on the assumption that the riskiness of the assets funded by the additional $300 of capital equals that of the original $400. If the riskiness of the new assets and existing assets are not equal, we would calculate a different cost of capital for the new investment. Our objective is to demonstrate how underproductive investments increase the company's cost of equity and decrease shareholder value. Working with two different asset classes would unnecessarily complicate the calculations.

Comments and Conclusions

We demonstrated how an underperforming investment reduced equity value, even though earnings increased. Our analysis began with the unlevered cost of capital because this is a relatively pure value unaffected by the capital structure. The capital structure (at market values) changed with the introduction of underperforming investments. With the capital expansion, the company found their equity value to be far different from what it used for calculating the WACC. Investments that earn less than the company's WACC will decrease equity value and increase the cost of equity. We estimated the correct equity value such that the Economic Profit Model derived equity value and the WACC equation equity were equal.

The Economic Profit Model is a valuable tool for identifying underperforming investments. If the new investment earns the WACC on an after–tax basis, we show that the initial estimate of the WACC continues to hold. If the new investment earns more than the WACC, we show that having the same equity value for the WACC equation and from the Economic Profit model requires that equity and company values be found simultaneously.

Do companies track their cost of equity and WACC as precisely as we did? Brealey et al. (2023) suggest that they do not, noting that in the long run, companies manage their capital toward their target value. Brigham and Daves (2017) argue that companies do not adjust their WACC if investments are approximately scale enhancing.

Fernandez (2020) argues that all valuation models except the Adjusted Present Value [(APV), see Brealey et al. (2023)] require iterating to arrive at correct valuations. We have provided an iterative approach for valuations as applied to the Economic Profit and Residual Earnings models. With minor adjustments, we could have also set up an iterative algorithm for the Free Cash Flow model (which is algebraically equivalent to the Economic Profit model). The same process of iterating until the equity value used to estimate the cost of capital and the valuation model equity value are equal works well with the Free Cash Flow model. We showed that a model based on discounting cash flows (Economic Profit) and a model based on returns (Residual Earnings) can be iterated to produce accurate corporate valuations. Thus, whichever type of valuation approach an analyst prefers can be set up to provide accurate valuations.

References

Brealey, Richard A., Stewart C. Myers, Franklin Allen, and Alex Edmans. 2023. *Principles of Corporate Finance* (14th edition). Irwin McGraw–Hill.

Brigham, Eugene F., and Philip R. Daves. 2017. *Intermediate Financial Management* (12th edition). Cengage.

Damodaran, Aswath. "Spreadsheet to Compute Current ERP for Current Month." 2021. *https://www.stern.nyu/~adamodar/pc/implprem/ERPSept21.xlsx*.

Fernandez, Pablo. 2020. "Valuing Companies by Cash Flow Discounting: Only APV Does Not Require Iteration." Available at SSRN: *https://ssrn.com/abstract=3682128*.

Jackson, Stewart E. 2007. "Creating Value Through Acquisitions." *Journal of Business Strategy* 28: 40–41.

Koller, Tim, Marc Goedhart, and David Wessels. 2020. *Valuation: Measuring and Managing the Value of Companies* (7th edition). McKinsey & Company Inc.

Lauenstein, Milton C. 1983. "To Grow... or Not to Grow...?" *Journal of Business Strategy* 4: 88–90.

Markou, Andreas, and Simon Taylor. 2014. "Peering Inside the Analyst "Black Box": How Do Equity Analysts Model Companies?" Available at SSRN: *https://ssrn.com/abstract=2410992*.

Ruback, Richard S. 2002. "Capital Cash Flows: A Simple Approach to Valuing Risky Cash Flows" *Financial Management* 31:85-103.

Teaching Foreign Exchange Rates: A Primer

Clark G. Ross and Jaya Jha[1]

ABSTRACT

Given the importance of foreign exchange in introductory economics courses, this paper aims at providing a full and intuitive teaching of foreign exchange. First, there is a review of how the major current introductory texts offer somewhat inadequate treatment of the topic. Then, international transactions within a balance of payments framework are explained. Importantly, the market equilibrium exchange rate between the dollar and the euro is concurrently derived. Finally, the paper introduces, within a comparative static framework, changes in exchange rates from changes in major macroeconomic variables: the GDP of trading partners, domestic price level, and real interest rates.

Introduction

The informed citizen in today's world must have some grasp of the principles of international economics, both those of trade and of finance. These topics should be taught in an introductory economics course. Given the importance of trade in today's world, for instance, the rhetoric about "trade conflicts" between the United States and China, a grasp of basic international economic principles is vital for an intelligent voter or for a professional working in the contemporary business world. Exchange rates also have practical import for many students. There are many international students studying in the United States, and U.S. students are accepting employment opportunities in a more globalized economy. Understanding the determination of exchange rates between different currencies is important.

A large percentage of college students will study or have studied basic economics, with some exposure to these international concepts. As college-level teachers, we can venture that student understanding of the microeconomic concepts, particularly comparative advantage and trade, often exceeds their knowledge of macroeconomic concepts, as applied in the foreign exchange market. There may be understandable reasons for this differential. First, many textbooks and many instructors like to use comparative advantage and gains from trade as part of their introduction to economics, with its serving as a powerful application of the production possibilities frontier and opportunity cost. Second, in introductory courses that combine both microeconomics and macroeconomics, maybe one-third of all college economics courses, microeconomic principles tend to consume a larger percentage of the course, leaving less time for macroeconomics, and even less time for international finance that is frequently taught at the end of the macroeconomics portion of the course. Finally, some instructors may have trouble grasping the principles of international finance; other faculty may not have kept current with this portion of economics. This difficulty can be compounded by the relatively closed nature of the U.S. economy compared to our European counterparts.

Before proceeding with this paper, the reader should know that the exchange rate represents the rate at which one currency can be exchanged for another currency. For example, there is an exchange rate between the British pound and the U.S. dollar. Some form of exchange rate is needed to facilitate international exchanges of goods and services. Europeans or holders of euros, for instance, need dollars to travel in the United States, to buy U.S. goods and services, and to buy U.S. financial assets. Also, some speculators might use Euros to buy dollars in the hope that in competitive foreign exchange markets the Euro price per dollar will increase or the dollar appreciate, generating a financial profit. For symmetric reasons, U.S. citizens or dollar holders will demand euros. Exchange rates facilitate these economic exchanges across countries that use different currencies.

[1] The authors are, respectively, the Frontis W. Johnston Professor of Economics and the Boone Assistant Professor of Economics at Davidson College, Davidson, NC. The authors express great appreciation to Dr. Shyam Gouri Suresh and Dr. Peter Hess for their helpful comments and to student researchers Raosaam SB Sharaheel, River Meng, Leo Hu, and Jonah Lane. An anonymous referee and the journal's editor also provided particularly helpful comments for revision. Corresponding author: clross@davidson.edu.

This paper serves as a 'primer' for an effective teaching of the exchange rate and important influences on exchange rates. The rest of the paper is organized as follows: the next section discusses how exchange rates, particularly competitive, market-determined exchange rates, are presented in the leading introductory economics textbooks. The limits or shortcomings of these approaches are mentioned, shortcomings that this paper attempts to overcome. Subsequently, there is an overview of international economic transactions, including introducing the concept of the balance of international payments, followed by a concurrent study of the foreign exchange markets for dollars and euros, with the market-determined competitive equilibrium exchange rates of each currency derived. These sections also reference, as appropriate, other exchange rate regimes.[2] Finally, partial equilibrium comparative statics are used to assess the impact on a country's equilibrium exchange rate from i) changes in the real income of a trading partner, ii) changes in the domestic price level, and, finally, iii) changes in relative real interest rates across countries.

Current Textbook Practices with Foreign Exchange Rates

In this section, we look at how six of the leading introductory economics textbooks incorporate international macroeconomic considerations, particularly the teaching of competitive market-determined exchange rates. Based on these summaries, the pedagogical advantages offered in this paper for the teaching of international finance and exchange rates should be clear.

Modern Principles of Economics, Fourth Edition by Tyler Cowen and Alex Tabarrok, 2018

Cowen and Tabarrok introduce international trade early in their book in two chapters, 2 and 9, linking comparative advantage, trade, and specialization first to the production possibilities frontier and opportunity cost. International finance follows the textbook's treatment of microeconomics and macroeconomics and is found in Chapter 38, the last chapter of the text. Clearly students receive an earlier, more thorough, and more visible treatment of international trade than of international finance.

To generate interest and relevance for international finance, the authors discuss the U.S. trade deficit as a means to introduce the study of the balance of payments, showing the current account and the capital/financial account. Their treatment of this topic is straightforward, except that their balancing of the two accounts with flexible exchange rates applies principally to the U.S. trade deficit (generating a current account deficit) being offset by the attractiveness of investing in the United States (the capital/financial account surplus). With such a specific example, students may not grasp the theoretical accounting equality of the balancing with flexible exchange rates.

For their exchange rate presentation, the authors show two foreign currency examples, independently: the market for yen (with a dollar price per yen) and the market for dollars (with a euro price per dollar). They do not show the simultaneous determination of exchange rates, either for the dollar and yen, or for the dollar and euro. They conclude the chapter with brief explanations of a variety of topics: the real exchange rate, purchasing power parity, monetary and fiscal policy affecting exchange rates and aggregate demand, fixed versus floating exchange rates, and international financial institutions like the International Monetary Fund. Their presentation of this myriad of topics in their last chapter may be overwhelming for an introductory economics course.

Economics, Fifth Edition, R. Glenn Hubbard and Anthony Patrick O'Brien, 2015

Hubbard and O'Brien conclude their textbook with Part 10 (pages 967-1021) entitled "The International Economy." This part has two chapters: Chapter 29 "Macroeconomics in an Open Economy" and Chapter 30 (the concluding chapter of the text): "The International Financial System."

Chapter 29 begins with a discussion of international transactions and the balance of payments. They introduce both the current account and the capital/financial account, showing the accounting need for a

[2] While overviews of the history of exchange-rate systems are covered in some introductory texts, more extensive treatments of the varieties of exchange rate systems can be found in older texts on international economics and macroeconomics. For example, chapter 4 in Charles P. Kindleberger's *International Economics* (R. D. Irwin, 1955) details three systems of international adjustment for regulating the foreign exchange market: fixed exchange rate, fluctuating exchange rate, and exchange controls. Similarly, chapters 6 and 20 in Rudiger Dornbusch and Stanley Fischer's *Macroeconomics* (McGraw Hill Education, 2005) provide detailed accounts of international adjustments under fixed and flexible exchange rate regimes.

balance between these accounts, with competitive exchange rates. They then show the derivation of the nominal exchange rate between the Japanese yen and the U.S. dollar. They use the market for dollars with the yen price per dollar on the vertical axis, but do not discuss the simultaneous determination of the dollar price per yen within the market for yen. They introduce the real exchange rate and the flow of funds into the loanable funds market for a country that has a trade deficit with another country.

Chapter 30, a relatively brief chapter, provides relevant topics including three key features: i) the dollar's floating against other major currencies, ii) the theory of purchasing power parity, including the effects of a tariff, and iii) the process and complications of 'pegging' a currency. This chapter may exceed what is needed at the introductory level.

Economics, Fifth Edition by Paul Krugman and Robin Wells, 2018

Krugman and Wells provide extensive coverage of international economic transactions and exchange rates in Chapter 33, entitled "International Macroeconomics." They begin by explaining international transactions and balance of payments accounts. They then discuss competitive exchange rates.

There are two concerns that could be raised here. First, there is an absence of any early discussion indicating that with today's definitions of international transactions, there is no official settlement adjustment. With market-determined exchange rates, there is an equality of demand and supply for the currency with no need for any official settlement adjustment. With fixed exchange rates, official settlements adjust with i) an outflow from a country that has a deficit in its overall transactions (with an overvalued exchange rate), and ii) an inflow to a country that has a surplus in its overall transactions (with an undervalued exchange rate). Second, they do not draw a clear connection between the concurrent determination of the exchange rate between the euro and the dollar, and the dollar and the euro. They show the determination of the euro price per U.S. dollar in the dollar market, but do not show the concurrent determination of the dollar price per euro in the euro market, as is carefully done in this article.

Krugman and Wells have one comparative static exercise where a change in preferences towards U.S. assets by foreign investors will increase the demand for dollars and appreciate the dollar. They also explain how this appreciation will tend to generate a current account deficit (if previously in balance) and an equal and opposite financial/capital account surplus that just offsets the current account deficit, assuming a market-determined competitive exchange rate. Krugman and Wells also consider some more sophisticated aspects of exchange rates, including the real exchange rate and the purchasing power parity exchange rate. Finally, there is discussion of different exchange rate regimes, including fixed exchange rates, and different implications for the real economy from exchange rate policy. These considerations are very useful.

Principles of Economics, Eighth Edition, N. Gregory Mankiw, 2018

Mankiw's text has a relatively unique approach to international finance. Part XI of the text, entitled "The Macroeconomics of Open Economies," has two full chapters, one introducing the basic concepts of the open macroeconomy, and the second applying those concepts in theory. Mankiw indeed stresses the importance of the open macroeconomy and places it less peripherally in his text (Chapters 31 and 32 out of 36 chapters).

The basic concepts that Mankiw introduces are the flows of goods and services, stressing the greater importance of the open contemporary U.S. economy to the world. He then discusses "the equality of net exports and the net capital outflow," or a revised balance of payments presentation, applicable with competitive exchange rates. Unlike other texts, Mankiw contrasts in a central way the nominal exchange rate with the real exchange rate. Mankiw then introduces purchasing power parity as a determining factor in deriving long-run equilibrium exchange rates. His discussion is more sophisticated and theoretical in comparison to developing the nominal exchange rate solely as a function of the forces of demand and supply. Mankiw shows how the forces of demand and supply for currencies can be influenced by the relative domestic prices of good and assets between trading partners. Over time, the nominal exchange rate should be the one that will equate the domestic prices of tradeable goods.

Mankiw's second chapter on the open macroeconomy introduces the loanable funds market, which stresses that international flows of funds can have significant impacts on real interest rates across countries. Concurrently, the flow of funds will influence market-determined exchange rates. Mankiw also introduces the role that tariffs can play in influencing exchange rates, with exchange rate changes frequently offsetting the role of the tariff. Mankiw provides an example of a more sophisticated teaching of the "open

macroeconomy," some of which may be more appropriate in an intermediate macroeconomic theory course than in an introductory economics course.

Principles of Economics, Second Edition by Dirk Mateer and Lee Coppock, 2018

Mateer and Coppock relegate the international economic coverage to the very end of the textbook, in Part X, with comparative advantage and international trade discussed in Chapter 32, followed by international finance in the book's concluding chapter, Chapter 33. With this sequencing, it may be difficult to find adequate class time to cover international finance. Moreover, with this sequencing, international finance is not integrated into the book's coverage of macroeconomic stabilization policy: fiscal and monetary policies with their secondary effects in an open economy.

Mateer and Coppock begin their international finance coverage by providing an extensive definition of the exchange rate and the changing value of the exchange rate over time. Their presentation of the derivation of the exchange rate is rather unique. They show a conventional downward-sloping demand for the yen at varying dollar prices per yen while their supply of yen is perfectly inelastic, as determined by the Central Bank of Japan. This more asset-model determination of the foreign exchange rate is less conventional and would require the instructor to augment the textbook presentation to link it more closely to the broader market-determined exchange rate model more commonly used. There is no discussion of the simultaneous determination of the dollar price per yen and the yen price per dollar, or the expected relative appreciations and depreciations of each currency.

They conclude their chapter with an explanation of purchasing power parity and then a conventional explanation of balance of payments. They do provide a helpful discussion of possible causes of a nation's trade deficit that likely generates a current account deficit that would then need to be offset by a financial and capital account surplus. In conclusion, however, the end-of-text placement of this chapter and the unique exchange rate model make this text less helpful in integrating international finance into the introductory economics course.

Economics: Principles, Problems and Policies, Twenty-first Edition, by Campbell R. McConnell, Stanley L. Brue, and Sean M. Flynn, 2018

McConnell et al.'s text reserves Part Eleven, the last two chapters of the text, for "International Economics." Chapter 34 concerns international trade and Chapter 35 is called "The Balance of Payments, Exchange Rates, and Trade Deficits." The ordering is similar to the presentation in Krugman and Wells, with a description of international financial transactions and the balance of payments (e.g., the current and the financial/capital flows account). Like Krugman and Wells, flexible exchange rates are introduced, but in a terse manner, with a single graph of the market for British pounds and a dollar price per British pound. No concurrent derivation of the dollar market and British pound price per dollar is mentioned. This text contains a helpful and exhaustive listing of comparative static changes with their effect on the demand or the supply of a currency. The chapter then addresses fixed exchange rates followed by an appendix with previous exchange rate regimes, including the Gold Standard.

Concerns with this presentation are two-fold: i) the inadequate derivation of market-determined competitive exchange rates, and ii) the lack of a solid discussion of the relative effects of fiscal and monetary policy on exchange rates and the real economy. In addition, the discussion of previous exchange rate regimes is not placed in an easily-followed historical context.

This review of these six major introductory textbooks provides a few important conclusions. First, with the exception of Mankiw's textbook, most authors introduce international finance at the end of their textbook, leaving it in a position that many introductory economics courses do not reach due to time considerations. Second, introducing international financial transactions and grouping them to discuss the balance of payments is an important concept, typically stressed in each textbook. The more current treatment of balance of payments equality, with international settlements included, is not clearly developed in all cases. The advantage of such a treatment is that it can be applied in both flexible and fixed exchange rate regimes. Third, while all textbooks introduce the nominal exchange rate and its derivation, none explicitly links the simultaneous derivation of both exchange rates, such as the euro price per dollar in the dollar market and the dollar price per euro in the euro market. This pedagogical technique shows with much more clarity the working of the foreign exchange market. Fourth, there is no consistent treatment of partial equilibrium

changes in the exchange rate following changes in key macroeconomic variables such as real income, the price level, and the real interest rate. This type of analysis is vital for the student to understand exchange rate determination and its importance. Fifth, while supplemental discussion of the real exchange rate, purchasing power parity, and the relationship of trade deficits to capital inflows is discussed in some fashion in some textbooks, there is no universal conclusion. Finally, the evolution and use of different exchange rate regimes, from the Gold Standard to Bretton Woods to the Smithsonian Agreement, is not adequately explained. Moreover, as mentioned, with fixed exchange rates, the role of international settlements, an outflow or an inflow is nowhere cogently developed.

This article addresses many of these concerns, aiding students in their understanding of an open macroeconomy. Using this article, students who study introductory economics will be familiar with important concepts in international finance. Students studying more advanced economics will know well basic concepts that can be applied at the intermediate macroeconomic level of analysis, as well as in a more detailed study of international finance. Also, students in elective courses like U.S. Economic History or Public Finance will find this article a very helpful supplement.

International Economic Transactions: The Balance of International Payments

Before looking at exchange rates and different exchange rate regimes, it is critical to understand the reasons that citizens or businesses of one country have economic interaction with citizens or businesses of another country. In this section, we review these economic and financial transactions across countries during a time period. Let us look at Mexico and the United States. To make these desired transactions, the different currencies used in the country, like the peso (Mexico's currency) and the U.S. dollar, must have an exchange rate or a price, like number of pesos per one dollar as the price of a U.S. dollar, or number of dollars per one peso as the price of a Mexican peso.

Foreign Transactions

Holders of Mexican pesos (frequently, but not always, Mexico's citizens) demand U.S. dollars for the following reasons, 1-6; reason 7 is relevant when there are fixed exchange rates and a balance of payments deficit that must be resolved or settled. In each of these cases the recipient of the funds prefers to have a dollar payment to use in the domestic economy of the United States. This desire for dollars generates the demand for the U.S. dollars by peso holders.

1. To travel in the United States, paying dollars for transportation, lodging, and meals in the United States.
2. To buy U.S. goods and services (exports from the United States which are imports into Mexico)
3. To pay interest on loans that dollar holders extended to Mexico and Mexicans
4. To send money or remittances from Mexico to U.S. citizens who wish to have their payment in dollars
5. To buy U.S. financial assets, such as bonds (either U.S. government bonds or private sector bonds) or to buy U.S. equities or stocks
6. To buy U.S. factories or physical assets or to buy real estate in the United States
7. Settlement Mechanism: To redeem (or acquire) domestic currency (the dollar) with an outflow of an official settlement: gold or other reserve currency to Mexico, following a balance of international transactions deficit for the United States (debits>credits). In this case, the central bank or government of Mexico is holding a stock of U.S. dollars.

In cases 1 through 6, there is a demand for dollars that results in pesos being converted to U.S. dollars. This action translates into a 'credit of dollars' in the sum of all international transactions needing dollars. The balance of payments for the U.S. dollar tracks all demands for dollars by holders of foreign currency (credits) and all supplying of dollars for other currencies (debits). Concurrently, since pesos are being sold to acquire dollars, there is an outflow of pesos from Mexico or a 'debit of pesos' in the sum of all international transactions needing dollars, called the balance of payments for the Mexican peso. All transactions listed as numbers 1 through 7 will generate a credit in the U.S. balance of payments and a debit in the Mexican balance of payments.

In a symmetric way, holders of U.S. dollars (frequently but not always U.S. citizens) will likely demand pesos for the following reasons:

1'. To travel in Mexico, paying pesos for transportation, lodging, and meals in Mexico
2'. To buy Mexican goods and services (exports from Mexico which are imports into the United States)
3'. To pay interest on loans that peso holders extended to the United States and/or its citizens
4'. To send money or remittances from the United States (frequently by Mexicans working in the United States) to Mexican citizens who wish to have their payment in pesos
5'. To buy Mexican financial assets, such as bonds (either Mexican government bonds or private sector bonds) or Mexican equities
6'. To buy Mexican factories or physical assets or to buy real estate in Mexico
7'. Settlement Mechanism: To redeem (or reduce holdings of) a foreign currency with an inflow of an official settlement: gold or other reserve currency, following a balance of international transactions surplus for the United States (credits>debits). In this case, the central bank or government of the United States is holding a stock of Mexican pesos.

In these cases, there is a demand for pesos that translates into a 'credit of pesos' in the sum of all international transactions needing pesos, called the balance of payments for the Mexican peso. Concurrently, since dollars are being used to acquire pesos there is a 'debit of dollars' in the sum of all international transactions needing pesos, called the balance of payments for the U.S. dollar. All transactions listed as numbers 1' through 6' (and 7', if appropriate) will generate a credit in the Mexican balance of payments and a debit in the United States balance of payments.

Magnitude of the Debits and Credits

The magnitude of the debits and credits will be a function of the exchange rate. Each credit in the U.S. balance of payments account will generate a debit in the Mexican balance of payments account. For instance, assume that there is a $5 million credit in the U.S. balance of payments from Mexican travel in the United States (#1 above). If the exchange rate were 25 Mexican peso per U.S. dollar or 0.04 dollar per peso, then the $5 million credit in the U.S. balance of payments would equal a 125-million-peso debit in the Mexican balance of payments account ($5 million x 25 peso/dollar = 125 million pesos).

Each credit in the Mexican balance of payments account will generate a debit in the U.S. balance of payments account. For instance, assume that Mexico sells (exports) 300 million pesos worth of goods to the U.S. (#2' above); this would generate a credit in the Mexican balance of payments of 300 million pesos and a debit (as it is an import to the United States) in the U.S. balance of payments of $12 million (300 million pesos x 0.04 dollars per peso = $12 million).

For a given time period, typically a year, all the credits in the U.S. balance of payments account, items 1-6 (treated as positives) are summed. All the debits in the U.S. balance of payments account, items 1'-6' (treated as negatives) are summed. If the sum of the credits (a positive value) exceeds the sum of the debits (a negative value), the United States has an overall international transactions surplus, with an overall positive value. If the sum of the debits exceeds the sum of the credits, the United States has an overall international transactions deficit, with an overall negative value. If the values just equal each other, and add to zero, the United States' international transactions are in balance, without a surplus or a deficit.

Three Categories of International Transactions

Frequently, the totality of these transactions is divided into three categories: Category I: Current account, Category II: Financial/capital flows account (nonofficial), and Category III: Changes in Reserve assets of official monetary authorities (central banks).[3]

The **current account** (Category I) shows as credit items "exports of goods and services, income (such as interest and dividends) received from investments abroad as well as other factor income (e.g., wages) earned abroad," items 1 through 4 in the list above.[4] The credits to the current account will be (for the United States, categories 1-4 listed above) earnings from services (like providing tourism to visitors from other currency zones), earnings from exporting of goods, like automobiles, earnings from investments like the paying of interest on foreign bonds held by citizens of the country, and remittances to residents within the country. The

[3] Appleyard and Field, pages 450 and 451.

[4] Ibid

debits to the U.S. current account will be (representing the credits for Mexico above, 1' to 4') expenses for services (like U.S. citizens touring in other currency zones), expenses from importing goods, earnings paid to those in other currency zones, like the paying of interest on U.S. bonds (private or public) held by citizens of other countries, and remittances from the United States to residents of another country. These debits and credits are likely to be unequal. If the sum of all the credits exceeds the sum of all the debits, a country has a current account surplus. In contrast, if the sum of all the debits exceeds the sum of all the credits, a country has a current account deficit.

Often, the trading status of a country is captured by comparing the credits associated with earnings from goods and services sold to foreigners as exports to the debits associated with payments for goods and services from abroad as imports (1 and 2 compared to 1' and 2') for the United States and Mexico. If the credits from this category (exporting goods and services, 1 and 2 for the United States) exceeds the debits from this category (importing goods and services, 1' and 2' for the United States), a country is said to have a balance of trade surplus. In contrast, if this sum of debits exceeds this sum of credits, the country has a balance of trade deficit.

In Category II, the **financial/capital flows account** (nonofficial) "includes changes in holdings of both long-term (maturity of one year or longer) and short-term (maturity of less than one year) real physical assets and financial assets," items 5 and 6 in the list above[5]. The purchase of U.S. financial assets by foreigners and U.S. physical assets, like plant and equipment represent a credit in the U.S. financial/capital account. When the United States sells bonds or equities to foreigners, they receive foreign currency which becomes a credit in the United States' financial/capital account and overall balance of payments.

When citizens or businesses of the United States buy bonds or equities from foreigners, they transfer dollars from the United States for these financial assets; such a transfer becomes a debit in the United States balance of international payments (category 5'). If they purchase physical assets from foreigners, (category 6'), there is also a debit in the U.S. balance of payments. The sum of these transactions, both credits and debits, becomes the balance on the financial/capital flows account. If the value of the credits exceeds the debits, there is a surplus in the financial/capital flows account; if the value of the credits is less than the debits, there is a deficit in the financial/capital flows account.

Category III or item 7, **changes in reserve assets of official authorities**, is relevant when a country has an imbalance in the sum of those transactions shown in Category I and Category II. If the U.S. has credits that exceed its debits in Categories I and II, then the U.S. will have a surplus that leads to its acquiring the currencies of other countries, for instance Mexico. The United States will settle that surplus by redeeming (effectively returning) the currencies of other country through an inflow of gold or other reserve currency from these deficit countries. As shown in Category III, this will represent an outflow of foreign currencies, or a debit in the United States' balance of payments, offsetting the overall surplus (credits>debits) from Categories I and II. With this settlement added, all credits will equal all debits, resulting in a zero balance in all international payments[6].

For instance, assume that the United States has an overall deficit when summing categories I and II. If the United States has a deficit of $300 million and an ounce of gold sold for $50 per ounce (as in the 1950s) the United States would need to transfer 6 million ounces of gold ($300 million/$50 per ounce of gold = 6 million ounces of gold) to redeem the $300 million of U.S. currency held by foreign central banks. Category III is particularly relevant when countries have fixed exchange rates and imbalances in the sum of transactions

[5] Ibid

[6] The Official settlements account (OSA) (or Category III above) measures the movement of official assets involving monetary authorities (typically the central banks) or between governments of nations. For most countries the stock of foreign currencies held by the monetary authorities constitutes the majority of the owned international reserves. Any imbalance on the current and capital/financial accounts is offset by the official settlements account: credits on the official settlements account reflect either a decrease in official assets held by the nation or an acquisition of the nation's assets by foreign officials. As an example, assume that Mexico has a balance of payments deficit, that is, an excess of debits over credits on the current and capital accounts. There will be an excess supply of Mexican pesos in the foreign currencies market to reflect this BoP deficit, causing a downward pressure on the value of the peso. If Mexico desired to maintain a fixed exchange rate and prevent the peso from falling, its central bank could purchase the surplus pesos with its holdings of foreign currencies, say, U.S. dollars. This transaction would be recorded as a credit on Mexico's official settlements account. Alternatively, other central banks, say, the Federal Reserve of the United States, could soak up the surplus pesos, which, when placed in central banks in Mexico, would count as official reserve assets for the United States; the increase in official liabilities would be recorded as a credit on Mexico's BoP account. [If the U.S. Federal Reserve used the newly purchased pesos to buy Mexican government issued bonds, the transaction would be recorded on Mexico's BoP account as an increase in foreign official assets (Mexican government-issued bonds) rather than official reserve assets (Mexican pesos).]

from Categories I and II can occur. We shall see that with competitive market-determined exchange rates, transactions from Categories I and II should balance, with no need for any change in reserve assets or activity in Category III.

International Settlements after the Gold Standard and World War I

Prior to World War I, the major economies were tied together internationally through their joint participation in the Gold Standard. During the time of the Gold Standard from the nineteenth century to the Great Depression of the 1930s and then partially from 1945 until 1973 with the Bretton Woods system of quasi-fixed exchange rates, each country's currency had a value that could be expressed in gold: e.g., the dollar value of an ounce of gold or the British pound value of an ounce of gold. Thus, each currency had an exchange rate with any other currency based on its parity value with gold. For instance, if an ounce of gold was fixed by the United States at $35 per ounce and an ounce of gold was fixed by the United Kingdom at 15 British pounds per ounce of gold, then $35 would equal 15 British pounds. The official exchange rates would be 0.429 pounds per dollar (15 pounds/$35 = 0.429 pounds per dollar) and $2.33 per British pound ($35/15 pounds = $2.33 per pound).

After World War I and with the Great Depression, the Gold Standard became relatively inoperable. Germany was not a truly sovereign economy, forced to pay significant monetary reparations to the United Kingdom and France. The British pound no longer could justify its pre-World War I parity ratio to gold. In the 1920s, the Pound was greatly overvalued, with Britain importing a much greater value of goods than it was exporting and having significant international transactions deficits. With the Great Depression and protectionist policies introduced in many countries, international trade became relatively nonexistent. The onset of World War II further strained international financial transactions.

The soon-to-be-victorious allied powers, principally the United Kingdom and the United States, agreed that the post-World War II international financial system should be one without a strict adherence to the Gold Standard. In 1945 at a conference in Bretton Woods, New Hampshire, policy makers developed a system of relatively fixed but adjustable exchange rates between the major economic powers. Of importance was that these exchange rates, referred to as the Bretton Woods system, while fixed, were not directly tied to gold with a fixed parity ratio in each country. With these fixed exchange rates, however, countries still could have trade imbalances. An overvalued exchange rate would lead to an overall balance of financial transactions deficit, necessitating an outflow of gold or reserve currency to cover the deficit. As an alternative to a country using protectionist policies likes tariffs to reduce such deficits in the future, the International Monetary Fund was created in 1944 to loan gold or other financial assets to countries with trade deficits who could not afford to pay those deficits.

Under the Bretton Woods system, countries could have significant and persistent combined surpluses or deficits in Categories I and II. For instance, during the 1950s, the U.S. tended to export much to the countries of Europe and Japan that were recovering from World War II and import little from those countries. With their credits from exports being large and exceeding all debits, the U.S. had annual surpluses, particularly in Category I. After the 1960s, the situation reversed. The U.S. began importing many goods, such as automobiles and steel, and exporting less. The U.S.'s trade balance reversed, with their having significant trade deficits that translated into overall deficits in Categories I and II.

With these deficits, other countries were then holding large amounts of U.S. dollars, demanding that these dollars be redeemed for gold. As the U.S. was losing significant quantities of gold, President Richard Nixon in August of 1971 suspended the international convertibility of the U.S. dollar for gold, effectively ending the last remnants of the Gold Standard and divorcing the U.S. dollar from any formal relationship to gold. It was after this decision that the major economies began moving towards competitive, market-determined exchange rates. The first step in the process of introducing competitive exchange rates was the December 1971 Smithsonian Agreement that revalued the currencies of the influential Group of Ten nations. After some financial turbulence, these economies moved to the use of competitive market exchange rates in 1973.

Following negotiations among the major economies, the Smithsonian Agreement of 1975 paved the way for the introduction of purely competitive, flexible foreign exchange markets among many of the major countries. Some countries still have forms of fixed exchange rates, but those countries producing the bulk of the world's output (the United States, Canada, the Euro Zone, Great Britain, Japan, and Russia, among others) have flexible exchange rates among themselves.

Impacts from Deficits and Surpluses in Categories I and II combined with Fixed and Competitive Exchange Rates

With a deficit, the desire of a country, call it the home country, to engage in foreign transactions, like importing goods, exceeds the desire of other countries to engage in international transactions with the home country. Assuming that relative prices drive the desires for such transactions, the international prices offered to the home country are attractive, leading them to purchase goods, services, and assets from the outside world. The reason that these prices are so attractive is that the exchange rate between the currency of the home country and the currency of other countries is not in balance or at an equilibrium. The purchasing power of the home country's currency is enhanced by the actual exchange rate. Economists will regard the home country as having an 'overvalued exchange rate.' Its demand for other currencies will exceed the supply of other currencies at this overvalued exchange rate.

With a surplus, the desire of a country, call it the home country, to engage in foreign transactions, like importing goods, is less than the desire of other countries to engage with the home country. Assuming that relative prices drive the desires for such transactions, the international prices offered to the home country are not so attractive, leading them to reduce their purchases of goods, services, and assets from the outside world. The reason that these prices are not so attractive is that the exchange rate between the currency of the home country and that of other countries is not in balance or at an equilibrium. The purchasing power of the home country's currency has been reduced by the actual exchange rate. Economists will regard the home country as having an 'undervalued exchange rate.' Its demand for other currencies will be less than the supply of other currencies at this overvalued exchange rate.

With fixed exchange rates, Category I and II deficits or surpluses can persist with exchange rates continuing to be overvalued or undervalued. Official settlements (Category III) will balance the accounts. With a deficit, a country will redeem its domestic currency with an outflow of gold or a reserve currency. With a surplus, a country will exchange the currency of other countries for gold or a reserve currency. Deficits can continue as long as a deficit country has the magnitude of official settlements to cover its deficit.

With competitive exchange rates that are derived and explained in the next section, there should be a balance of credits and debits with Categories I and II; there should not be any persistent deficit or surplus within these two categories. The value of a currency will adjust to ensure an equality between demand for the use of the currency in international transactions (credits in the home country's balance of payments) and the supply offered of the currency for international transactions (debits in the home country's balance of payments).

As an example, assume a competitive exchange rate without any deficit or surplus (of Categories I and II) with Mexico for the United States is 23 pesos per dollar (or 0.044 dollars per peso). At 25 pesos per dollar (or 0.04 dollars per peso), the dollar is overvalued, and Mexican goods will seem inexpensive for dollar holders. The demand for pesos will exceed the supply of peso. For pesos' holders, the exchange rate leaves the peso undervalued and dollar-denominated goods will seem expensive. Thus, the U.S. will have a deficit in Categories I and II with Mexico; the U.S. will be importing from Mexico more than it will be exporting to Mexico. In contrast, Mexico will have a surplus with the United States, exporting more to the U.S. than it is importing from the U.S. At 20 pesos per dollar or (or 0.05 dollars per peso), the dollar is undervalued, and Mexican goods will seem expensive for dollar holders. The supply of pesos will exceed the demand for pesos. For pesos' holders, the exchange rate leaves the peso overvalued (and the dollar undervalued) and dollar-denominated goods will seem inexpensive. Thus, the U.S. will have a surplus in Categories I and II with Mexico, while Mexico will have a deficit with the U.S. At a market equilibrium exchange rate of 23 pesos per dollar (or 0.044 dollars per peso), there will be an equality of the demand for pesos by dollar holders (demand for dollars by pesos holder) and the supply of pesos (supply of dollars). There will be an overall balance of credits and debits with Categories I and II. With such a balance there will be no need for any Change in Reserve Assets (Category III) for either country.

Also of interest with competitive, market-determined exchange rates is the relationship between the current and the financial account. In this exchange-regime, with competitive market-determined exchange rates, the sum of all transactions from Categories I and II will be such that credits equal debits, and there will be neither a deficit nor a surplus for the country. In that case, if transactions are grouped as a current account and a financial/capital flows account, their sum must balance (all credits equal all debits), as they represent all transactions. Thus, if there is a deficit in the current account, the capital/financial flows account must have a surplus such that their sum balances. If there is a surplus in the current account, conversely, there must be a deficit in the capital/financial flows account such that their sums balance.

In recent years the U.S. has had a deficit in their current account, principally because U.S. imports (a debit in the current account) exceed U.S. exports (a credit in the current account). In these cases, foreign countries hold excess U.S. dollars which they have used to purchase U.S. assets, such as U.S. government bonds, generating an offsetting surplus in the U.S. financial account. The sum of the two accounts, the U.S. current account with a deficit and the U.S. financial/capital flows account with a surplus, will balance, as all credits from both accounts should equal all deficits from both accounts. Overall, the U.S. will not have any need for an official settlement (Category III will equal zero). Thus, with competitive, market-determined exchange rates, countries do not encounter deficits or surpluses that persist.

Countries attempting to maintain fixed exchange rates can experience persistent deficits or surpluses on their balance of payments accounts, but with competitive, market-determined exchange rates countries do not encounter deficits or surpluses that require international settlements. As a country drifts towards a deficit, with debits exceeding credits, the country exchange rate should depreciate, reducing its imports (which become more expensive) and stimulating its exports (which become less expensive to foreigners), thereby reducing any deficit in Categories I and II combined. As a country's balance of payments drifts towards a surplus, with credits exceeding deficits, the country's exchange rate should appreciate, increasing its imports (which become less expensive) and reducing its exports (which become more expensive to foreigners), thereby reducing any surplus in Categories I and II combined.

The Dollar and Euro Markets with Equilibria Exchange Rates

We now study the competitive foreign exchange market with the dollar and the euro, highlighting the relationship between the two foreign exchange markets. We look concurrently at both the dollar market and the euro market with an initial recognition that the exchange rate of one currency (i.e., the dollar) can be expressed in a predictable manner to the exchange rate of the other currency (i.e., the euro). While historically not applicable, consider that if one euro costs two dollars then one dollar will cost 0.5 euro. The market exchange rate of one currency will simply be the reciprocal of the price of the other currency. Table 1 shows exchange rates between the U.S. dollar and the euro that have been more applicable over the last few decades.

Table 1: Exchange Rates

Dollar price (or cost) of one euro	*Euro price (or cost) of one dollar*
$ 1.05	0.952 Euro
$ 1.10	0.909 Euro
$ 1.15	0.870 Euro
$ 1.20	0.833 Euro
$ 1.25	0.800 Euro

As the dollar price of one euro rises, the dollar is less valuable and euro more valuable. First, the euro is more expensive, costing increasing amounts of dollars from $1.05 per euro to $1.25 per euro; therefore, the dollar is weaker, and the euro is more valuable. Second, as the dollar price of a euro increases, note that for a euro, more dollars are given. Again, the euro is more valuable and the dollar weaker. As the dollar price per euro rises, the euro price per dollar falls. Thus, when the dollar price per euro is $1.25, the euro price per dollar is only 0.80 euro, showing again a stronger euro and a weaker dollar. When a currency strengthens compared to another currency, economists say the currency has appreciated; when the currency weakens, economists say that the currency has depreciated. Thus, as the market-determined dollar price of a euro rises, the euro price per dollar falls; thus, the dollar has depreciated (weakened) and the euro has appreciated (strengthened)[7].

We now examine how the competitive foreign exchange market works and leads to the determination of an equilibrium market clearing exchange rate. First, note that this market tends to be a near ideal example of

[7] Some students may have heard of a currency devalued or revalued. Devaluation and revaluation occur when a currency has a fixed exchange rate (not a market determined exchange rate) between itself and other currencies or the value of the currency is expressed between itself and a commodity like gold. When the currency's official rate weakens, like the official price of a currency, say the rab, to gold increases from 500 to 550 rabs, the rab is less valuable and has been devalued. As a result, other currencies will also have a higher rab price, signifying a devaluation of the rab. In contrast, were the rab price of an ounce of gold to decrease, say from 500 to 475, the rab would strengthen and be revalued (upward).

perfect competition. There are many relatively small buyers and many relatively small sellers of the product (the dollar or the euro). Thus, no one buyer or seller can influence for any significant period of time the equilibrium exchange rate. Second, it is relatively easy and costless for an individual buyer or seller to enter into this market, lowering transactions costs and eliminating sustained economic profits for being a dealer or agent in foreign exchange. The product of each seller is identical, whether dollars are being sold in the dollar market or euros in the euro market. The offering of each seller is identical, creating a standard product. Finally, in today's electronic world, perfect information about prices or exchange rates tends to exist and be readily available to all buyers and sellers. Thus, all conditions for a perfectly competitive market seem to be fulfilled in the foreign exchange market.

In Figure 1(a) and 1(b), we view adjacently the market for the dollar and the market for the euro. This presentation permits us to see clearly three important relationships i) the relationship between the demand for dollars and the supply of euros, ii) the relationship between the demand for euros and the supply of dollars, and iii) the relationship between the two exchange rates, *i.e.*, the euro price per dollar and the dollar price per euro. These exchange rates are not fixed, but constantly changing in response to changes in market forces. Thus, these specific values, dollars per euro and euros per dollar, are relevant for an instance of time. We now look at the specific demand and supply functions influencing these exchange rates.

Figure 1(a): The market for U.S. dollars

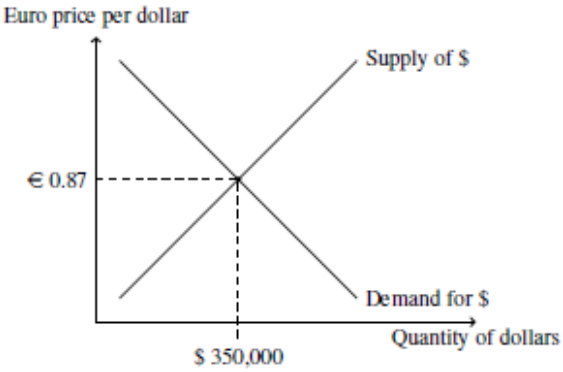
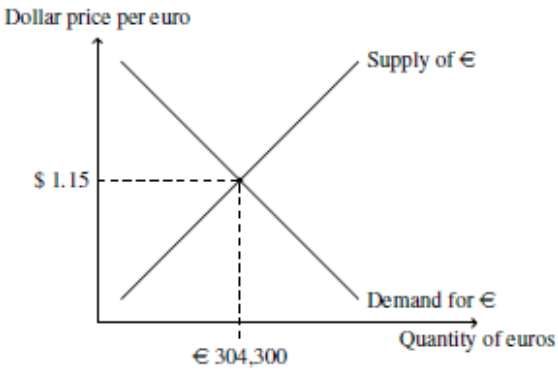


Figure 1(b): The market for euros



First, recall that the demanders of dollars are those holding other currencies who wish to exchange them for dollars for four general purposes: traveling in the United States, purchasing U.S. goods, purchasing U.S. assets, and speculating that the dollar will increase in value, gaining a profit in euros. With currencies

changing value frequently, even if by small amounts, speculation can be profitable with small gains over short time periods yielding an attractive annualized rate of return. Speculation, of course, is risky. In this case, were the dollar to depreciate or lose value, when sold back to euros, there will be a loss. Banks, brokerage houses, and private investors all engage in this type of speculation in search of profit.

Why is this demand for dollars downward sloping? Note that in Figure 1(a), the market for dollars, the vertical axis shows the euro price of one dollar. This is the market for dollars, so the vertical axis will show the price (expressed in euros) per one dollar (just like the vertical axis in the market for apples shows the price of one single apple). The horizontal axis shows the total quantity of dollars demanded. As the euro price per dollar falls, recall the euro strengthens or appreciates. At lower euro prices per dollar, goods priced in dollars effectively cost less in euros. As shown in Table 2, a hotel room in the U.S. that costs $150 per night will cost the following in euros, as the euro price per dollar falls, as above, from 0.952 euros to 0.80 euros.

Table 2: Euro Price of a Hotel Room

Euro price per dollar	*Price in euros of a $150 per night hotel*	*Dollar price per euro*
0.952	142.80 Euros	$ 1.05
0.909	136.35 Euros	$ 1.10
0.870	130.05 Euros	$ 1.15
0.833	124.95 Euros	$ 1.20
0.800	120.00 Euros	$ 1.25

As the price per night of the U.S. hotel falls in euros for the individual holding euros, we assume that she will opt to stay longer as the effective price per night (in euros, but not in dollars) falls. We assume a downward sloping demand curve for hotel rooms, the price of which is expressed in euros to the customer initially holding euros. Thus, if at 0.952 euro price per dollar she stays one night in the hotel, she will demand $150 dollars; if at 0.870 euro price per dollar she stays two nights in the hotel, she will demand $300 dollars (2 nights x $150 per night); at 0.80 euro price per dollar, let us assume she stays three nights, demanding $450 dollars. Plotting these prices (euro price per dollar) and quantity combinations (aggregate quantity demanded of dollars) gives a downward sloping demand curve for dollars. The same rationale will hold for purchasing U.S. goods and U.S. assets. As the euro price per dollar falls, those goods and assets become cheaper to euro holders, who will increase their demand for those goods; the total quantity demanded of dollars will increase, again showing the downward sloping nature of the demand curve.

Let us think of the relationship between the demand for dollars and the supply of euros. The act of demanding dollars, at a bank for instance, requires the concurrent supplying of euros by the individual. If an individual holding euros requests (or demands) $1000, the bank expects the individual to provide (or supply) an equivalent amount of euros (952, if the euro price per dollar is 0.952). Thus, in a simple exchange market, looking only at dollars and euros, the demand for dollars corresponds at exchange rates to a given supply of euros. As an example, in Table 3, note the dollar price per euro, the total quantity demanded of euros (assumed values for this exercise), the corresponding euro price per dollars, and the corresponding quantity supplied of euros.

Table 3: Dollar Demand and Euro Supply

Euro price per dollar	*Dollar demand*	*Dollar price per euro*	*Euro supply*
0.952 Euro	$ 250,000	$ 1.05	238,095 Euros
0.909 Euro	$ 300,000	$ 1.10	272,727 Euros
0.870 Euro	$ 350,000	$ 1.15	304,300 Euros
0.833 Euro	$ 400,000	$ 1.20	333,333 Euros
0.800 Euro	$ 450,000	$ 1.25	360,000 Euros

Thus, the downward sloping demand for dollars generates an upward sloping supply of euros. These price and quantity combinations are graphed in Figure 1(a) for dollars and 1(b) for euros. Note that as the euro price per dollar falls (dollar market), the euro appreciates, and the dollar depreciates. Concurrently, as the dollar price per euro increases (euro market), the dollar depreciates, and the euro appreciates.

Now let us look at the demand for euros and the corresponding supply of dollars. The demand for euros is downward sloping. As the dollar price per euro falls, the dollar strengthens and goods, services, and assets

denominated in euros become cheaper for holders of dollars. We assume they buy more of these items and assets, thus demanding more euros. For instance, a hotel in Paris that costs 160 euros per night will cost $200 when a euro costs $1.25. If the dollar appreciates such that a euro costs only $1.05, that same hotel room will only cost $168. At this lower dollar price per euro and resulting lower dollar price for the hotel room in Paris, the traveler will stay more nights, demanding more euros to pay the hotel bill. Concurrently, as the dollar price per euro falls, European goods and assets become less expensive in dollars; and more European goods and assets will be purchased, increasing the demand for euros to pay for these goods and assets.

Recall the relationship of demanding one currency and supplying another currency. In this case the demanding of euros leads to a supplying of dollars to buy the euros. As an example, Table 4 lists the demand for euros at different dollar prices per euro. As the dollar price per euro falls, the quantity demanded of euros increases. To the right is the corresponding supply of dollars. As the quantity demanded of euros increases, the quantity supplied of dollars increases.

Table 4: Euro Demand and Dollar Supply

Dollar price per euro	*Euro demand*	*Euro price per dollar*	*Dollar supply*
$ 1.25	275,000 Euros	0.800 Euros	$ 343,750
$ 1.20	290,000 Euros	0.833 Euros	$ 348,139
$ 1.20	304,300 Euros	0.870 Euros	$ 350,000
$ 1.15	325,000 Euros	0.909 Euros	$ 357,535
$ 1.05	345,000 Euros	0.952 Euros	$ 362,250

Table 5, using data from Tables 3 and 4, shows both the demand and supply in the dollar market, as well as the demand and supply in euro market at selected euro prices per dollar and dollar prices per euro. As with any competitive market, the equilibrium market price will be that which equates the quantity demanded with the quantity supplied. In the dollar market, that price will be 0.870 euros per dollar; in the euro market, that price will be $1.15 (dollars) per euro.

When the price of a currency is not at its equilibrium level, then demand will not equal supply. As in other markets, if the current price is above the equilibrium price, e.g., 0.952 euros per dollar in the dollar market, there will be an excess supply of dollars; at this price, quantity supplied of dollars ($362,250) exceeds the quantity demanded ($250,000). If the current price is below the equilibrium price, e.g., $1.05 per euro in the euro market, there will be an excess demand for euros; at this price quantity demanded of euros (345,000 euros) exceeds the quantity supplied (238,095 euros). With an excess supply of U.S. dollars, the euro price per dollar will fall towards the equilibrium of 0.870 euros per dollar in the dollar market; with an excess demand for euros, the dollar price per euro will rise towards the equilibrium of $1.15 per euro in the euro market.

Table 5: Dollar Market and Euro Market

Euro price per dollar	*Demand*	*Supply*	*Dollar price per euro*	*Demand*	*Supply*
0.952 Euro	$ 250,000	$ 362,250	$1.25	275,000 Euro	360,000 Euro
*0.870 Euro	$ 350,000	$ 350,000	*$1.15	304,300 Euro	304,300 Euro
0.800 Euro	$ 450,000	$ 343,750	$1.05	345,000 Euro	238,095 Euro

*Note: * denotes equilibrium in the dollar and euro markets.*

As with all competitive markets, periodic shifts in demand and supply will affect the market price and quantity. In this case, shifts in one market, say the dollar market, will lead to a concurrent shift in the related market, i.e., the euro market. As a specific example, let us assume that European tastes for U.S. goods increase with the result that Europeans wish to import more U.S. goods and services. In that case Europeans will need more dollars to pay for these imports. There will be an increase in the demand for dollars, or an outward (rightward) shift in the dollar demand curve. An increase in the demand will lead to an increase in the price, the euro price per dollar, and an increase in the equilibrium quantity of dollars traded. The new euro price per dollar might rise from 0.87 to 0.90 euros as shown in Figures 2(a) and 2(b). Note that the supply curve of dollars has not changed; however, there is a movement up and along the existing supply curve, as demand for dollars increases. Thus, the quantity supplied of dollars increases, even though the supply of dollars remains unchanged.

Figure 2(a): The effect of an increase in European demand for U.S. goods on the market for U.S. dollars

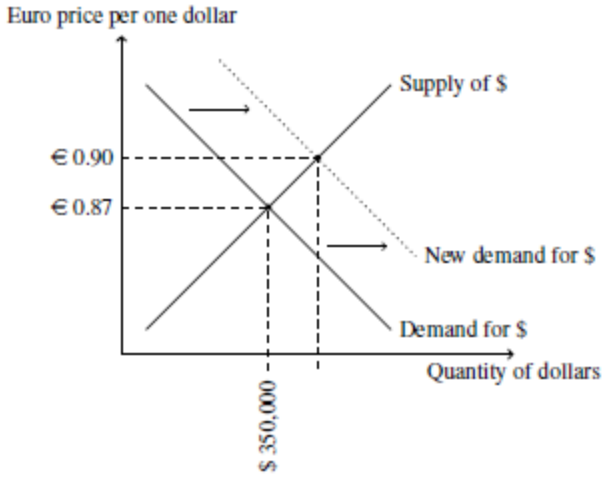
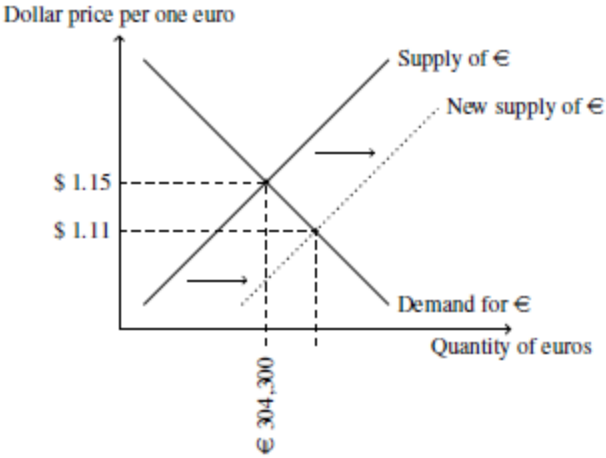


Figure 2(b): The effect of an increase in European demand for U.S. goods on the market for euros



Given that an increase in dollar demand necessitates an increase in the supply of euros to pay for these dollars, then the supply of euros increases, as shown in Figures 2(a) and 2(b). The dollar price of the euro may fall from $1.15 to $1.11 per euro. The number of euros transacted increases. In this case, note that in the euro market there is an increase in supply, no change in demand, but an increase in the quantity demanded. Of course, the expected relationship between the two exchange rates continues to hold. The new dollar price of a euro, $1.11 per euro, is the reciprocal of the new euro price of a dollar, 0.90 euro (1/.90 = $1.11). The dollar has appreciated, and the euro has depreciated.

Finally, as demand and supply vary, causing a currency to appreciate or depreciate, students often ask if we prefer for a currency to appreciate or to depreciate. There is no simple answer. The economic impact, whether favorable or unfavorable, depends on one's role in the economy. Let us assume that the dollar appreciates, and the euro depreciates, shown by a higher euro price per dollar and a lower dollar price per euro. With an appreciated dollar, goods and assets priced in euros become cheaper. Thus, dollar-holding tourists travelling to Europe, importers of European goods into the United States, and purchasers of European assets benefit by paying less in U.S. dollars for their purchases. Also, we assume that they will increase their purchases from these categories. However, goods denominated in dollars will cost more to Europeans. Thus, exports from the United States to Europe will cost Europeans more, leading to reduced exports sales. U.S.

producers will be hurt as their foreign sales fall. Also, with net exports as a component of GDP, a decrease in U.S. exports coupled with an increase in imports to the United States will reduce U.S. aggregate demand, short-run GDP, and employment. Thus, when a currency's value changes, there will be those within that country who benefit and those who are hurt economically. In general, an appreciation of a currency leads to a fall in net exports (total exports minus imports), reduced aggregate demand, reduced GDP, and reduced employment. In contrast, a deprecation of the currency leads to a rise in net exports, increased aggregate demand, increased GDP, and increased employment.

Varying Approaches to the Equilibrium Exchange Rate

The broad market-determined exchange rate approach explained in this paper is the principal model currently used to explain exchange rates in the post-1975 or post-Smithsonian period. In this case the exchange rate is the price of a currency that equates the overall demand and overall supply of a currency, for those uses previously explained in the current and financial accounts. Thus, this model incorporates the roles of trade and financial flows within an open economy into the demand and supply of a currency. This approach to exchange rate determination is often referred to as the Mundell-Fleming model and is the approach principally taught within international economics.

There are two other models that equilibrate the demand and supply of foreign currencies to generate an equilibrium exchange rate.[8] These approaches tend to emphasize financial factors in determining the equilibrium exchange rate. For instance, the monetary approach or monetarist model stresses the role of money supply changes affecting balance of payments imbalances or exchange rates. In this model, a money supply increase will generate increased spending on imported goods and assets; the converse will occur with a decrease in the money supply. With flexible or market-determined exchange rates, a money supply increase will lead to a depreciation of the currency; the converse is also true.

Another approach to exchange rate determination, sometimes called the asset market approach, stresses changes in asset markets more broadly, including other assets (particularly both domestic and foreign issued bonds), not just money as in the monetary approach. Any change in underlying financial conditions, such as a decrease in the domestic money supply, increasing domestic interest rates, will unleash a portfolio readjustment that will have an impact on the demand for foreign financial assets and foreign currencies. With market-determined exchange rates there will be an appreciation of this currency as portfolios are rebalanced to a new equilibrium. While potentially relevant under certain circumstances, these two models are not further included or discussed in this paper.

Exchange Rate Changes and Changes in the Balance of Payments Categories

Before turning to changing macroeconomic circumstances under which a country's international currency value is likely to change, let us join together our discussion of the balance of payments and exchange rate changes. In this section we have shown that shifts in the demand for a currency, e.g., dollars, will affect the equilibrium exchange rate in the foreign exchange market. Consider, for example, that there is a change in preferences of American investors towards Mexican assets, causing capital flows from the United States to Mexico to increase. The resulting increase in the demand for Mexican pesos (and the decrease in demand for U.S. dollars) would, ceteris paribus, cause the peso to appreciate relative to the U.S. dollar (that is, the dollar price of the peso will increase, and the peso price of the dollar will fall.) The increased capital inflow which causes an increase in the balance of payments on Mexico's financial account must be offset by a compensating decline in the balance of payments on Mexico's current account, which is ensured by the appreciation of the peso vis-à-vis the dollar.

Suppose that, as a result of the increased capital inflow into Mexico, the peso appreciates from 20 pesos per dollar to 16 pesos per dollar. This is equivalent to a depreciation of the dollar, from $0.05 per peso to $0.0625 per peso. Thus, Mexican goods and services selling for 20 pesos that previously cost $1.00 now cost $1.25. Consequently, as the peso appreciates relative to the dollar (that is, the peso price of dollar-denominated American goods and services fall) and the dollar appreciates relative to the peso (that is, the dollar price of peso-denominated Mexican goods and services rise), we would expect fewer exports to the

[8] Appleyard and Field, pp. 537-543.

Unites States from Mexico and an increase in imports to Mexico from the United States. As Mexican exports decline and imports rise, Mexican balance of payments on the current account would be expected to decline.

To summarize, movements in the exchange rate ensure that any movement in the balance of payments on a country's capital account will be matched by an equal and opposite movement in the balance of payment on that country's current account.

Comparative Statics

The comparative static exercises that follow aim to facilitate the teaching of exchange rates and initial, short-run effects on the exchange rate from changes to key macroeconomic variables: real income (or real GDP), the price level, and the real interest rate. These are three changes, common occurrences in our contemporary economies, that have predictable and immediate effects on the exchange rate. Economists refer to the initial assessment of these effects as a partial equilibrium, short-run analysis. This type of analysis has the advantage of isolating the predominant, immediate effect from a change in an underlying influence, helping a student to understand better the basic Mundell-Fleming exchange-rate model developed in this paper. More advanced study of economics and more complete models of the effect on an exchange rate and other macroeconomic variables from a singular change, like an increase in real income, would look at secondary effects and feedback mechanisms, as all endogenous variables within the economy readjust to a new equilibrium. That approach is a 'general equilibrium' approach that would be useful, really vital, in the case of predicting actual exchange rate changes for policy makers and investors. However, the approach does obscure the initial major impact of a change, which is the teaching function being stressed in this article.

A significant problem with using the general-equilibrium approach is that few *a priori* predications can often be advanced. For instance, an increase in real income or real GDP from expansionary fiscal policy will lead to an increase in imports and a depreciation of the domestic currency. However, that same increase in real GDP, assuming an increase in the domestic price level, will increase the demand for money and raise the nominal interest rate, and real interest rate in the short run, leading to an appreciation of the domestic currency. Thus, there are now two more secondary impacts on both real GDP and the exchange rate, from price level changes and interest rate changes (which will affect domestic private investment), leaving the ultimate set of changes for real GDP, employment, investment, and the exchange rate indeterminate, and subject principally to the strengths of empirical relationships incorporated in the aggregate macroeconomic model of the economy. As important as this general equilibrium approach is for policy makers and investors, it is a subsequent approach beyond the teaching function for introductory economics, which is more thoroughly explained with the partial equilibrium analysis in this paper.

In this section we look at three significant macroeconomic changes and their influences on a county's exchange rate, using the dollar and the euro to illustrate. Each of the three macroeconomic changes has the same effect on the dollar (an appreciation) and on the euro (a depreciation). These macroeconomic changes with their resulting effects are summarized in Table 6 below. We begin with the same equilibrium exchange rate as before: 0.870 euros per dollar and $1.15 per euro.

Increase in Real Incomes in the Euro Zone

Assume that the U.S. and the countries of the Euro zone are trading partners, importing and exporting from each other. Assume that there is strong economic growth in the Euro zone and the real GDP of the European countries, leading to an increase in the per capita real income of Europeans. European citizens will now demand more of all normal goods, including some that will be imported from the U.S. and other countries. To pay for these additional U.S. imports, there will be an increased demand for dollars and an increased supply of euros. As a result of these changes, there will be an increase in the euro price per dollar and a decrease in the dollar price per euro.

For instance, following the increase in the demand for dollars, assume that the euro price per dollar increased to 0.910 euros per dollar. In that case, the dollar price euro would fall to $1.10 (or 1/0.910.) Thus, the dollar would appreciate, and the euro depreciate.

As a result of this increased demand for U.S. goods, U.S. net exports will increase (exports increase with no initial change in imports), U.S. aggregate demand increases, short-run real GDP will increase, and employment will increase. Having an economically healthy trading partner is thus economically beneficial for a country-- in this example, the U.S. benefitting from the economic strength of the European countries.

In an economically integrated world, the economic health of one country can transfer quickly and significantly to the economic health of a trading partner.

Increase in the Price Level and Inflation in the Euro Zone

Assume that there is significant increase in labor costs in the countries of the Euro Zone, leading to an increase in the domestic price level, i.e., inflation. In this case, the goods (and services and assets) produced in these countries will increase in price and be less attractive both to the citizens of the country (those in Europe) and to those from other countries (the U.S.) Within the Euro zone, net exports will decrease, as the European countries export less of their now more expensive goods and import more goods from the U.S. With respect to the currencies, foreigners, as from the U.S., will be less inclined to purchase goods from the Euro zone (reduced exports from the Euro zone), reducing the demand for euros and equivalently reducing the supply of dollars. In this case, the dollar price per euro will fall and the euro price per dollar will increase, causing the dollar to appreciate and the euro to depreciate.

Europeans will be inclined to purchase more goods from outside the Euro Zone (more imports into the Euro zone) and fewer goods from within the Euro Zone, given the increase in the price of European goods. Thus, there will be an increased demand for dollars and an increased supply of euros to purchase dollars. In this case, the euro price per dollar will increase, and the dollar price per euro will fall, causing the dollar to appreciate further and the euro to depreciate further.

Changes in the demand for euros and supply of dollars, as well as changes in the demand for dollars and supply of euros, will each tend to appreciate the dollar and depreciate the euro. These changes reinforce each other, with both appreciating the dollar and depreciating the euro, as shown in the Table 6.

Decrease in Real Interest Rates within the Euro Zone

Let us assume that real interest rates decrease in the Euro Zone relative to those in the United States or increase in the United States relative to those in Europe. In this case there will be an increase in financial flows from Europe to the United States, seeking the higher real return in the United States. There will concurrently be a decrease in financial flows from the United States to Europe, as funds will remain in the United States to benefit from the higher relative real rate of return. The increased financial flows from Europe to the United States will increase the demand for dollars, raising the euro price per dollar and increase the supply of euros for dollars, reducing the dollar price per euro. Thus, the dollar will appreciate, and the euro will depreciate. Also, the decreased financial flows from the United States to Europe will decrease the demand for euros and lower the dollar price per euro; also, there will be a decrease in the supply of dollars for euros, increasing the euro price per dollar. These two actions, the increased financial flows to the United States and the reduced financial flows to Europe, both appreciate the dollar and depreciate the euro, reinforcing each other with respect to the influence on each exchange rate.

Table 6: Summary of Three Comparative Static Changes

	Demand for dollars	*Supply of dollars*	*Demand for euros*	*Supply of euros*
Increased income in Europe	Increase	No Change	No Change	Increase
	⟹ Dollar Appreciates		⟹ Euro Depreciates	
Increased price level in Europe	Increase	Decrease	Decrease	Increase
	⟹ Dollar Appreciates		⟹ Euro Depreciates	
Decreased real interest rate in Europe	Increase	Decrease	Decrease	Increase
	⟹ Dollar Appreciates		⟹ Euro Depreciates	

This interest rate effect on an exchange rate is quite relevant when the choice between fiscal and monetary policy to stabilize a country's economy is being made. For instance, if a country wishes to pursue expansionary policy to lower unemployment during a recession, the choice of fiscal versus monetary policy can have a secondary effect on the exchange rate and on the real economy itself. Expansionary fiscal policy (i.e., increased government spending and/or decreased taxes) that reduces government savings and necessitates borrowing tends to raise interest rates. As we have seen above, higher interest rates will tend to attract foreign financial investment and appreciate the currency. An appreciated currency would then have a second-round effect of increasing imports and lowering exports. This reduction in net exports would mitigate against the stimulating effects on aggregate demand from the expansionary fiscal policy on the economy.

In contrast, expansionary monetary policy (i.e., an open market operation of buying bonds by the central bank) increases the money supply and lowers interest rates. Lower interest rates will discourage foreign investment and lead to domestic financial outflows, depreciating the currency. A depreciated currency would then have a second-round effect of decreasing imports and increasing exports. The increase in net exports would reinforce the stimulating effects of the expansionary monetary policy on the economy.

Table 6 summarizes the effects on a country's currency exchange rate from an increase in economic activity of a trading partner, an increase in the domestic rate of inflation, and a relative decrease in real interest rates outside the country (or relative increase in the real interest rate within the country). In this partial equilibrium analysis, we have shown how changes to macroeconomic variables such as real income or GDP, the price level, and the interest rate can affect, in the short run, the value of a currency.

Additionally, exogenous supply-side changes, more microeconomic in effect, can affect the value of a currency. Frequently these effects are more 'general equilibrium' in nature, more complicated and multidimensional. For instance, an increase in energy prices will reduce aggregate supply, leading to a reduced real domestic GDP, an increase in the price level, and an indeterminate effect on interest rates. In particular, the change in nominal GDP which affects the demand for money is indeterminate, complicating any prediction with respect to nominal interest rates. In any case, the reduced real domestic GDP reduces the demand for imports and for foreign currency, appreciating the domestic currency. The increase in the domestic price level encourages a substitution of imported goods for domestic goods and increases the demand for imports and for foreign currency (and reduces the demand for the domestic currency), leading to a depreciation of the domestic currency. Thus, the income and price level effects counter each other, leading to an indeterminate effect on the domestic exchange rate. The indeterminacy of the interest rate change further complicates this analysis of the effect on the exchange rate from any change in aggregate supply.

As shown for countries with flexible, market-determined exchange rates, changes in macroeconomic performance can affect the country's exchange rate. In a more general equilibrium model, there can be important second-round effects that impact the performance of the real economy: with effects on net exports, investment, aggregate demand, real GDP, and employment.

Conclusion

This paper introduced the undergraduate student of economics to the importance of international finance in today's world. Whether for a student of introductory economics or of intermediate macroeconomic theory, the topics raised in this paper are vital to understanding the contemporary economy. A survey of the coverage of international finance in our leading introductory economics textbooks shows that our pedagogy in introductory economics can benefit from a concise, straightforward treatment of international finance, with some of the same consistency shown in our teaching of microeconomic topics such as perfect competition or monopoly.

References

Appleyard, Dennis R., and Alfred J. Field. 2017. *International Economics*, Ninth Ed. New York: McGraw Hill.

Cowen, Tyler, and Alex Tabarrok. 2018. *Modern Principles of Economics*, Fourth Ed. New York: Worth Publishers, Macmillan Learning.

Dornbusch, Rudiger, and Stanley Fischer. 2005. *Macroeconomics*, Sixth Ed. New York: McGraw Hill.

Hubbard, Glenn, R., and Anthony Patrick O'Brien. 2015. *Economics*, Fifth Ed. New Jersey: Pearson.

Kindleberger, Charles P. 1955. *International Economics*, Second Ed. Illinois: Richard D. Irwin, Inc.

Krugman, Paul, and Robin Wells. 2018. *Economics*, Fifth Ed. New York: Worth Publishers, Macmillan Learning.

Mankiw, N. Gregory. 2018. *Principles of Economics*, Eight Ed. Massachusetts: CENGAGE Learning.

Mateer, Dirk, and Lee Coppock. 2018. *Principles of Economics*, Second Ed. W.W. New York: Norton.

McConnell, Campbell R., Stanley L. Brue, and Sean M. Flynn. 2018. *Economics: Principles, Problems and Policies*, Twenty-first Ed. New York: McGraw Hill.

Excel Templates for Illustrating TVM Calculations and a Financial Calculator with Associated Excel Functions

***Maura Alexander, Tom Arnold, and Joseph Farizo*[1]**

ABSTRACT

Time value of money calculations are illustrated through developing a timeline with cash flow graphics in Excel. The graphic is readily adjustable to different scenarios making it useful for multiple time value of money topics (e.g., NPV, MIRR, etc.). Further, a second graphic illustrates a financial calculator that also provides the associated Excel functions. Both graphics can be used in the live or virtual classroom and as a resource for students outside of the classroom.

Introduction

Widely adopted corporate finance and investment texts make frequent use of timelines to demonstrate time value of money (TVM) calculations and applications.[2] Instructors also use timelines extensively in the classroom, and have done so for decades. Yet, students have a reluctance to use timelines initially or abandon using timelines altogether, despite the great amount of intuition one can garner by using them. In this presentation, Excel programing with a graphic is used to illustrate the timeline and the associated TVM calculations. A simple graphic is generated by merging and adding cell borders to establish the discrete time periods. An instructor can display the Excel timeline graphic on an overhead screen or through virtual screen share for lecture, and post the file as a resource for students to use on their own outside the classroom.

A second Excel graphic produces a financial calculator simulation with the associated Excel functions. Although financial calculator simulations are readily available, none of the simulations connect the financial calculator function to an associated Excel function. This is critical for students because the input criteria differ between the financial calculator and Excel often leading to confusion and frustration. Like the previous graphic, the simulated financial calculator can be used as part of a lecture or as a class resource.

The next section presents the programing for working with a single cash flow. The third section presents a similar graphic utilizing multiple cash flows. Although the multiple cash flows graphic technically subsumes the single cash flow graphic, the former is necessary to generate intuition and should not be overlooked. The fourth section uses multiple timelines to illustrate modified cash flows for computing the modified internal rate of return (MIRR) and the modified net present value (MNPV). The fifth section presents a basic financial calculator simulation and is followed by a more advanced version in the sixth section to connect the calculator and associated Excel functions. The seventh section concludes the article.

Single Cash Flow Graphic in Excel

Exhibit 1 displays a three-year timeline with the associated Excel programming. (A ten-year version of the graphic is available for download at: *https://scholarship.richmond.edu/finance-faculty-publications/46/*.) The programming prevents the user from entering multiple cash flows and from entering multiple points in time for determining the time value of money calculation.

[1] Maura Alexander: Department of Finance, University of Richmond, 102 UR Drive, Richmond, VA 23173, malexand@richmond.edu. Tom Arnold: Department of Finance, University of Richmond, 102 UR Drive, Richmond, VA 23173, tarnold@richmond.edu. Joseph Farizo: Department of Finance, University of Richmond, 102 UR Drive, Richmond, VA 23173, jfarizo@richmond.edu. The authors wish to thank Bill Yang and an anonymous referee for helpful suggestions.

[2] See, for example, Chapter 14 of Bodie et al. (2018), Chapter 4 of Ross et al. (2016a), and Chapter 5 of Ross et al. (2016b).

Exhibit 1: Single Cash Flow Graphic in Excel

	A	B	C	D	E	F	G	H	I
1	Periodic interest rate:	10.00%							
2									
3									
4	Enter a single cash flow above associated moment in time:					$121.00			
5									
6									
7		**0**		**1**		**2**		**3**	
8	Type in "X" to indicate the point in time at which the time value of money is calculated:	X							
9									
10	Answer:	$100.00							
11									
12	Calculation:	$121 ÷ (1 + 10%)^2 = $100.000							
13									
14	FV = cash flow × (1 + interest rate)^N								
15	PV = cash flow ÷ (1 + interest rate)^N								
16									
17									
18		0		0		1		0	
19		1		0		0		0	
20		2	0	÷	×				
21		2	÷	121					
22									
Formulas are in the Appendix									

The single cash flow graphic displays the timeline and the associated TVM calculation. The instructor should show the student how to determine the present and future values based on when a cash flow appears in time relative to the point in time the cash flow is to be evaluated (i.e., the position of "X" in the graphic). Further, the instructor should demonstrate the effect of changing the discount rate on the present and future values. The student should then become comfortable with reproducing these computations on their own.

For example, increasing the discount rate to 12% in Exhibit 1 makes the $121.00 cash flow received in Year 2 worth $96.46 today. The student should be informed that the present value equation has a larger denominator as the discount rate increases and results in a lower present value for the cash flow.

An additional feature of the calculator is that it does not require time value to be calculated relative to time 0. That is, one can find the future value of some future cash flow at a point in time beyond when the future cash flow is received. For example, if "X" is moved to Year 3, the $121.00 in Year 2 earns one year of interest and will be worth $135.52 in Year 3.

What is important is that the student begins to understand that TVM calculations are about finding the value of cash flows *relative to a desired point in time*. Students tend to think present value is always a calculation for determining the value of future cash flows at Year 0 and that future value is only applied at the point in time when the final cash flow is received. While a number of TVM calculations including security pricing and saving for a retirement goal are computed in this manner, there are many TVM calculations that do not follow this format. A student's failure to understand this intuition early in the process of learning time value of money concepts may hinder understanding more advanced computations. The easiest way to build this intuition is by considering a single cash flow and then changing the relative moments in time between the cash flow and when the cash flow is to be valued.

Multiple Cash Flows Graphic in Excel

When considering multiple cash flows on a timeline, a number of new educational possibilities emerge (see Exhibit 2).

In the current form, the student can see what cash flows are being evaluated at Year 0 (i.e., "X" is set at Year 0) and how the overall value is calculated by summing individual discounted cash flows. In this example, having all the cash flows constant allows a discussion about a present value annuity.

Exhibit 2: Multiple Cash Flows Graphic in Excel

	A	B	C	D	E	F	G	H	I
1	Periodic interest rate:	10.00%							
2									
3									
4	Enter a single cash flow above associated moment in time:			$100.00		$100.00		$100.00	
5									
6									
7		**0**		**1**		**2**		**3**	
8	Type in "X" to indicate the point in time at which the time value of money is calculated:	X							
9									
10	Answer:	$248.69							
11									
12	Calculation (Equation):	Year 0:	= $ 0.00 ÷ (1 + 10%)^0						
13		Year 1:	+ $ 100.00 ÷ (1 + 10%)^1						
14		Year 2:	+ $ 100.00 ÷ (1 + 10%)^2						
15		Year 3:	+ $ 100.00 ÷ (1 + 10%)^3						
16									
17	Calculation (Values):	=		$ 0.00					
18		+		$ 90.91					
19		+		$ 82.64					
20		+		$ 75.13					
21									
22	FV = cash flow × (1 + interest rate)^N								
23	PV = cash flow ÷ (1 + interest rate)^N								
24									
25		1		0		0		0	
26		0		1		2		3	
27		÷		÷		÷		÷	
28		$ 0.00		$ 90.91		$ 82.64		$ 75.13	
29		0	÷	×					
30									
Formulas are in the Appendix									

Another set of cash flows allows for the discussion of the net present value (NPV) of a project. Assume a project costs $400 and generates three annual cash flows of $100, $200, and $300 over the next three years with an associated discount rate of 15% annually. The timeline in Exhibit 3 illustrates the cash flows and the calculation of the NPV as $35.44.

Next, one can introduce the "Goal Seek" feature to change the discount rate to a value that sets the NPV to zero: go to the "Data" menu on the Excel dashboard and then to the "What-If Analysis" pulldown menu and select "Goal Seek" to set cell B10 = 0 by changing cell B1.

After iterating, the NPV becomes zero and the discount rate of 19.44% is revealed to be the internal rate of return (IRR). This illustrates that the IRR is a calculation found through iteration rather than a solution found directly by applying a formula. Demonstrating this visually helps the student understand how the IRR is determined.

By changing the cash flows in the following manner: Year 0 = - $118,525.54, Year 1 = $424,321.44, Year 2 = - $505,155.86, and Year 3 = $200,000.00, a different type of project is illustrated that requires additional investment in Year 2. For example, the project could be the purchase of a mine that needs safety maintenance in Year 2 and is then expected to be sold in Year 3. Assuming an annual discount rate of 8.00%, the project has an NPV = $41.40 making it potentially an acceptable project (see Exhibit 4).

Increasing the annual discount rate to 15% generates an NPV = -$17.53. This is not necessarily surprising given the intuition that a higher discount rate leads to lower valuations relative to Year 0. However, increasing the annual discount rate to 25% generates a positive NPV of $31.86, which is very counterintuitive. In this case, the higher discount rate is affecting the value of the investment being made in Year 2 more so than the inflows in Years 1 and 3.

At this point, the instructor can introduce the student to the concept of multiple project IRRs (in this case: 10%, 18%, and 30%), and that such a situation occurs whenever the cash flows change sign through time, from positive to negative or vice versa. Further, the instructor can graph the NPV based on different discount

rates that incorporate the multiple IRRs to demonstrate how the NPV has positive and negative regions based on the discount rate.

Exhibit 3: Net Present Value (NPV) Illustrated

	A	B	C	D	E	F	G	H	I
1	Periodic interest rate:	15.00%							
2									
3									
4	Enter a single cash flow above associated moment in time:	($400.00)		$100.00		$200.00		$300.00	
5									
6									
7		**0**		**1**		**2**		**3**	
8	Type in "X" to indicate the point in time at which the time value of money is calculated:	X							
9									
10	Answer:	$35.44							
11									
12	Calculation (Equation):	Year 0:	= -$ 400.00 ÷ (1 + 15%)^0						
13		Year 1:	+ $ 100.00 ÷ (1 + 15%)^1						
14		Year 2:	+ $ 200.00 ÷ (1 + 15%)^2						
15		Year 3:	+ $ 300.00 ÷ (1 + 15%)^3						
16									
17	Calculation (Values):	=	($400.00)						
18		+	$ 86.96						
19		+	$ 151.23						
20		+	$ 197.25						
21									
22	FV = cash flow × (1 + interest rate)^N								
23	PV = cash flow ÷ (1 + interest rate)^N								

Exhibit 4: Net Present Value (NPV) with Cash Flows that Change Sign

	A	B	C	D	E	F	G	H	I
1	Periodic interest rate:	8.00%							
2									
3									
4	Enter a single cash flow above associated moment in time:	($118,525.54)		$424,321.44		($505,155.86)		$200.000.00	
5									
6									
7		**0**		**1**		**2**		**3**	
8	Type in "X" to indicate the point in time at which the time value of money is calculated:	X							
9									
10	Answer:	$41.40							
11									
12	Calculation (Equation):	Year 0:	= -$ 118,525.54 ÷ (1 + 8%)^0						
13		Year 1:	+ $ 424,321.44 ÷ (1 + 8%)^1						
14		Year 2:	+ -$ 505,155.86 ÷ (1 + 8%)^2						
15		Year 3:	+ $ 200,000.00 ÷ (1 + 8%)^3						
16									
17	Calculation (Values):	=	($118,525.54)						
18		+	$ 392,890.22						
19		+	($ 433.089.73)						
20		+	$ 158,766.45						
21									
22	FV = cash flow × (1 + interest rate)^N								
23	PV = cash flow ÷ (1 + interest rate)^N								

If desired, the concepts of modified internal rate of return (MIRR) and modified net present value (MNPV, McClure and Girma 2004) can be introduced as a potential solution for multiple IRRs. To introduce this topic, multiple timelines are instructive to illustrate how cash flows are "modified" using a re-investment rate to find the MIRR and MNPV.

Multiple Timelines to Illustrate MIRR and MNPV

Although MIRR and MNPV are being developed here as a solution to working with multiple IRRs, there is significant literature that discusses the "reinvestment assumption" within the internal rate of return calculation (IRR) and technically, the calculation of the net present value (NPV) (see Arnold and Nixon 2006 and 2013, and Rich and Rose 2014 concerning IRR). In Exhibit 5, a three-year version of the MIRR-MNPV graphic is presented with the associated Excel programming. A ten-year version of the graphic is available for download at: *https://scholarship.richmond.edu/finance-faculty-publications/47/.*

Exhibit 5: MIRR-MNPV Graphic in Excel

	A	B	C	D	E	F	G	H	I
1	Re-investment Rate (usually set at the Weighted Average Cost of Capital, WACC):	5.00%							
2	Project Discount Rate:	8.00%							
3									
4	Enter cash flows above associated moment in time:	($118,525.54)		$424,321.44		($505,155.86)		$200,000.00	
5									
6									
7		**0**		**1**		**2**		**3**	
8									
9	STEP 1: Present Value all cash outflows using re-investment rate	($118,525.54)				(505,155.86)			
10									
11									
12		**0**		**1**		**2**		**3**	
13									
14		PV(Outflows):		($576,716.80)					
15									
16		= -$118,525.54		+ -$0.00		+ -$458,191.26			
17									
18		= -$118,525.54 / (1 + 5%) ^ 0		+ -$0.00 / (1 + 5%) ^ 1		= -$505,155.86 / (1 + 5%) ^ 2			
19									
20	STEP 2: Future Value all cash inflows using re-investment rate	$0.00		$424,321.44		$0.00		$200,000.00	
21									
22									
23		**0**		**1**		**2**		**3**	
24									
25		FV(Inflows):		$667,814.39					
26									
27		= $0.00		+ $467,814.39		+ $0.00		+ $200,000.00	
28									
29		= $0.00 *(1 + 5%) ^ 3		+ $424,321.44 *(1 + 5%) ^ 2		+ $0.00 *(1 + 5%) ^1		+ 200,000.00 *(1 + 5%) ^ 0	
30									
31	STEP 3: Display modified cash flows	($576,716.80)						$667,814.39	
32									
33									
34		**0**		**1**		**2**		**3**	
35									

As in the previous section, the project illustrated has two investments, $118,525.54 initially and $505,155.86 two years later. The project produces positive cash flows of $424,321.44 and $200,000.00 in Years 1 and 3 respectively. To generate modified cash flows, a re-investment rate (usually set at the weighted average cost of capital and set to 5% in Exhibit 5) is applied to the inflows and outflows of the project. The outflows are discounted and summed to produce a single outflow of $576,716.80 at Year 0 (STEP 1 in Exhibit 5). The inflows are appreciated at the re-investment rate to the end of the project (Year 3 in Exhibit 5) to produce a terminal inflow of $667,814.39 (STEP 2 in Exhibit 5).

Exhibit 5: MIRR-MNPV Graphic in Excel (continued)

	A	B	C	D	E	F	G	H	I
36	STEP 4: Compute Modified Internal Rate of Return (MIRR) as the holding period return of the STEP 3 modified cash flows:	MIRR:		5.01%		= [$ 667,814.39 / $ 576,716.80] ^ (1 / 3) -1			
37									
38	Compute Modified Net Present Value (MNPV) based on the project discount rate and the STEP 3 modified cash flows:	MNPV:		($46,584.20)		= $ 667,814.39 / (1 + 8%) ^ 3 – $ 576,716.80 / (1 + 8%) ^ 0			
39									
40	Internal rate of Return (IRR) based on initial cash flows:	IRR:		10.00%					
41									
42	Net Present Value:	NPV:		$41.40					
43									
44									
45									
46		0		1		0		0	
47		0		1		2		3	
48		1							
49		3							
50									
Formulas are in the Appendix									

The modified cash flows are displayed on the timeline in STEP 3 of Exhibit 5. These cash flows may then be used for the MIRR and MNPV calculations, presented in STEP 4 of Exhibit 5. The MIRR is the annual holding period return (HPR) of the modified cash flows.

$$HPR = \left[\frac{\$667{,}814.39}{\$576{,}716.80}\right]^{1/3} - 1 = 5.01\% = MIRR \quad (1)$$

The MNPV is the NPV of the modified cash flows using the project discount rate.

$$MNPV = \frac{\$667{,}814.39}{(1+8\%)^3} - \frac{\$576{,}716.80}{(1+8\%)^0} = -\$46{,}584.20 \quad (2)$$

The MIRR, MNPV, IRR and NPV are also calculated within the graphic with the latter two being 10.00% and $41.40 respectively. Changing the discount rate will not result in the MNPV switching signs due to multiple IRRs as was the case with the NPV in the previous section.

In the classroom (live or virtual), the graphic allows the student to see the steps necessary for computing the modified cash flows and the results of the MIRR, MNPV, IRR, and NPV calculations. The instructor can then lead a discussion of the differences between, and the potential benefits of, each technique. The graphic can also be used as a resource for the student to use on their own or as an illustration of the MIRR and MNPV techniques to be programmed on another platform in a fintech class.

Although not demonstrated above, changing the re-investment rate to equal the discount rate of 8% sets the MNPV to equal the NPV and setting the re-investment rate to the IRR of 10% sets the MIRR to equal the IRR. This illustrates the "re-investment assumption" within the NPV and IRR metrics if there is a desire to discuss the topic.

Once the intuition of the time value of money is solidified with using timelines, students are, usually, then introduced to using the financial calculator. Just as an Excel template can be developed for timelines, an Excel template can be developed for the financial calculator.

TI BAII-Plus Simulation in Excel

In Exhibit 6, the TI BAII-Plus financial calculator is simulated using Excel functions with written instructions within the spreadsheet to allow the student to use the calculator independently of the classroom. Although the instructions are basic in nature, they are critical in helping the student overcome any apprehension of using the simulation, and eventually the financial calculator. A further goal might be using Excel financial functions that are similar to the financial calculator functions (developed in the next section), but this is not necessary initially.

Exhibit 6: TI BAII-Plus Financial Calculator Simulation in Excel

	A	B	C	D	E	F	G	H	I	J	K
1	**Financial Calculator**										
2											
3	***Enter an "X" in the "ENTRY:" column for the "KEY:" you desire to compute. In the financial calculator, [CPT] key + [associated "KEY:"] key are pressed after other entries are completed.***										
4											
5	**Set cash flows to arrive at the end of the period "END" or at the beginning of the period "BEG"**										
6	To adjust this setting: [2ND] key + [PMT] key and then switch between "END" and "BEG" using [2ND] key + [ENTER] key, use [2ND] key + [CPT] key to exit the setting.										
7											
8	"END" is the default						END				
9											
10							**ENTRY:**		**KEY:**		**RESULT:**
11	**Number of periods** (single cash flow), **Number of payments** (annuity)						10.00		**N**		
12											
13	**Period-adjusted Interest rate in terms of percentage**						5.00		**I/Y**		
14	(e.g. 8% is entered as 8.00 = rate% × 100)										
15	**Present value**						X		**PV**		-613.91
16	(default sets cash flow to its negative, 100 becomes -100)										
17	**Constant payment received each period**						0		**PMT**		
18	(default sets cash flow to its negative, 100 becomes -100)										
19	**Future value**						1000		**FV**		
20	(default sets cash flow to its negative, 100 becomes -100)										
21											
Formulas are in the Appendix. This file can be downloaded from: *https://scholarship.richmond.edu/finance-faculty-publications/48/*											

The actual programming is not extensive, but can be tedious. In the exhibit, a calculation is displayed for the present value of $1,000 received ten periods in the future assuming a periodic interest rate of 5%.

Although one may be tempted to omit some of the instructions or information presented in Exhibit 6, one should be careful about making changes. The fact that payments are set to the "end of period" at the top of the exhibit is to emphasize a setting that students often neglect. Further, having numerical entries to the left of the financial calculator key is to emphasize the value must be inputted into the calculator before assigning the value to the associated input key.

Exhibit 7: Financial Calculator Simulation in Excel with a Bond

	A	B	C	D	E	F	G	H	I	J	K
1	**Financial Calculator**										
2											
3	***Enter an "X" in the "ENTRY:" column for the "KEY:" you desire to compute. In the financial calculator, [CPT] key + [associated "KEY:"] key are pressed after other entries are completed.***										
4											
5	**Set cash flows to arrive at the end of the period "END" or at the beginning of the period "BEG"**										
6	To adjust this setting: [2ND] key + [PMT] key and then switch between "END" and "BEG" using [2ND] key + [ENTER] key, use [2ND] key + [CPT] key to exit the setting.										
7											
8	"END" is the default						END				
9											
10							**ENTRY:**		**KEY:**		**RESULT:**
11	**Number of periods** (single cash flow), **Number of payments** (annuity)						10.00		**N**		
12											
13	**Period-adjusted Interest rate in terms of percentage**						X		**I/Y**		6.00
14	(e.g. 8% is entered as 8.00 = rate% × 100)										
15	**Present value**						-1000		**PV**		
16	(default sets cash flow to its negative, 100 becomes -100)										
17	**Constant payment received each period**						60		**PMT**		
18	(default sets cash flow to its negative, 100 becomes -100)										
19	**Future value**						1000		**FV**		
20	(default sets cash flow to its negative, 100 becomes -100)										
21											

Other pieces of information are also critical; for example, the periodic interest is entered as a whole number assumed to be a percentage and certain values are assumed negative even through a positive number is entered. These are the "small things" that students find baffling initially.

Once the simulation is presented, it is easy to demonstrate time value of money computations. The simulation could be displayed virtually through screen sharing or in the classroom and then utilized within a live/virtual class when a need arrives.

For example, one can easily demonstrate how a bond's yield decreases as its price increases. In Exhibit 7, a ten-year $1,000 bond with 6% annual coupons (i.e. $60.00 = 6% × $1,000) that sells for par (i.e. $1,000) is displayed. From Column K in the simulator, the result demonstrates that the yield is 6.00% in this scenario. When the bond price is increased to $1,050 by modifying cell G15, the yield to maturity decreases to 5.34%, which is shown in the result column (Column K) of the simulator.

From here, an instructor can begin a discussion on how a central bank has to engage in market transactions to "set" interest rates or how a bond with relatively high coupons becomes more desirable as interest rates fall, making its price increase until the yield deceases to a level equivalent to current market rates. Many other visual presentations of finance concepts are certainly possible.

Transition from the Financial Calculator to Excel

In Exhibit 8, programming is introduced to the right of the financial calculator simulation to display the corresponding Excel financial function. This programming is tedious and repetitive, however, as mentioned previously, a downloadable version of the calculator is available.

Exhibit 8: Financial Calculator Simulation with Excel Formulas

	G	H	I	J	K	L	M	N	O	P	Q
8	END										
9											
10	**ENTRY:**		**KEY:**		**RESULT:**		**EXCEL FORMULA:**				
11	10.00		**N**								
12											
13	5.00		**I/Y**								
14											
15	X		**PV**		-613.91		**= PV (5% , 10 , 0.00 , 10,000.00 , 0)**				
16							**= PV (I/Y , N , PMT , FV , 0)**				
17	0		**PMT**								
18											
19	1000		**FV**								
20											
21											
Formulas are in the Appendix											

The benefit of viewing the equivalent Excel financial function (and generally, the same function in Google Sheets) relative to the financial calculator is not available in other simulations. Depending on the instructor's intentions, this portion of the simulation can become part of a classroom exercise or provided as a resource for students to try the Excel programing on their own.

Further, in an online testing situation, it may be beneficial to have students compute answers to test problems using Excel and Excel functions rather than with a financial calculator in order to send the instructor a file with computational work for partial credit. If this is the case, there is a great benefit for using the simulation to help students make the transition to Excel from the financial calculator.

Conclusion

The use of a timeline to illustrate TVM concepts is not a new practice. Textbooks and instructors have done so for decades. Yet, students often abandon this powerful tool early when developing TVM skills, perhaps by shifting too quickly to using calculators or paying too much attention to written formulas without understanding the intuition behind them. Using Excel to create a timeline within a cash flow graphic enhances the benefit of using timelines by displaying cash flows through time as well as the associated TVM

calculations. The graphic is readily adaptable to different scenarios to make the file useful for multiple in-person or virtual lectures. It is also suitable as a standalone resource for students to use outside the classroom. Although examples are provided for NPV, IRR, and modified techniques, a number of other possibilities exist that can be readily illustrated using the cash flow graphic.

The financial calculator simulation is useful for teaching how to use the calculator (and as a student resource) because instructions/formatting are provided that are applicable to the financial calculator. If desired, the financial calculator simulation can also provide the associated Excel functions to help the student transition to using Excel.

When all sets of graphics are implemented, the student can start with using timelines, transition to using the financial calculator, and then transition to using Excel functions. This process can start in the classroom (live or virtual) with the graphics being made available as resources for students to use to reinforce these skills.

References

Arnold, Tom, and Terry D. Nixon. 2006. "Adding Depth to the Discussion of Capital Budgeting Techniques." *Journal of Financial Education* 32: 78 – 89.

Arnold, Tom, and Terry D. Nixon. 2013. "An Easy Method to Introduce MIRR into Introductory Finance Classes." *Advances in Financial Education* 11: 70 – 74.

Bodie, Zvi, Alex Kane, and Alan Marcus. 2018. *Investments* (11th Ed.). New York: McGraw-Hill.

McClure, Kenneth G., and Paul Berhanu Girma. 2004. "Modified Net Present Value (MNPV): A New Technique for Capital Budgeting." *Zagreb International Review of Economics & Business* 1: 67-82.

Rich, Steven P., and John T. Rose. 2014. "Reexamining an Old Question: Does the IRR Method Implicitly Assume a Reinvestment Rate?" *Journal of Financial Education* 40: 152 – 166.

Ross, Stephen, Randolph Westerfield, Jeffrey Jaffe, and Bradford Jordan. 2016a. *Corporate Finance* (11th Ed.). New York: McGraw-Hill.

Ross, Stephen, Randolph Westerfield, and Bradford Jordan. 2016b. *Fundamentals of Corporate Finance* (11th Ed.). New York: McGraw-Hill.

Appendix

Exhibit 1 formulas:

Merge cells: B4 and C4, D4 and E4, F4 and G4, H4 and I4
Provide associated cell borders in rows 5 and 6 to mimic a timeline.
Merge cells: B7 and C7, D7 and E7, F7 and G7, H7 and I7
Merge cells: B8 and C8, D8 and E8, F8 and G8, H8 and I8
Merge cells: B10 and C10
Cell B10: = D21 / (1 + B2)^(B20 – C20)
Merge cells: B12 to E12
Cell B12: = CONCAT(TEXT(D21,"$ #,###.00")," ", C21," ","(1 + ",B1*100,"%) ^ ",B21," = ",TEXT(B10, "$ #,###.00"))

These formulas can be "hidden" or "cut and pasted" to another worksheet to hide from view:
Merge cells: B18 and C18, D18 and E18, F18 and G18, H18 and I18
Merge cells: B19 and C19, D19 and E19, F19 and G19, H19 and I19
Cell B18: =IF(B4 = 0, 0, 1)
Cell D18: =IF(D4 = 0, 0, 1)
Cell F18: =IF(F4 = 0, 0, 1)

Cell H18: =IF(H4 = 0, 0, 1)

Cell B19: =IF(B8 = “X”, 1, 0)
Cell D19: =IF(D8 = “X”, 1, 0)
Cell F19: =IF(F8 = “X”, 1, 0)
Cell H19: =IF(H8 = “X”, 1, 0)

Cell B20: =IF(SUM(B18:I18)=1,SUMPRODUCT(B7:I7,B18:I18), “ERROR”)
Cell C20: =IF(SUM(B19:I19)=1,SUMPRODUCT(B7:I7,B19:I19), “ERROR”)
Cell B21: = ABS(B20 – C20)
Cell C21: =IF((B20 – C20) > 0, D20, E20)
Cell D21: = SUM(B4:I4)

Additional features if desired:

Highlight cells B4 to I4, go to “Data” menu on Excel dashboard, go to “Data Validation” pulldown menu and an input box displays with three sub-menus:
In “Settings” menu: select “Custom” and enter =COUNT($B$4:$I$4) <= 1 as the Formula
In “Input Message” menu: activate “Show input message when cell is selected” box (this is usually the default), and enter “Enter a single cash flow above associated moment in time” into “Input message” text box
In “Error Alert” menu: activate “Show error alert after invalid data is entered” box (this is usually the default), set “Style” as “Stop” (this is usually the default), and enter “Only enter one cash flow on the timeline.” into the “Error message” text box

Highlight cells B8 to I8, go to “Data” menu on Excel dashboard, go to “Data Validation” pulldown menu and an input box displays with three sub-menus:
In “Settings” menu: select “Custom” and enter =COUNTA($B$8:$I$8) <= 1 as the Formula
In “Input Message” menu: activate “Show input message when cell is selected” box (this is usually the default) and enter “Type in “X” to indicate the point in time at which the time value of money is calculated” into “Input message” text box
In “Error Alert” menu: activate “Show error alert after invalid data is entered” box (this is usually the default), set “Style” as “Stop” (this is usually the default), and enter “Only enter “X” at a single point in time.” into the “Error message” text box.

Exhibit 2 formulas:

Merge cells: B4 and C4, D4 and E4, F4 and G4, H4 and I4
Provide associated cell borders in rows 5 and 6 to mimic a time line.
Merge cells: B7 and C7, D7 and E7, F7 and G7, H7 and I7
Merge cells: B8 and C8, D8 and E8, F8 and G8, H8 and I8
Merge cells: B10 and C10
Cell B10: = SUM(B28:I28)
Merge cells: C12 to F12
Cell C12: = CONCAT(“= “,TEXT(B4,”$ #,###.00”),” “, B27,“ “,”(1 + “,B1*100,”%) ^ ”,ABS(B26))
Merge cells: C13 to F13
Cell C13: = CONCAT(“+ “,TEXT(D4,”$ #,###.00”),” “, D27,“ “,”(1 + “,B1*100,”%) ^ ”,ABS(D26))
Merge cells: C14 to F14
Cell C14: = CONCAT(“+ “,TEXT(F4,”$ #,###.00”),” “, F27,“ “,”(1 + “,B1*100,”%) ^ ”,ABS(F26))
Merge cells: C15 to F15
Cell C15: = CONCAT(“+ “,TEXT(H4,”$ #,###.00”),” “, H27,“ “,”(1 + “,B1*100,”%) ^ ”,ABS(H26))
Merge cells: C17 to D17
Cell C17: = B28
Merge cells: C18 to D18
Cell C18: = D28
Merge cells: C19 to D19

Cell C19: = F28
Merge cells: C20 to D20
Cell C15: = H28

These formulas can be "hidden" or "cut and pasted" to another worksheet to hide from view:
Merge cells: B25 and C25, D25 and E25, F25 and G25, H25 and I25
Merge cells: B26 and C26, D26 and E26, F26 and G26, H26 and I26
Merge cells: B27 and C27, D27 and E27, F27 and G27, H27 and I27
Merge cells: B28 and C28, D28 and E28, F28 and G28, H28 and I28
Cell B25: =IF(B8 = "X", 1, 0)
Cell D25: =IF(D8 = "X", 1, 0)
Cell F25: =IF(F8 = "X", 1, 0)
Cell H25: =IF(H8 = "X", 1, 0)
Cell B26: = B7 – B29
Cell D26: = D7 – B29
Cell F26: = F7 – B29
Cell H26: = H7 – B29
Cell B27: = IF(B26<0,D29,C29)
Cell D27: = IF(D26<0,D29,C29)
Cell F27: = IF(F26<0,D29,C29)
Cell H27: = IF(H26<0,D29,C29)
Cell B28: = B4/(1 + B1)^B26
Cell D28: = D4/(1 + B1)^D26
Cell F28: = F4/(1 + B1)^F26
Cell H28: = H4/(1 + B1)^H26
Cell B29: =IF(SUM(B25:I25)=1,SUMPRODUCT(B7:I7,B25:I25),"ERROR")

Additional feature if desired:

Highlight cells B8 to I8, go to "Data" menu on Excel dashboard, go to "Data Validation" pulldown menu and an input box displays with three sub-menus:
In "Settings" menu: select "Custom" and enter =COUNTA($B$8:$I$8) <= 1 as the Formula
In "Input Message" menu: activate "Show input message when cell is selected" box (this is usually the default) and enter "Type in "X" to indicate the point in time at which the time value of money is calculated" into "Input message" text box
In "Error Alert" menu: activate "Show error alert after invalid data is entered" box (this is usually the default), set "Style" as "Stop" (this is usually the default), and enter "Only enter "X" at a single point in time." into the "Error message" text box.

Exhibit 5 formulas:

Merge cells: B4 and C4, D4 and E4, F4 and G4, H4 and I4
Provide associated cell borders in rows 5 and 6 to mimic a time line.
Merge cells: B7 and C7, D7 and E7, F7 and G7, H7 and I7

Merge cells: B9 and C9, D9 and E9, F9 and G9, H9 and I9
Provide associated cell borders in rows 10 and 11 to mimic a timeline.
Merge cells: B12 and C12, D12 and E12, F12 and G12, H12 and I12
Cell B9: =IF(B4<0,B4,0)
Cell D9: =IF(D4<0,D4,0)
Cell F9: =IF(F4<0,F4,0)
Cell H9: =IF(H4<0,H4,0)

Merge cells: B14 and C14 and D14 and E14
Cell D14: =NPV(B1,B9:I9)*(1+B1)

Merge cells: B16 and C16, D16 and E16, F16 and G16, H16 and I16
Cell B16: =IF(B12>B48," ",CONCAT("= ",TEXT(B9/(1 + B1)^B12,"$ #,##0.00")))
Cell D16: =IF(D12>B48," ",CONCAT("= ",TEXT(D9/(1 + B1)^D12,"$ #,##0.00")))
Cell F16: =IF(F12>B48," ",CONCAT("= ",TEXT(F9/(1 + B1)^F12,"$ #,##0.00")))
Cell H16: =IF(H12>B48," ",CONCAT("= ",TEXT(H9/(1 + B1)^H12,"$ #,##0.00")))

Merge cells: B18 and C18, D18 and E18, F18 and G18, H18 and I18
Cell B18: =IF(B12>B48," ",CONCAT("= ",TEXT(B9,"$ #,##0.00")," / (1 + ",B1*100,"%) ^ ",B12))
Cell D18: =IF(D12>B48," ",CONCAT("+ ",TEXT(D9,"$ #,##0.00")," / (1 + ",B1*100,"%) ^ ",D12))
Cell F18: =IF(F12>B48," ",CONCAT("+ ",TEXT(F9,"$ #,##0.00")," / (1 + ",B1*100,"%) ^ ",F12))
Cell H18: =IF(H12>B48," ",CONCAT("+ ",TEXT(H9,"$ #,##0.00")," / (1 + ",B1*100,"%) ^ ",H12))

Merge cells: B20 and C20, D20 and E20, F20 and G20, H20 and I20
Provide associated cell borders in rows 21 and 22 to mimic a time line.
Merge cells: B23 and C23, D23 and E23, F23 and G23, H23 and I23
Cell B20: =IF(B4>0,B4,0)
Cell D20: =IF(D4>0,D4,0)
Cell F20: =IF(F4>0,F4,0)
Cell H20: =IF(H4>0,H4,0)
Merge cells: B25 and C25, D25 and E25
Cell D25: =NPV(B1,B20:I20)*(1+B1)^(B49+1)

Merge cells: B27 and C27, D27 and E27, F27 and G27, H27 and I27
Cell B27: =IF(B23>B49," ",CONCAT("= ",TEXT(B20*(1 + B1)^(B49 – B23),"$ #,##0.00")))
Cell D27: =IF(D23>B49," ",CONCAT("= ",TEXT(D20*(1 + B1)^(B49 – D23),"$ #,##0.00")))
Cell F27: =IF(F23>B49," ",CONCAT("= ",TEXT(F20*(1 + B1)^(B49 – F23),"$ #,##0.00")))
Cell H27: =IF(H23>B49," ",CONCAT("= ",TEXT(H20*(1 + B1)^(B49 – H23),"$ #,##0.00")))

Merge cells: B29 and C29, D29 and E29, F29 and G29, H29 and I29
Cell B29: =IF(B23>B49," ",CONCAT("= ",TEXT(B20,"$ #,##0.00")," * (1 + ",B1*100,"%) ^ ",B49-B23))
Cell D29: =IF(D23>B49," ",CONCAT("+ ",TEXT(D20,"$ #,##0.00")," * (1 + ",B1*100,"%) ^ ",B49-D23))
Cell F29: =IF(F23>B49," ",CONCAT("+ ",TEXT(F20,"$ #,##0.00")," * (1 + ",B1*100,"%) ^ ",B49-F23))
Cell H29: =IF(H23>B49," ",CONCAT("+ ",TEXT(H20,"$ #,##0.00")," * (1 + ",B1*100,"%) ^ ",B49-H23))

Merge cells: B31 and C31, D31 and E31, F31 and G31, H31 and I31
Provide associated cell borders in rows 32 and 33 to mimic a time line.
Merge cells: B34 and C34, D34 and E34, F34 and G34, H34 and I34
Cell B31: =D14
Cell D31: =IF(B49=D34,D25,0)
Cell F31: =IF(B49=F34,D25,0)
Cell H31: =IF(B49=H34,D25,0)

Merge cells: B36 and C36, D36 and E36, F36 through I36
Cell D36: =(D25/-D14)^(1/B49)-1
Cell F36: =CONCAT("= [",TEXT(D25, "$ #,###.00")," / ",TEXT(-D14,"$ #,###.00"),"] ^ (1 / ",B49,") – 1")

Merge cells: B38 and C38, D38 and E38, F38 through I38
Cell D38: =NPV(B2,B31:I31)*(1+B2)
Cell F38: =CONCAT("= ",TEXT(D25, "$ #,###.00")," / (1 + ",B2*100,"%) ^ ",B49," – ",TEXT(-D14,"$ #,###.00")," / (1 + ",B2*100,"%) ^ 0")

Merge cells: B40 and C40, D40 and E40
Cell D40: =IRR(B4:I4,B2)

Merge cells: B42 and C42, D42 and E42

Cell D42: =NPV(B2,B4:I4)*(1+B2)

These formulas can be “hidden” or “cut and pasted” to another worksheet to hide from view:
Merge cells: B46 and C46, D46 and E46, F46 and G46, H46 and I46
Cell B46: =IF(B9 <> 0, B12, 0)
Cell D46: =IF(D9 <> 0, D12, 0)
Cell F46: =IF(F9 <> 0, F12 0)
Cell H46: =IF(H9 <> 0, H12, 0)

Merge cells: B47 and C47, D47 and E47, F47 and G47, H47 and I47
Cell B47: =IF(B4 <> 0, B23, 0)
Cell D47: =IF(D4 <> 0, D23, 0)
Cell F47: =IF(F4 <> 0, F23, 0)
Cell H47: =IF(H4 <> 0, H23, 0)

Cell B48: =MAX(B46:I46)
Cell B49: =MAX(B47:I47)

Exhibit 6 formulas:

Merged cells: A1:B1, A3:I3, A5:I5, A6:I6, A8:F8, A11:F11, A13:F13, A14:F14, A15:F15, A16:F16, A17:F17, A18:F18, A19:F19, A20:F20

Cells I11, I13, I15, I17, and I19 are set with “font color” as white and “fill color” as black to have the cells resemble financial calculator keys

Cell K11 formula: =IFERROR(NPER(G13/100, G17, G15, G19, IF(G8=”BEG”, 1, IF(G8= “END”,0,”A”))), “ ”)

Cell K13 formula: =IFERROR(RATE(G11, G17, G15, G19, IF(G8=”BEG”, 1, IF(G8= “END”,0,”A”)))*100, “ ”)

Cell K15 formula: =IFERROR(PV(G13/100, G11, G17, G19, IF(G8=”BEG”, 1, IF(G8= “END”,0,”A”))), “ ”)

Cell K17 formula: =IFERROR(PMT(G13/100, G11, G15, G19, IF(G8=”BEG”, 1, IF(G8= “END”,0,”A”))), “ ”)

Cell K19 formula: =IFERROR(FV(G13/100, G11, G17, G15, IF(G8=”BEG”, 1, IF(G8= “END”,0,”A”))), “ ”)

Alternative formulas for cells K11, K13, K15, K17, and K19 that will default to end-of-period cash flows unless cell C8 is set as “BEG”

Cell K11 formula: =IFERROR(NPER(G13/100, G17, G15, G19, IF(G8= “BEG”, 1, 0)), “ ”)

Cell K13 formula: =IFERROR(RATE(G11, G17, G15, G19, IF(G8= “BEG”, 1, 0))*100, “ ”)

Cell K15 formula: =IFERROR(PV(G13/100, G11, G17, G19, IF(G8= “BEG”, 1, 0)), “ ”)

Cell K17 formula: =IFERROR(PMT(G13/100, G11, G15, G19, IF(G8= “BEG”, 1,0)), “ ”)

Cell K19 formula: =IFERROR(FV(G13/100, G11, G17, G15, IF(G8= “BEG”, 1, 0)), “ ”)

Exhibit 8 formulas:
Merged cells: M11 through Q11, M12 through Q12, M13 through Q13, M14 through Q14, M15 through Q15, M16 through Q16, M17 through Q17, M18 through Q18, M19 through Q19, M20 through Q20

Cell M11: =IF(K11=" "," ",CONCAT("= NPER (",G13,"% , ",TEXT(G17,"#,##0.00")," , " ,TEXT(G15,"#,##0.00")," , ",TEXT(G19,"#,##0.00")," , ",IF(G8="BEG",1,0),")"))

Cell M12: =IF(K11=" "," ",CONCAT("= NPER (I/Y , PMT , PV , FV , ",IF(G8="BEG",1,0),")"))

Cell M13: =IF(K13=" "," ",CONCAT("= RATE (",G11," , ",TEXT(G17,"#,##0.00")," , " ,TEXT(G15,"#,##0.00")," , ",TEXT(G19,"#,##0.00")," , ",IF(G8="BEG",1,0),")*100"))

Cell M14: =IF(K13=" "," ",CONCAT("= RATE (N , PMT , PV , FV , ",IF(G8="BEG",1,0),")*100"))

Cell M15: =IF(K15=" "," ",CONCAT("= PV (",G13,"% , ",G11," , ",TEXT(G17,"#,##0.00")," , " ,TEXT(G19,"#,##0.00")," , ",IF(G8="BEG",1,0),")"))

Cell M16: =IF(K15=" "," ",CONCAT("= PV (I/Y , N , PMT , FV , ",IF(G8="BEG",1,0),")"))

Cell M17: =IF(K17=" "," ",CONCAT("= PMT (",G13,"% , ",G11," , ",TEXT(G15,"#,##0.00")," , " ,TEXT(G19,"#,##0.00")," , ",IF(G8="BEG",1,0),")"))

Cell M18: =IF(K17=" "," ",CONCAT("= PMT (I/Y , N , PV , FV , ",IF(G8="BEG",1,0),")"))

Cell M19: =IF(K19=" "," ",CONCAT("= FV (",G13,"% , ",G11," , ",TEXT(G17,"#,##0.00")," , " ,TEXT(G15,"#,##0.00")," , ",IF(G8="BEG",1,0),")"))

Cell M206: =IF(K19=" "," ",CONCAT("= FV (I/Y , N , PMT , PV , ",IF(G8="BEG",1,0),")"))

Teaching Margin Trading and Financial Leverage Together

Serkan Karadas and Minh Tam Tammy Schlosky[1]

ABSTRACT

Margin trading and financial leverage are typically taught separately from each other. In this study, we present a pedagogical approach with a dual focus on margin trading and financial leverage. We borrow the insights and tools from financial leverage in a corporate setting and apply them to margin trading as a special case of financing an investment using debt and equity. In addition to producing a simple and intuitive formula for performance calculations on a margin trade, our approach offers the benefit of improving students' understanding of how margin trading and financial leverage operate in general.

Introduction

"*Three things ruin people: drugs, liquor or leverage.*"
Charlie Munger, Vice Chairman, Berkshire Hathaway.

Margin trading and financial leverage are typically taught separately in business curriculum. For example, in the twelfth edition of their popular book *Investments*, Bodie et al. (2021, hereafter BKM) introduce margin trading in Chapter 3 on pp. 77-80 with numerical and Excel-based exercises and cover the relationship between financial leverage, return on assets (ROA), and return on equity (ROE) in Chapter 19 on pp. 622-624. This paper presents a new pedagogical approach to teaching margin trading and financial leverage by relating these two topics with each other. Using this paper's pedagogical approach, instructors teaching financial leverage can examine margin trading as a special case of financial leverage and help students understand how buying stocks with borrowed money affects ROE. Similarly, instructors teaching margin trading can use financial leverage formulas to further students' understanding of how margin trading works and when it is beneficial to investors.

Our focus on margin trading, a leveraged transaction in which investors use equity- and debt-financing to purchase stocks, is motivated by two main reasons. First, margin trading is a popular choice by investors. Based on data from the New York Stock Exchange (NYSE), Figure 1 shows the level of margin debt in the United States from January 1980 ($11.9 billion) to March 2017 ($536.9 billion).[2] The figure suggests that margin debt has been increasing since 1980s with occasional steep drops. More recent data from the Financial Industry Regulatory Authority (FINRA) reveal that margin debt reached its highest level ($882.1 billion) in June 2021.[3] Second, margin trading, while undertaken to boost returns, could lead to major losses if not used properly. Anecdotal evidence shows that a wide range of investors such as experienced traders (Zuckerman et al. 2021), financial advisors (Wursthorn and Dulaney 2018), corporate executives (Casselman 2008; Davies 2017), and students with a business degree (Ensign 2021) suffer from major losses exacerbated by margin trading. However, investors seem to ignore lessons from such outcomes

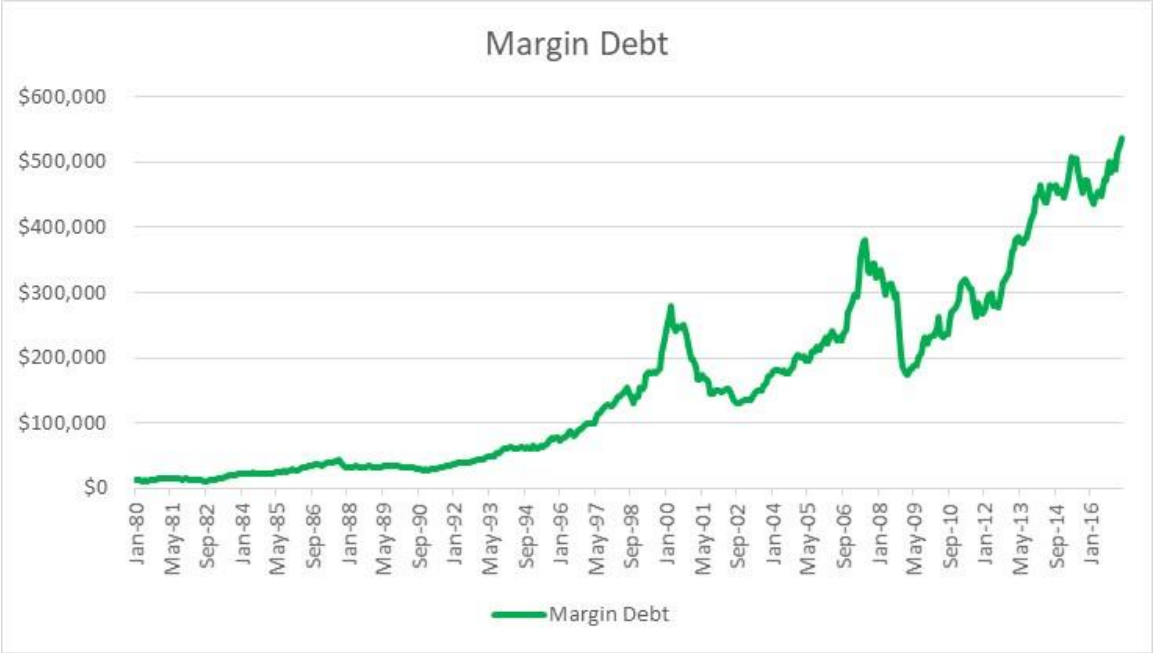
[1] Serkan Karadas, Assistant Professor (skara6@uis.edu) and Minh Tam Tammy Schlosky, Assistant Professor (mschl7@uis.edu), Department of Accounting, Economics, and Finance, College of Business and Management, University of Illinois Springfield, One University Plaza, Springfield, IL 62703.

[2] NYSE discontinued publishing the margin data. Figure 1 is the authors' graph using data from NYSE. Margin debt data are in millions of dollars.

[3] The NYSE and FINRA data differ from each other. However, the NYSE data have longer time series (starting in 1980) than FINRA data (starting in 1997). FINRA data can be accessed here: *https://www.finra.org/investors/learn-to-invest/advanced-investing/margin-statistics*.

(Davies and Chatterjee 2020). Young and inexperienced investors seem especially at a high risk to misunderstand margin trading and to default on margin loans (Gandel 2021).

Figure 1: Margin Debt Levels (1980-2017)



We argue that a thorough understanding of margin trading with the aid of the tools and insights from the pedagogy on financial leverage may help students avoid undesirable trading outcomes during their college as well as post-college years. This is essential as trading without fully understanding the employed strategies may lead to more than monetary losses as in the case of Alex Kearns who committed suicide after mistakenly believing that he lost about $750,000 in an options trade (Wigglesworth et al. 2020).

Our focus on financial leverage is motivated by the research on the importance of using "hooks" to help students understand complex topics (Burney et al. 2007). Finance textbooks typically cover financial leverage by showing how it affects earnings per share (EPS) and ROE for corporations. We believe that margin trading, as a special case of financing policy, serves as a hook to get students interested in financial leverage. We put this hook into action by applying the formulas and insights from financial leverage in *Investments* by BKM (2021) in the derivation of return on equity on margin trading. Using numerical examples and visual aids, we show that margin trading (hence the use of financial leverage) is beneficial to an investor if the return on asset exceeds the cost of the margin debt.

Our dual focus on margin trading and financial leverage has multiple benefits. First, it produces a simple formula for performance calculations (ROE on a margin trade) and strengthens students' understanding of how margin trading works. Second, it introduces how financial leverage affects ROE earlier in the semester than a typical Investments class. Third, it helps students better grasp how financial leverage operates. Anecdotally, we found that students understand reasonably well that the returns generated on the capital from the margin loan should exceed the interest rate of the loan if they want margin trading to work for them.

The rest of this study proceeds as follows. First, we summarize related studies in the literature. Second, we discuss the main framework behind our analysis. Third, we provide extensions on our approach. Fourth, we conclude the paper.

Related Studies

This paper is related to the pedagogical studies on financial leverage, which predominantly focus on firm performance such as (EPS) and on how the effect of financial leverage on firm performance can be better communicated to finance (or business) students. Luoma and Spiller (2002) point out the scant

treatment of financial leverage in accounting textbooks, and provide simple, yet illustrative examples on how financial leverage may enhance or hurt return on equity. In one of their examples, the return on asset is 5.4%, whereas the cost of capital is 4.8% (after-tax) for debt and 10% for preferred stock. Hence, the use of debt has a positive effect on ROE, but the use of preferred stock adversely affects ROE. These two different costs of capital clearly convey to students that the benefit of financial leverage hinges on the relationship between the return on assets and the cost of capital.

Burney et al. (2007) argue that finance instructors should use "hooks", simplified and general examples or stories, to motivate complex topics to students prior to using comprehensive examples. They demonstrate how financial leverage affects accounting rates of returns in a simplified example to convey the crux of their paper without covering the changes in EPS from the use of financial leverage. Our paper is in the same spirit with their study. It uses margin trading as a hook, which does not have the complexities of a fully-developed financial leverage example. Furthermore, Burney et al. (2007) cite survey results that business students prefer specific and hands-on examples to complex examples when they start learning a new subject. Margin trading, especially for those in class where stock trading simulation is used, provides a setup that students can easily relate to. Instructors can use paid (e.g., StockTrak) or free (e.g., MarketWatch) trading platforms to allow students to trade with borrowed money.

Rose (2010) demonstrates the effect of financial leverage in the Earnings Before Interest and Taxes (EBIT)-EPS context by appropriately taking risk into account. The risk here refers to the EPS sensitivity to EBIT for a firm that uses financial leverage. The author superimposes a probability distribution on the EBIT-EPS graph and provides numerical examples to demonstrate that levered firms can accomplish higher EPS at the expense of higher probability of negative EPS. Cheng et al. (2010, p. 12)) also focus on the risk-return tradeoff within the financial leverage context. They provide a model to assess whether firms should issue debt or equity to finance a given project. Their model suggests that "[i]f the change in expected return exceeds the change in required return, then the debt issuance will increase the stock price and thus is preferred to the equity issuance."

Our paper is also very similar to the study by Liang and Singh (2001) in its spirit. These authors simplify the treatment of financial leverage in corporate finance textbooks by relating the operating return on investment (operating ROI) to the cost of debt. Their highly intuitive framework shows that financial leverage benefits the firm if the cost of debt is below the operating ROI and that firms break even when the cost of debt equals the operating ROI.

Main Framework

An investor makes an investment amounting to *V* dollars in a stock at time *t=0*. Initial percentage margin equals *m*, which represents the fraction of investor's equity investment in *V*. Hence, the investor uses *mV* dollars in equity and *(1-m)V* dollars in margin debt to finance her investment of *V* dollars at time *t=0*. In this context, a higher value of *m* implies less financial leverage. The investor's balance sheet will look like the following at time *t=0*:

Assets	Liabilities and Owners' Equity
V	(1-m)V
	mV
V	V

We assume the investor has a one-year investment horizon that ends at time *t=1*. Over this investment horizon, the investor pays *y* percent annual interest on margin debt, and the value of the stock changes by *x* percent. For simplicity, we assume that the stock does not pay any dividends, that the investor's tax rate is *0* percent, and that there are no additional transaction costs. To assess the effect of leverage on the return on equity acquired by this margin trader, we now invoke the ROE formula that BKM (2021, p. 623) provides:

$$ROE = (1 - t_c)\left[ROA + (ROA - Interest\ Rate)\frac{Debt}{Equity}\right] \quad (1)$$

For a company subject to a tax rate of t_c percent, Equation (1) summarizes the relationship between ROE, ROA, and financial leverage in a broad setup: corporate financing policy. We apply this formula to a more specific case of financing: financing the purchase of stocks with a margin loan.[4]

BKM (2021) calculate ROA as the ratio of EBIT to Assets in Equation (1).[5] For the investor in our exercise who is subject to 0 percent tax rate, EBIT is simply the difference between the ending value of her investment, *(1+x)V*, and the beginning value of her investment, *V*:

$$EBIT = Ending\ value\ of\ investment - Beginning\ value\ of\ investment$$

$$EBIT = (1+x)\,V - V$$

$$EBIT = xV$$

As a result, ROA then equals to:

$$ROA = \frac{EBIT}{Assets} \quad \text{or} \quad ROA = \frac{xV}{V}$$

$$ROA = x$$

Furthermore, debt-to-equity ratio for the investor equals to the following:

$$\text{Debt-to-equity} = \frac{Debt}{Equity} \quad \text{or} \quad \text{Debt-to-equity} = \frac{(1-m)V}{mV}$$

$$\text{Debt-to-equity} = \frac{1-m}{m}$$

As a result, we re-express Equation (1) as the following:

$$ROE = (1 - t_c)\left[ROA + (ROA - Interest\ Rate)\frac{Debt}{Equity}\right]$$

$$ROE = (1 - 0)\left[x + (x - y)\frac{(1\text{-m})}{\text{m}}\right]$$

The further simplification of the expression above gives us Equation (2):

$$ROE = x + (x - y)\left[\frac{1-m}{m}\right] \tag{2}$$

where *y* is the interest rate on the margin loan and *m* is the initial percentage margin. In this setup, the effect of leverage on the return on equity depends on the relationship between the return on asset and the interest rate on the debt. This is the key insight that BKM (2021) emphasize in Equation (1) and Liang and Singh (2001) highlight in their study. Here, the investor captures *x* (the percentage change in the value of the asset or ROA) plus the difference between *x* and *y*. However, depending on the debt-to-equity ratio (hence the amount of leverage), the investor may capture the entire difference, a fraction of the difference or a

[4] Instructors who cover margin trading before financial leverage may prefer the current exposition in the main framework, which starts with the layout of the margin trading setup and proceeds to the financial leverage formula. On the other hand, instructors who cover financial leverage before margin trading may prefer to start with Equation (1) and then proceed to the discussion of the margin setup.

[5] It is important to note that ROA is traditionally calculated as Net income divided by Total assets, and EBIT/Total assets is commonly known as the basic earning power. We recommend that instructors mention the lack of standardization in the ROA formula to students prior to exposing them to Equation (1). We also would like to note that how ROA is calculated does not affect the final result: Equation (2). In Extensions section, we derive Equation (2) using the traditional formula for ROA.

multiple of the difference. The higher the leverage (i.e., the smaller *m*) is, the more of (*x-y*) the investor will keep. When there is no leverage used (i.e., *m=1*), the investor does not keep any of *(x-y)*. This is an intuitive result: if an investor does not use any leverage, the leverage does not affect the investor's returns.

When leverage is used (i.e., for any positive value of *m* that is less than 1), Equation (2) represents the ROE under leverage (ROE_L). When there is no leverage used (i.e., *m=1*), ROE in this unlevered case (ROE_U) equals *x*. At the break-even point, we will have the following outcome:

$$ROE_L = ROE_U$$

$$x + (x - y)\left[\frac{1-m}{m}\right] = x$$

$$(x - y)\left[\frac{1-m}{m}\right] = 0$$

Since the expression inside the bracket is non-zero under a leveraged position, then it must be that *(x-y) =0* or *x=y* at the break-even point. Next, we discuss the margin investor's return expectations. To induce the investor to purchase the stock in the first place, *x* must exceed *0*. However, this is not sufficient in the levered case. The investor will take on the margin loan if she anticipates that $ROE_L > ROE_U$:

$$ROE_L > ROE_U$$

$$x + (x - y)\left[\frac{1-m}{m}\right] > x$$

$$(x - y)\left[\frac{1-m}{m}\right] > 0$$

Since the expression inside the bracket is a positive number in the presence of the margin loan, then *(x-y)* must be positive as well, suggesting that *(x-y)>0* or *x>y*. In other words, the investor expects that the percentage change in the value of the asset will exceed the interest rate on the margin loan prior to taking on the margin debt. When *x* exceeds *y*, then ROE exceeds ROA, which implies that the leverage enhances the return on equity. If *x* is less than *y*, ROE falls short of ROA, which implies that leverage hurts the return on equity. If *x* is equal to *y*, then ROE will equal ROA.[6] For example, suppose a margin loan costs an investor 8% (*y=8%*). The invested asset needs to go up by more than 8% (*x > 8%*) for the investor to benefit from the margin loan. When this happens, the investor earns more than the interest on the borrowed capital, which boosts ROE. If the asset changes by the same amount as the interest on the margin loan, the investor simply breaks even with no effect on the ROE. If the asset's value changes by less than 8%, the borrowed capital does not produce sufficient income to make the interest payments, hurting ROE.

Extensions

In this section, we first provide a general example that uses Equation (2) and then analyze this equation via comparative statics. Further, we compare different approaches to solving textbook-style questions. We finish this section by demonstrating an alternative way of deriving Equation (2) based on DuPont analysis.

General Example

In this general example, an investor is interested in buying shares of a company on margin. The current share price is $9.77, and the initial percentage margin, *m*, is 70% (the investor is using 70% equity financing). The investor is interested in buying a total of 1,000,000 shares and will be subject to 10% interest (per annum) on the margin loan.[7] Table 1 summarizes the key information for this example.

To keep this exercise simple, we come up with 81 different values for the rate of price change (*x*), starting at -50%, increasing at 1.25% increments, thus ending at 50%. Next, we calculate ROE based on Equation (2). Table 2 provides a *portion* of the results from the sample data. This example does not deal with margin calls. Alternatively, we can have this investor dynamically manage her position by making

[6] The equality also holds when there is no leverage used (i.e., *m=1*).

[7] Currently, the interest rate on margin loans starts around 1.55% and can exceed 7.75% depending on the broker. A comparison of different brokerage firms can be found here: *https://www.optimizedportfolio.com/lowest-margin-rates-brokers/*.

margin calls. Here instead, we use these possible returns in a static sense: for a given ROA, investor closes out her position and settles her gains and losses. This simplification is consistent with the textbook example in BKM (2021) and the hook strategy in Burney et al. (2007).

Table 1: Key Data for General Example

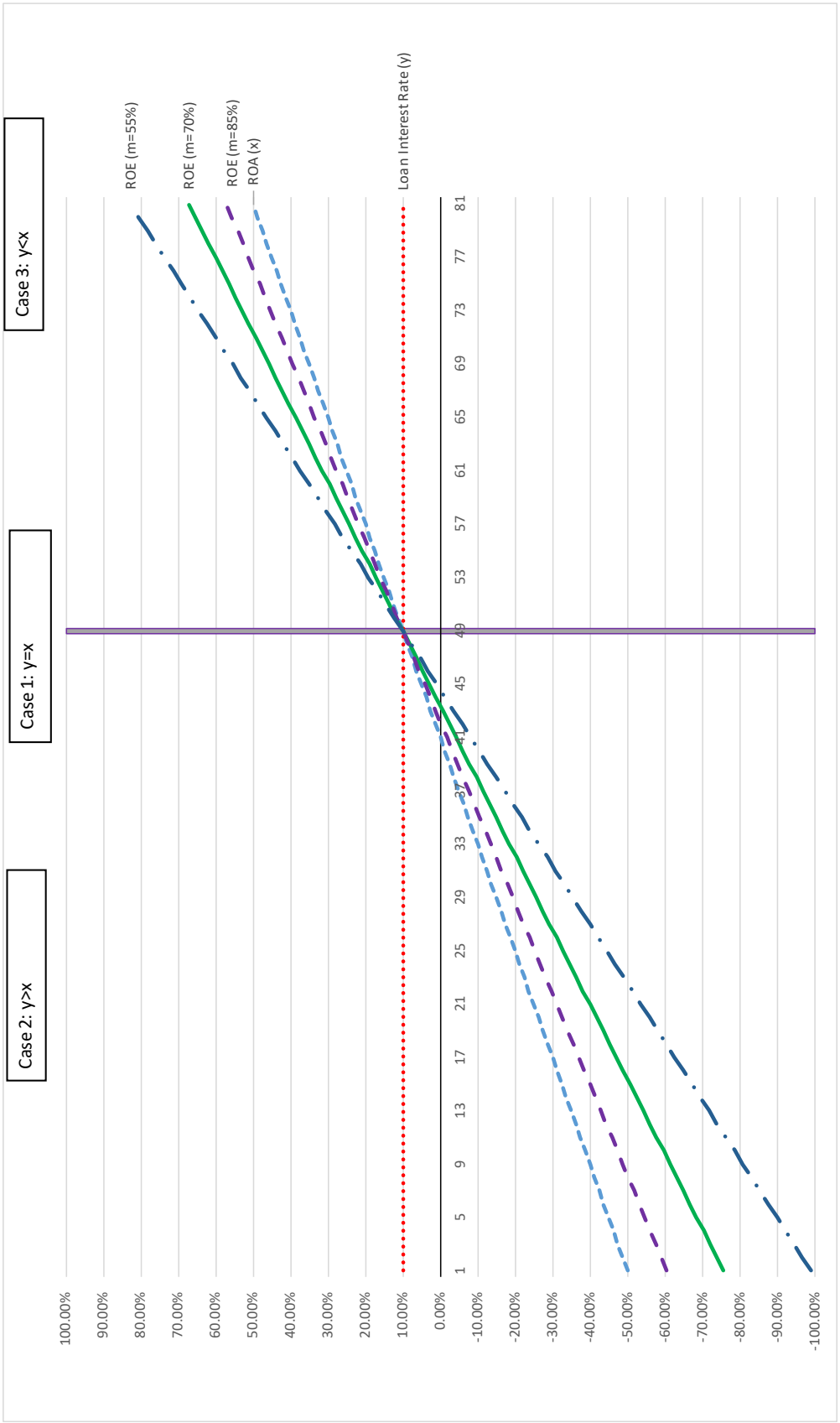
Share Price	9.77
# Shares Purchased	1,000,000
Total Cost	9,770,000.00
Initial % Margin (*m*)	70.00%
Initial Equity	6,839,000.00
Loan	2,931,000.00
Interest	10.00%

Table 2: Return on Equity (ROE) from Sample Data

ROA (x)	Loan Interest Rate (y)	ROE
-50.00%	10.00%	-75.71%
-48.75%	10.00%	-73.93%
-47.50%	10.00%	-72.14%
-46.25%	10.00%	-70.36%
-45.00%	10.00%	-68.57%
-43.75%	10.00%	-66.79%
-42.50%	10.00%	-65.00%
-41.25%	10.00%	-63.21%
-40.00%	10.00%	-61.43%
.	.	.
.	.	.
.	.	.
10.00%	10.00%	10.00%
.	.	.
.	.	.
.	.	.
40.00%	10.00%	52.86%
41.25%	10.00%	54.64%
42.50%	10.00%	56.43%
43.75%	10.00%	58.21%
45.00%	10.00%	60.00%
46.25%	10.00%	61.79%
47.50%	10.00%	63.57%
48.75%	10.00%	65.36%
50.00%	10.00%	67.14%

We present ROA, interest rate, and ROE from this general example in Figure 2. At the vertical line, ROA (x) equals the interest rate (y). As a result, at this point, ROE equals ROA. This is the break-even point for the margin trader. This point is also consistent with Liang and Singh (2001) who show that the operating ROI, calculated as EBIT over Assets, equals the cost of debt at the break-even point. To the right of the vertical line, the loan interest rate is less than ROA on the margin trade. This area represents a set of favorable outcomes for the margin trader, as the asset generates sufficient income to meet the obligations of the loan and to leave residual income to the trader. Hence, in this area of Figure 2, ROE exceeds ROA. On the other hand, the area to the left of the vertical line represents a set of unfavorable outcomes since ROA is less than the interest rate. Here, the income generated by investing the proceeds from the margin loan is not sufficient to make the interest payments. Therefore, ROE is less than ROA. This graphical representation shows to the students that debt is not inherently a bad or good thing. Its effect on financial performance depends on the returns generated from the capital financed by issuing debt.

Figure 2: Performance of Margin Trading at Different Initial Percentage Margins



Comparative Statics

To help students understand the full effect of leverage on the return on equity, the partial derivative of ROE with respect to *m* presents an important pedagogical opportunity. In a functional form of *ROE=f(x,y,m)*, we determine this partial derivative to be the following:

$$\frac{\partial ROE}{\partial m} = \frac{y - x}{m^2} \tag{3}$$

We analyze Equation (3) under three different cases: (i): $y=x$, (ii) $y>x$, and (iii) $y<x$. To see how the changes in initial percentage margin (*m*) affect ROE, we also augment these cases by providing two additional scenarios: one for *m*=85% (lower leverage) and another one for *m*=55% (higher leverage). We treat the general example (*m*=70%) as the base case and demonstrate the outcome of these additional scenarios in Figure 2 as well. It is important to recall that the investor chooses *m* ex-ante, prior to the trade. Therefore, these different cases should be considered as part of a what-if analysis.

Case 1: y=x

The first case refers to a scenario under which the interest rate on the margin loan equals the percentage change in the value of the asset. Under this case, Equation (3) equals zero, implying that increasing or decreasing the amount of financial leverage does not have an effect on ROE. Figure 2 shows that when Case 1 holds, increasing the initial percentage margin (implying a reduction in the leverage) to 85% or decreasing the initial percentage margin (implying an increase in the leverage) to 55% does not affect ROE.

Case 2: y>x

This case refers to a scenario in which the interest rate on the margin loan exceeds the percentage change in the value of the asset. Under this unfavorable scenario, the sign of Equation (3) is positive, implying a positive relationship between the initial percentage margin and ROE. If the investor had less leverage (i.e., greater *m*), her return would be higher relative to a case where she had more leverage (i.e., smaller *m*). The area left of the vertical line in Figure 2 represents Case 2 where at all points, the loan interest rate (*y*) exceeds ROA (*x*). Under this case, increasing the initial percentage margin to 85% from a base value of 70% leads to smaller losses. On the other hand, decreasing the initial percentage margin to 55% from a base value of 70% leads to much greater losses.

Case #3: y<x

This case refers to a scenario in which the interest rate on the margin loan is below the percentage change in the value of the asset. Under this favorable scenario, the sign of Equation (3) is negative, implying a negative relationship between initial percentage margin (*m*) and ROE. Here, for the borrowed part of her investment, the investor has money left over after paying the principal and the interest to the broker. Under such circumstances, the investor would be better off having more leverage and less equity (a smaller *m*). The area right of the vertical line in Figure 2 represents Case 3 where at all points, the loan interest rate (*y*) is below ROA (*x*). Under this case, increasing the initial percentage margin to 85% from a base value of 70% leads to smaller returns. On the contrary, lowering the initial percentage margin to 55% from a base value of 70% leads to much larger returns.[8]

Numerical Exercise

Our approach yields a simpler formula for determining the ROE on a margin trade than the textbook solution based on BKM (2021). We demonstrate this point with a simple example. Suppose that an investor purchases a non-dividend paying stock when a share is trading at $100. The investor puts $12,000 of her own money and borrows the remaining $8,000 from her broker to purchase a total of 200 shares. The

[8] Using partial derivatives may not be appropriate in all teaching settings such as in introductory-level courses. In these cases, we suggest that instructors skip Equation (3), and build their discussion on Figure 2 only. They may further support their discussion with the following intuitive explanations. The left of the vertical line in Figure 2 represents a range of undesirable outcomes for the investor in which case having less leverage (a larger *m*) is advantageous. On the other hand, the right of the vertical line represents a range of desirable outcomes for the investor in which case having more leverage (a smaller *m*) is advantageous.

interest rate on the margin loan is 9%. The task is to calculate the rate of return on this position if the stock goes up by 30% in the following 12 months. Table 3 compares the solutions from the textbook approach and this paper's approach. Equation (2) offers a shorter and simpler solution to the return calculation.

Table 3: Comparing Different Approaches to Textbook Problems

The approach in the textbook:

At time *t=1*:

- Value of the Stock Position = Number of Shares Purchased *x* Price per share

  = 200 x ($100 + $100 x 0.30)

  = $26,000

- Payment to the broker = Principal + Interest

  = $8,000 + $8,000 x 0.09

  = $8,720

- Proceeds to the investor = Value of the stock position – Payment to the broker

  = $26,000 - $8,720

  = $17,280

- Return on Equity = [Proceeds to the investor – Investor's initial equity] / Investor's initial equity

  = [$17,280 – $12,000] / $12,000

  = 0.44 or 44.00%

The approach in this paper:

m=60% (12,000/20,000) x=30% y=9%

At time *t=1*:

$$ROE = x + (x - y)\left[\frac{1-m}{m}\right]$$

$$ROE = 0.30 + (0.30 - 0.09)\left[\frac{1-0.60}{0.60}\right]$$

$$ROE = 44\%$$

DuPont Analysis

Though we follow a popular investments textbook in our derivations, instructors of Corporate Finance can find the pedagogical techniques in this paper helpful as well. This will be especially the case if instructors also adopt stock simulation games, which may provide an additional boost to student learning. For example, Karadas and Hoffer (2017) present evidence on the benefits of simulation games in Corporate Finance in terms of student engagement and satisfaction. The authors also provide a list of sample Corporate Finance topics that are related to the stock trading simulation. For introductory finance courses, Harrington and Tan (2018) provide a template for a stock trading game with assessable learning outcomes.

Students in Corporate Finance classes are typically exposed to the relationship between ROE, and ROA, and the financial leverage in the form of DuPont analysis:

$$ROE = ROA\left(1 + \frac{Debt}{Equity}\right) \text{ where } ROA = \frac{Net\ Income}{Assets}$$

In our margin setup, net income equals:

$$Net\ income = Ending\ value\ of\ investment - Beginning\ value\ of\ investment - Interest\ expense - Taxes$$

$Net\ income = (1 + x)V - V - y(1 - m)V - Taxes$. Under the assumption of zero taxes,

$$Net\ income = [x - y(1 - m)]V$$

Hence, ROE equals:

$$ROE = \frac{[x - y(1-m)]V}{V}\left(1 + \frac{1-m}{m}\right)$$

$$ROE = [x - y(1 - m)]\left(\frac{1}{m}\right)$$

$$ROE = \frac{x}{m} - y\left(\frac{1-m}{m}\right)$$

By adding and subtracting *x* on the right-hand side of the expression above leads to Equation (2):

$$ROE = \frac{x}{m} - y\left(\frac{1-m}{m}\right) + x - x$$

$$ROE = x + \frac{x}{m} - x - y\left(\frac{1-m}{m}\right)$$

$$ROE = x + x\left(\frac{1}{m} - 1\right) - y\left(\frac{1-m}{m}\right)$$

$$ROE = x + x\left(\frac{1-m}{m}\right) - y\left(\frac{1-m}{m}\right)$$

$$ROE = x + (x - y)\left[\frac{1-m}{m}\right]$$

Even though *ROA* is calculated differently in the DuPont analysis here than it is in BKM (2021), we still arrive at the same equation (i.e., Equation (2)). Furthermore, the regardless of the starting point to derive Equation (2), we still have the same intuition at work: margin trading will boost ROE if the percentage change in the value of the invested asset (*x*) exceeds the interest rate on the margin loan (*y*), leaving investors with residual income after paying the principal and the interest back to the broker.

Discussion and Conclusions

This paper focuses on margin trading and financial leverage and advocates teaching them together. We accomplish this dual focus by applying the formulas for financial leverage to margin trading. Under certain assumptions, we show that the percentage change in the value of a stock is the same as the return on assets (ROA) for an investor who invested in this stock. Furthermore, we show that the use of financial leverage (margin debt) enhances the return on equity (ROE) acquired by a margin trader as long as the ROA on the margin trade exceeds the interest rate on the margin debt. In further analysis, we present comparative statics by taking the partial derivative of ROE with respect to the initial percentage margin (the opposite of financial leverage) and analyze how changes in financial leverage affect ROE conditional on the relationship between ROA and the interest rate on the margin loan.

Our approach has the benefit of simplifying margin trading calculations and of serving as a hook to get students interested in the topic of financial leverage. Instructors can amplify learning outcomes for margin trading and financial leverage by having students perform leverage-related calculations using Excel.

Students can easily manipulate such spreadsheets to examine how factors such as interest rate and initial percentage margin can affect ROE. Furthermore, students are likely to have an improved understanding of margin trading and leverage thanks to the use of spreadsheets (Cagle et al. 2010).

References

Bodie, Zvi, Alex Kane, and Alan J. Marcus. 2021. *Investments* (12th ed.). New York: McGraw-Hill Education.

Burney, Robert B., John G. Marcis, and Gerald V. Boyles. 2007. "The Pedagogy of Financial Leverage: Using a Hook to Improve Learning." *Journal of Economics and Finance Education* 6: 57-69.

Cagle, Julie A.B., Philip W. Glasgo, and David C. Hyland. 2010. "Spreadsheets: Do They Improve Student Learning in the Introductory Finance Course?" *Journal of Financial Education* 36: 35-52.

Casselman, Ben. 2008. "Margin Calls Bite Insiders; Chesapeake Energy Boss, Among Others, Forced into Selling." *The Wall Street Journal*, October 13.

Cheng, Joseph, Julie Fitzpatrick, and Mojtaba Seyedian. 2010. "A Model for Evaluating the Tradeoff Between Earnings Per Share and Financial Leverage." *The BRC Journal of Advances in Business* 1: 1-13.

Davies, Paul J. 2017. "A Massive Margin Loan that Went Really Wrong for the Banks." *The Wall Street Journal*, December 19.

Davies, Megan, and Sumeet Chatterjee. 2020. "Once Bitten, Not Shy: Investors Again Seek Margin Loans as Stocks Rally." *Reuters*, June 9.

Ensign, Rachel L. 2021. "Robinhood, Three Friends, and the Fortune that Got Away." *The Wall Street Journal*, April 22.

Gandel, Stephen. 2021. "Robinhood Offers Loans to Buy Stocks – They Were 14 Times More Likely to Default." *CBS News*, February 5.

Harrington, Christine, and Tih K. Tan. 2018. "A Trading Game for an Introductory Finance Course with Assessable Learning Outcomes." *Advances in Financial Education* 2018: 34-55.

Karadas, Serkan, and Adam Hoffer. 2017. "Teaching Corporate Finance Using a Stock Trading Simulation: Student Expectations, Engagement, Performance, and Satisfaction." *Journal of Economics and Finance Education* 16: 69-84.

Liang, Bing, and Ajai K. Singh. 2001. "Effects of Financial Leverage: A Simpler Pedagogical Approach." *Journal of Financial Education* 27: 99-103.

Luoma, Gary A., and Earl A. Spiller. 2002. "Financial Accounting Return on Investment and Financial Leverage." *Journal of Accounting Education* 20: 131-138.

Rose, John T. 2010. "Extending the Presentation of the EBIT-EPS Relationship for Introducing Financial Leverage in the Classroom." *Journal of Financial Education* 36: 137-147.

Wigglesworth, Robin, Richard Henderson, and Eric Platt. 2020. "The Lockdown Death of a 20-Year-Old Day Trader." *Financial Times*, July 2.

Wursthorn, Michael, and Chelsey Dulaney. 2018. "Margin Bets Fueled Selloff – Stock Traders Borrow a Record $642.8 Billion, Stirring Fears Volatility Will Be More Common." *The Wall Street Journal*, February 26.

Zuckerman, Gregory, Juliet Chung, and Maureen Farrell. 2021. "Inside Archegos's Epic Meltdown." *The Wall Street Journal*, April 1.

Editor's Note

E. Frank Stephenson

Although a few papers he supervised, as well as one he wrote, will be appearing in future issues of the journal, Professor Bill Yang is retiring in December 2022 and will no longer be serving as co-editor of JEFE. I am extremely grateful for Professor Yang's assistance and thank him for serving as the co-editor for finance for the last decade. He's served the journal well and authors have greatly benefitted from his guidance. Best wishes for your retirement, Bill!